Computer algebra independent integration tests

4-Trig-functions/4.3-Tangent/4.3.1.3-d-sin-^m-a+b-tan-^n

Nasser M. Abbasi

July 17, 2021 Compiled on July 17, 2021 at 11:57am

Contents

1 Introduction **3**
- 1.1 Listing of CAS systems tested . . . 3
- 1.2 Results . . . 3
- 1.3 Performance . . . 7
- 1.4 list of integrals that has no closed form antiderivative . . . 8
- 1.5 list of integrals solved by CAS but has no known antiderivative . . . 8
- 1.6 list of integrals solved by CAS but failed verification . . . 8
- 1.7 Timing . . . 9
- 1.8 Verification . . . 9
- 1.9 Important notes about some of the results . . . 9
  - 1.9.1 Important note about Maxima results . . . 9
  - 1.9.2 Important note about FriCAS and Giac/XCAS results . . . 10
  - 1.9.3 Important note about finding leaf size of antiderivative . . . 10
  - 1.9.4 Important note about Mupad results . . . 11
- 1.10 Design of the test system . . . 11

2 detailed summary tables of results **13**
- 2.1 List of integrals sorted by grade for each CAS . . . 13
  - 2.1.1 Rubi . . . 13
  - 2.1.2 Mathematica . . . 13
  - 2.1.3 Maple . . . 13
  - 2.1.4 Maxima . . . 13
  - 2.1.5 FriCAS . . . 14
  - 2.1.6 Sympy . . . 14
  - 2.1.7 Giac . . . 14
  - 2.1.8 Mupad . . . 14
- 2.2 Detailed conclusion table per each integral for all CAS systems . . . 15
- 2.3 Detailed conclusion table specific for Rubi results . . . 30

3 Listing of integrals **35**
- 3.1 $\int \frac{\sin^4(x)}{i+\tan(x)}\,dx$ . . . 35
- 3.2 $\int \frac{\sin^3(x)}{i+\tan(x)}\,dx$ . . . 38
- 3.3 $\int \frac{\sin^2(x)}{i+\tan(x)}\,dx$ . . . 41
- 3.4 $\int \frac{\sin(x)}{i+\tan(x)}\,dx$ . . . 44

3.5 $\int \frac{\csc(x)}{i+\tan(x)}\,dx$ . . . 47
3.6 $\int \frac{\csc^2(x)}{i+\tan(x)}\,dx$ . . . 50
3.7 $\int \frac{\csc^3(x)}{i+\tan(x)}\,dx$ . . . 52
3.8 $\int \frac{\csc^4(x)}{i+\tan(x)}\,dx$ . . . 55
3.9 $\int \frac{\csc^5(x)}{i+\tan(x)}\,dx$ . . . 58
3.10 $\int \frac{\csc^6(x)}{i+\tan(x)}\,dx$ . . . 62
3.11 $\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx$ . . . 65
3.12 $\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx$ . . . 68
3.13 $\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx$ . . . 72
3.14 $\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx$ . . . 78
3.15 $\int \sin(c+dx)(a+b\tan(c+dx))\,dx$ . . . 81
3.16 $\int \csc(c+dx)(a+b\tan(c+dx))\,dx$ . . . 85
3.17 $\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx$ . . . 87
3.18 $\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx$ . . . 89
3.19 $\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx$ . . . 92
3.20 $\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx$ . . . 95
3.21 $\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx$ . . . 98
3.22 $\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx$ . . . 101
3.23 $\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx$ . . . 108
3.24 $\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx$ . . . 111
3.25 $\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx$ . . . 115
3.26 $\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx$ . . . 120
3.27 $\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx$ . . . 123
3.28 $\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx$ . . . 125
3.29 $\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx$ . . . 129
3.30 $\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx$ . . . 132
3.31 $\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx$ . . . 137
3.32 $\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx$ . . . 140
3.33 $\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx$ . . . 144
3.34 $\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx$ . . . 149
3.35 $\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx$ . . . 159
3.36 $\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx$ . . . 162
3.37 $\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx$ . . . 165
3.38 $\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx$ . . . 169
3.39 $\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx$ . . . 172
3.40 $\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx$ . . . 177
3.41 $\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx$ . . . 180
3.42 $\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx$ . . . 184
3.43 $\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx$ . . . 189
3.44 $\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx$ . . . 193
3.45 $\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx$ . . . 197
3.46 $\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx$ . . . 200
3.47 $\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx$ . . . 205
3.48 $\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx$ . . . 208
3.49 $\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx$ . . . 213
3.50 $\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx$ . . . 216
3.51 $\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx$ . . . 222
3.52 $\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx$ . . . 227

3.53 $\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx$ . . . 231
3.54 $\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx$ . . . 235
3.55 $\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx$ . . . 239
3.56 $\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx$ . . . 242
3.57 $\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx$ . . . 245
3.58 $\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx$ . . . 248
3.59 $\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx$ . . . 253
3.60 $\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx$ . . . 256
3.61 $\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$ . . . 259
3.62 $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$ . . . 264
3.63 $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$ . . . 269
3.64 $\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$ . . . 273
3.65 $\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$ . . . 276
3.66 $\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$ . . . 279
3.67 $\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$ . . . 283
3.68 $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$ . . . 289
3.69 $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$ . . . 294
3.70 $\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$ . . . 298
3.71 $\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$ . . . 301
3.72 $\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$ . . . 304
3.73 $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$ . . . 308
3.74 $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$ . . . 314
3.75 $\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$ . . . 319
3.76 $\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$ . . . 322
3.77 $\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx$ . . . 326
3.78 $\int \frac{\csc(x)}{1+\tan(x)}\,dx$ . . . 330
3.79 $\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx$ . . . 333
3.80 $\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx$ . . . 336
3.81 $\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx$ . . . 339
3.82 $\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx$ . . . 342
3.83 $\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx$ . . . 346
3.84 $\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx$ . . . 353
3.85 $\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx$ . . . 364
3.86 $\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx$ . . . 367
3.87 $\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx$ . . . 369
3.88 $\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx$ . . . 372
3.89 $\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx$ . . . 374
3.90 $\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx$ . . . 376
3.91 $\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx$ . . . 378

4 Listing of Grading functions **381**
4.0.1 Mathematica and Rubi grading function 381
4.0.2 Maple grading function 383
4.0.3 Sympy grading function 386
4.0.4 SageMath grading function 388

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [91]. This is test number [102].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (91)	% 0.00 (0)
Mathematica	% 98.90 (90)	% 1.10 (1)
Maple	% 91.21 (83)	% 8.79 (8)
Maxima	% 86.81 (79)	% 13.19 (12)
Fricas	% 91.21 (83)	% 8.79 (8)
Sympy	% 8.79 (8)	% 91.21 (83)
Giac	% 82.42 (75)	% 17.58 (16)
Mupad	% 91.21 (83)	% 8.79 (8)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

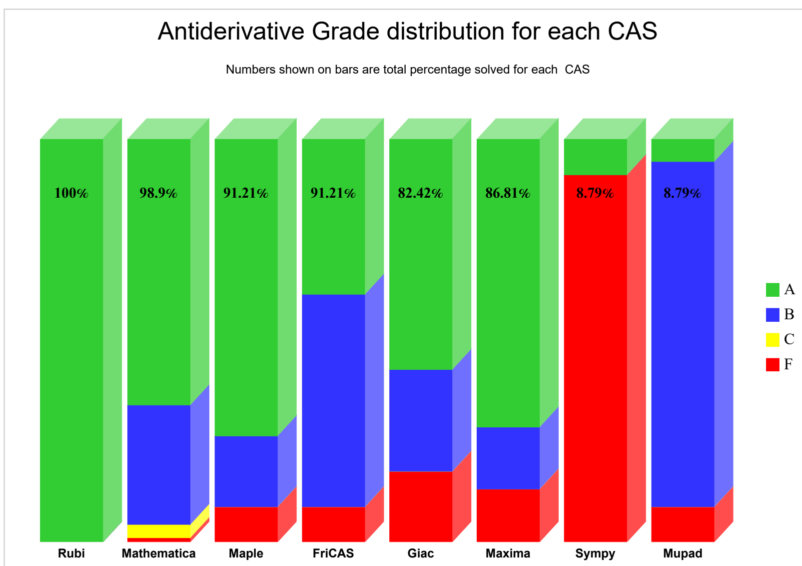
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

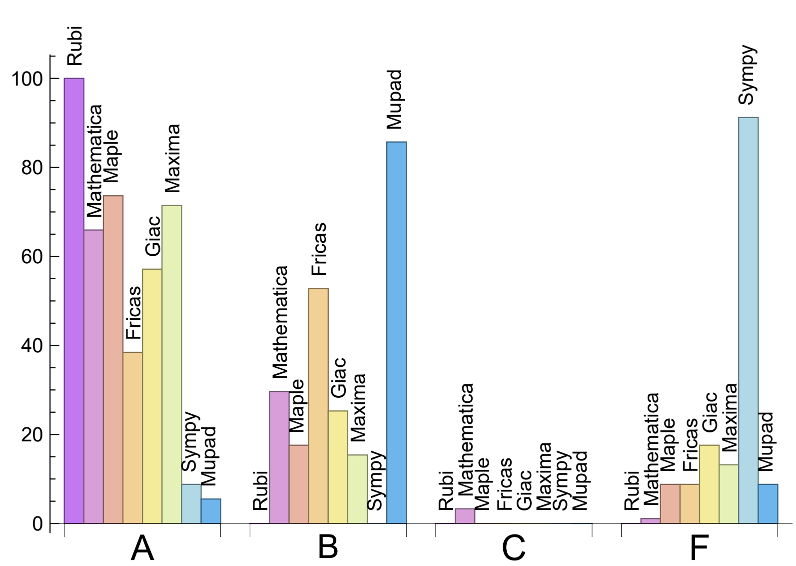
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	65.93	29.67	3.30	1.10
Maple	73.63	17.58	0.00	8.79
Maxima	71.43	15.38	0.00	13.19
Fricas	38.46	52.75	0.00	8.79
Sympy	8.79	0.00	0.00	91.21
Giac	57.14	25.27	0.00	17.58
Mupad	5.49	85.71	0.00	8.79

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	100.00 %	0.00 %	0.00 %
Maple	8	100.00 %	0.00 %	0.00 %
Maxima	12	66.67 %	0.00 %	33.33 %
Fricas	8	100.00 %	0.00 %	0.00 %
Sympy	83	79.52 %	14.46 %	6.02 %
Giac	16	43.75 %	31.25 %	25.00 %
Mupad	8	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

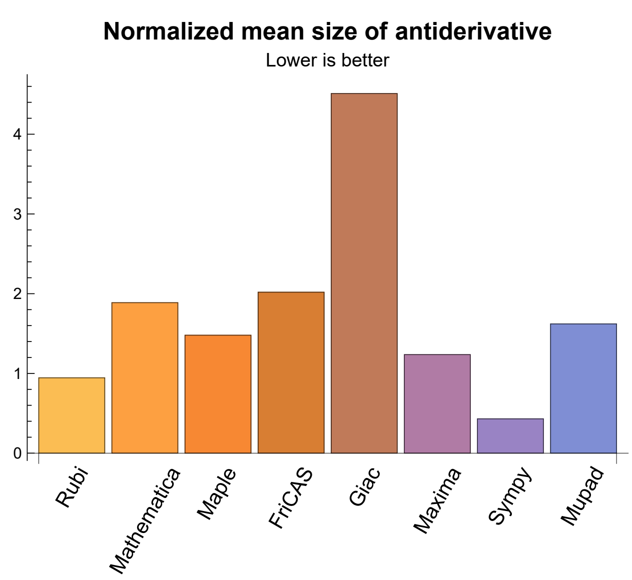
1.3 Performance

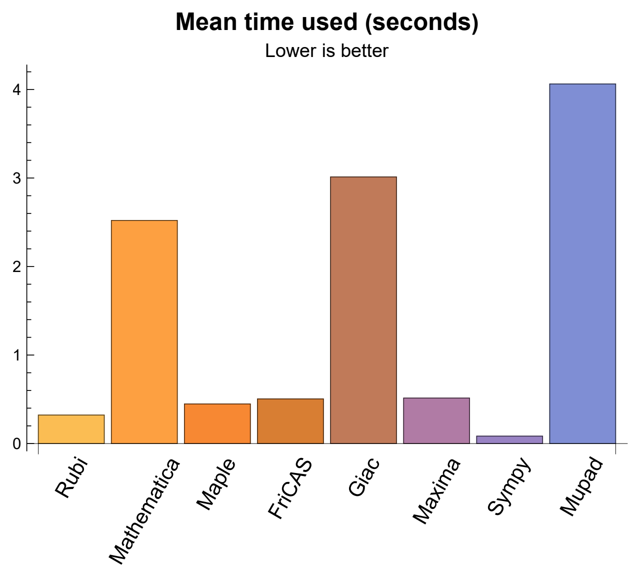
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.32	136.40	0.95	109.00	1.00
Mathematica	2.52	278.01	1.89	162.00	1.64
Maple	0.45	222.58	1.48	141.00	1.32
Maxima	0.51	185.22	1.24	120.00	1.08
Fricas	0.50	285.40	2.02	176.00	1.97
Sympy	0.08	17.00	0.43	8.50	0.31
Giac	3.01	437.39	4.51	177.00	1.55
Mupad	4.06	234.31	1.62	146.00	1.25

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{83, 88, 89, 90, 91}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {30, 39, 48}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

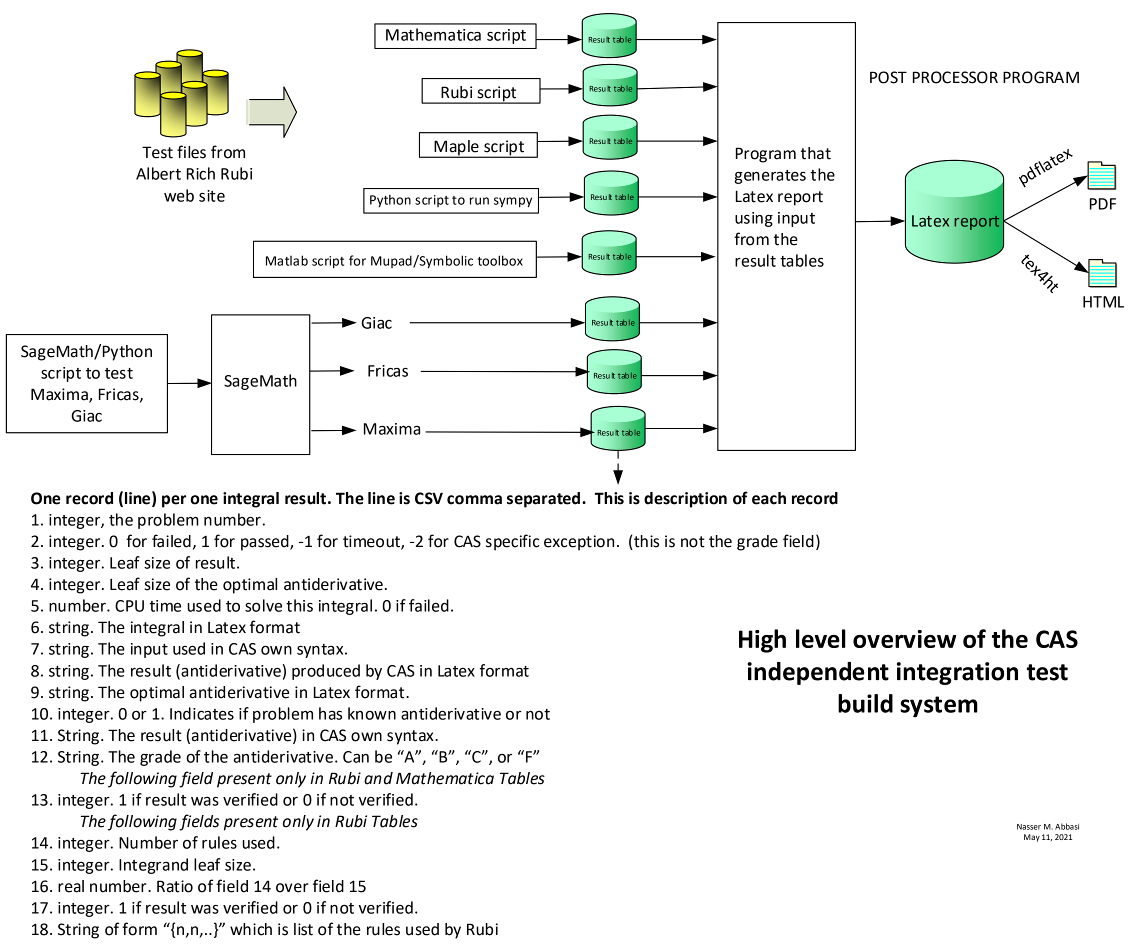
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 19, 21, 23, 25, 29, 31, 33, 36, 38, 42, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 67, 68, 69, 72, 73, 74, 79, 80, 81, 83, 85, 86, 87, 88, 89, 90, 91 }

B grade: { 7, 9, 22, 24, 26, 27, 28, 30, 32, 34, 35, 37, 39, 40, 41, 43, 44, 46, 48, 61, 66, 70, 71, 75, 76, 77, 84 }

C grade: { 18, 20, 78 }

F grade: { 82 }

2.1.3 Maple

A grade: { 1, 3, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 55, 56, 57, 58, 59, 60, 64, 65, 66, 70, 71, 72, 75, 76, 77, 78, 83, 88, 89, 90, 91 }

B grade: { 2, 4, 7, 24, 33, 42, 52, 54, 61, 62, 63, 67, 68, 69, 73, 74 }

C grade: { }

F grade: { 79, 80, 81, 82, 84, 85, 86, 87 }

2.1.4 Maxima

A grade: { 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 70, 71, 72, 75, 76, 77, 83, 88, 89, 90, 91 }

B grade: { 5, 7, 9, 51, 53, 61, 62, 63, 67, 68, 69, 73, 74, 78 }

C grade: { }

F grade: { 1, 2, 3, 4, 79, 80, 81, 82, 84, 85, 86, 87 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 6, 11, 12, 13, 14, 15, 22, 23, 24, 25, 32, 33, 34, 35, 41, 42, 43, 44, 45, 50, 51, 52, 53, 54, 57, 59, 83, 88, 89, 90, 91 }

B grade: { 5, 7, 8, 9, 10, 16, 17, 18, 19, 20, 21, 26, 27, 28, 29, 30, 31, 36, 37, 38, 39, 40, 46, 47, 48, 49, 55, 56, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

C grade: { }

F grade: { 79, 80, 81, 82, 84, 85, 86, 87 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 83, 89, 90, 91 }

B grade: { }

C grade: { }

F grade: { 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88 }

2.1.7 Giac

A grade: { 1, 3, 5, 6, 8, 10, 16, 17, 19, 20, 21, 26, 27, 28, 29, 30, 31, 35, 36, 38, 39, 40, 44, 45, 46, 47, 48, 49, 50, 51, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 70, 71, 72, 75, 76, 77, 78, 88, 89, 90, 91 }

B grade: { 2, 4, 7, 9, 12, 13, 14, 15, 18, 24, 25, 33, 37, 42, 52, 54, 61, 62, 67, 68, 69, 73, 74 }

C grade: { }

F grade: { 11, 22, 23, 32, 34, 41, 43, 79, 80, 81, 82, 83, 84, 85, 86, 87 }

2.1.8 Mupad

A grade: { 83, 88, 89, 90, 91 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

C grade: { }

F grade: { 79, 80, 81, 82, 84, 85, 86, 87 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	67	66	0	39	51	53	49
normalized size	1	1.00	0.86	0.85	0.00	0.50	0.65	0.68	0.63
time (sec)	N/A	0.066	0.086	0.175	0.000	0.529	0.200	0.365	3.708

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	51	81	0	26	37	71	59
normalized size	1	1.00	1.76	2.79	0.00	0.90	1.28	2.45	2.03
time (sec)	N/A	0.137	0.021	0.172	0.000	0.456	0.191	0.342	3.865

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	39	47	0	25	31	41	35
normalized size	1	1.00	0.78	0.94	0.00	0.50	0.62	0.82	0.70
time (sec)	N/A	0.054	0.096	0.169	0.000	0.482	0.149	0.448	3.745

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	33	47	0	14	17	33	39
normalized size	1	1.00	1.74	2.47	0.00	0.74	0.89	1.74	2.05
time (sec)	N/A	0.089	0.013	0.165	0.000	0.466	0.131	0.757	3.828

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	31	21	28	25	0	22	20
normalized size	1	1.00	1.94	1.31	1.75	1.56	0.00	1.38	1.25
time (sec)	N/A	0.086	0.019	0.157	0.314	0.471	0.000	0.309	3.800

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	15	20	17	25	0	18	19
normalized size	1	1.00	0.83	1.11	0.94	1.39	0.00	1.00	1.06
time (sec)	N/A	0.033	0.019	0.174	0.315	0.465	0.000	0.316	3.738

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	75	42	59	73	0	46	41
normalized size	1	1.00	3.12	1.75	2.46	3.04	0.00	1.92	1.71
time (sec)	N/A	0.132	0.022	0.177	0.318	0.471	0.000	0.303	3.729

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	29	15	12	36	0	12	13
normalized size	1	1.00	1.53	0.79	0.63	1.89	0.00	0.63	0.68
time (sec)	N/A	0.039	0.019	0.184	0.574	0.459	0.000	0.292	3.624

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	139	58	83	121	0	62	57
normalized size	1	1.00	3.48	1.45	2.08	3.02	0.00	1.55	1.42
time (sec)	N/A	0.151	0.025	0.189	0.511	0.469	0.000	0.200	3.696

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	41	28	24	54	0	24	27
normalized size	1	1.00	1.11	0.76	0.65	1.46	0.00	0.65	0.73
time (sec)	N/A	0.047	0.020	0.186	0.415	0.449	0.000	0.323	3.601

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	103	113	91	97	0	0	121
normalized size	1	1.00	1.02	1.12	0.90	0.96	0.00	0.00	1.20
time (sec)	N/A	0.076	0.048	0.320	0.374	0.515	0.000	0.000	3.804

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	82	92	87	74	0	1066	155
normalized size	1	1.00	0.99	1.11	1.05	0.89	0.00	12.84	1.87
time (sec)	N/A	0.168	0.082	0.313	0.530	0.460	0.000	0.884	3.787

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	71	79	70	74	0	5350	87
normalized size	1	1.00	1.03	1.14	1.01	1.07	0.00	77.54	1.26
time (sec)	N/A	0.068	0.028	0.299	0.580	0.509	0.000	31.086	3.828

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	56	58	52	47	0	413	50
normalized size	1	1.00	1.14	1.18	1.06	0.96	0.00	8.43	1.02
time (sec)	N/A	0.084	0.050	0.184	0.566	0.544	0.000	0.544	3.815

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	48	45	46	49	0	1236	53
normalized size	1	1.00	1.30	1.22	1.24	1.32	0.00	33.41	1.43
time (sec)	N/A	0.036	0.027	0.182	0.413	0.453	0.000	0.798	3.820

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	52	42	46	58	0	49	86
normalized size	1	1.00	2.00	1.62	1.77	2.23	0.00	1.88	3.31
time (sec)	N/A	0.029	0.018	0.280	0.539	0.523	0.000	0.489	3.745

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	36	26	25	62	0	35	25
normalized size	1	1.00	1.44	1.04	1.00	2.48	0.00	1.40	1.00
time (sec)	N/A	0.078	0.062	0.366	0.318	0.552	0.000	0.388	3.634

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	107	75	83	142	0	118	149
normalized size	1	1.00	1.78	1.25	1.38	2.37	0.00	1.97	2.48
time (sec)	N/A	0.068	0.028	0.386	0.535	0.463	0.000	0.512	3.716

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	72	60	50	122	0	62	49
normalized size	1	1.00	1.26	1.05	0.88	2.14	0.00	1.09	0.86
time (sec)	N/A	0.088	0.255	0.377	0.522	0.521	0.000	0.502	3.672

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	151	109	123	213	0	177	211
normalized size	1	1.00	1.54	1.11	1.26	2.17	0.00	1.81	2.15
time (sec)	N/A	0.092	0.039	0.392	0.514	0.515	0.000	2.926	3.846

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	104	94	72	174	0	84	70
normalized size	1	1.00	1.20	1.08	0.83	2.00	0.00	0.97	0.80
time (sec)	N/A	0.108	0.556	0.401	0.586	0.487	0.000	1.803	3.930

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	240	204	128	137	0	0	127
normalized size	1	1.00	2.12	1.81	1.13	1.21	0.00	0.00	1.12
time (sec)	N/A	0.186	3.507	0.418	0.892	0.550	0.000	0.000	3.820

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	152	167	104	126	0	0	174
normalized size	1	1.00	1.25	1.37	0.85	1.03	0.00	0.00	1.43
time (sec)	N/A	0.131	0.989	0.398	0.470	0.469	0.000	0.000	6.706

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	162	145	82	101	0	1061	75
normalized size	1	1.00	2.13	1.91	1.08	1.33	0.00	13.96	0.99
time (sec)	N/A	0.113	2.497	0.338	0.435	0.503	0.000	2.171	3.688

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	111	108	67	90	0	2837	93
normalized size	1	1.00	1.63	1.59	0.99	1.32	0.00	41.72	1.37
time (sec)	N/A	0.074	0.469	0.311	0.366	0.515	0.000	11.780	4.042

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	97	61	60	102	0	74	125
normalized size	1	1.00	2.26	1.42	1.40	2.37	0.00	1.72	2.91
time (sec)	N/A	0.056	0.250	0.327	0.547	0.452	0.000	0.933	3.721

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	91	43	39	96	0	51	44
normalized size	1	1.00	2.17	1.02	0.93	2.29	0.00	1.21	1.05
time (sec)	N/A	0.047	0.579	0.472	0.468	0.464	0.000	0.870	3.641

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	250	120	122	230	0	172	292
normalized size	1	1.00	2.63	1.26	1.28	2.42	0.00	1.81	3.07
time (sec)	N/A	0.118	1.918	0.527	0.350	0.626	0.000	1.593	3.845

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	127	104	69	174	0	91	72
normalized size	1	1.00	1.61	1.32	0.87	2.20	0.00	1.15	0.91
time (sec)	N/A	0.070	1.434	0.556	0.621	0.447	0.000	0.783	3.787

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	994	183	187	333	0	269	378
normalized size	1	1.00	6.02	1.11	1.13	2.02	0.00	1.63	2.29
time (sec)	N/A	0.160	6.219	0.431	0.466	0.513	0.000	0.811	3.909

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	114	166	104	240	0	131	107
normalized size	1	1.00	0.93	1.36	0.85	1.97	0.00	1.07	0.88
time (sec)	N/A	0.101	1.509	0.522	0.738	0.444	0.000	0.706	4.060

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	771	271	173	188	0	0	291
normalized size	1	1.00	3.76	1.32	0.84	0.92	0.00	0.00	1.42
time (sec)	N/A	0.189	6.305	0.437	0.707	0.474	0.000	0.000	6.436

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	203	226	113	149	0	2590	151
normalized size	1	1.00	1.97	2.19	1.10	1.45	0.00	25.15	1.47
time (sec)	N/A	0.145	4.524	0.373	0.450	0.455	0.000	7.281	3.695

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	637	193	128	144	0	0	193
normalized size	1	1.00	4.79	1.45	0.96	1.08	0.00	0.00	1.45
time (sec)	N/A	0.116	6.172	0.345	0.319	0.449	0.000	0.000	5.799

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	241	125	111	148	0	144	278
normalized size	1	1.00	2.80	1.45	1.29	1.72	0.00	1.67	3.23
time (sec)	N/A	0.086	2.289	0.347	0.327	0.530	0.000	7.583	4.209

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	126	63	56	127	0	70	62
normalized size	1	1.00	1.97	0.98	0.88	1.98	0.00	1.09	0.97
time (sec)	N/A	0.053	1.009	0.524	0.318	0.475	0.000	1.796	3.662

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	897	170	171	299	0	304	581
normalized size	1	1.00	6.36	1.21	1.21	2.12	0.00	2.16	4.12
time (sec)	N/A	0.135	6.194	0.515	0.334	0.557	0.000	2.106	3.966

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	212	141	98	237	0	133	103
normalized size	1	1.00	1.88	1.25	0.87	2.10	0.00	1.18	0.91
time (sec)	N/A	0.086	2.247	0.545	0.457	0.456	0.000	2.512	3.719

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	1229	254	250	427	0	373	698
normalized size	1	1.00	5.37	1.11	1.09	1.86	0.00	1.63	3.05
time (sec)	N/A	0.208	6.217	0.484	0.514	0.566	0.000	5.326	4.038

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	515	230	142	343	0	189	146
normalized size	1	1.00	3.08	1.38	0.85	2.05	0.00	1.13	0.87
time (sec)	N/A	0.134	1.838	0.569	0.324	0.464	0.000	1.850	3.877

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	1017	412	218	224	0	0	319
normalized size	1	1.00	3.70	1.50	0.79	0.81	0.00	0.00	1.16
time (sec)	N/A	0.248	6.270	0.533	0.538	0.494	0.000	0.000	7.213

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	263	368	154	186	0	3931	161
normalized size	1	1.00	1.89	2.65	1.11	1.34	0.00	28.28	1.16
time (sec)	N/A	0.175	6.286	0.469	0.452	0.468	0.000	22.339	3.794

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	383	309	166	176	0	0	268
normalized size	1	1.00	2.13	1.72	0.92	0.98	0.00	0.00	1.49
time (sec)	N/A	0.157	5.260	0.438	0.695	0.505	0.000	0.000	7.285

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	352	214	139	175	0	193	496
normalized size	1	1.00	2.98	1.81	1.18	1.48	0.00	1.64	4.20
time (sec)	N/A	0.114	5.000	0.464	0.681	0.514	0.000	6.001	4.144

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	162	90	72	159	0	86	81
normalized size	1	1.00	1.95	1.08	0.87	1.92	0.00	1.04	0.98
time (sec)	N/A	0.058	1.207	0.537	0.417	0.453	0.000	4.038	3.663

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	1128	192	188	346	0	300	670
normalized size	1	1.00	7.01	1.19	1.17	2.15	0.00	1.86	4.16
time (sec)	N/A	0.157	6.207	0.515	0.476	0.554	0.000	12.818	4.069

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	188	184	120	267	0	161	132
normalized size	1	1.00	1.37	1.34	0.88	1.95	0.00	1.18	0.96
time (sec)	N/A	0.104	3.844	0.657	0.404	0.460	0.000	7.802	3.761

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	1491	317	304	547	0	479	857
normalized size	1	1.00	5.44	1.16	1.11	2.00	0.00	1.75	3.13
time (sec)	N/A	0.241	6.302	0.504	0.633	0.685	0.000	8.970	4.171

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	233	301	171	386	0	235	181
normalized size	1	1.00	1.20	1.55	0.88	1.99	0.00	1.21	0.93
time (sec)	N/A	0.157	4.012	0.594	0.629	0.479	0.000	3.636	3.827

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	660	442	387	697	0	647	990
normalized size	1	1.00	1.64	1.10	0.96	1.73	0.00	1.61	2.46
time (sec)	N/A	0.306	6.266	0.535	0.401	0.709	0.000	3.187	4.035

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	289	361	658	370	0	464	683
normalized size	1	1.00	1.05	1.32	2.40	1.35	0.00	1.69	2.49
time (sec)	N/A	0.354	3.186	0.404	0.799	0.493	0.000	2.842	6.830

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	249	565	280	216	0	334	313
normalized size	1	1.00	1.58	3.58	1.77	1.37	0.00	2.11	1.98
time (sec)	N/A	0.339	2.811	0.365	0.625	0.463	0.000	3.867	4.269

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	139	205	364	261	0	241	324
normalized size	1	1.00	0.83	1.22	2.17	1.55	0.00	1.43	1.93
time (sec)	N/A	0.223	0.891	0.359	0.643	0.463	0.000	0.985	6.497

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	170	238	144	122	0	184	147
normalized size	1	1.00	1.81	2.53	1.53	1.30	0.00	1.96	1.56
time (sec)	N/A	0.163	0.771	0.362	0.579	0.457	0.000	0.617	3.865

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	79	100	141	185	0	118	110
normalized size	1	1.00	0.88	1.11	1.57	2.06	0.00	1.31	1.22
time (sec)	N/A	0.106	0.366	0.315	0.788	0.476	0.000	0.612	3.913

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	75	65	107	183	0	94	174
normalized size	1	1.00	1.14	0.98	1.62	2.77	0.00	1.42	2.64
time (sec)	N/A	0.129	0.108	0.364	0.471	0.483	0.000	1.619	4.092

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	47	53	47	95	0	60	39
normalized size	1	1.00	0.94	1.06	0.94	1.90	0.00	1.20	0.78
time (sec)	N/A	0.060	0.134	0.353	0.585	0.442	0.000	0.702	3.737

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	179	162	215	270	0	209	764
normalized size	1	1.00	1.47	1.33	1.76	2.21	0.00	1.71	6.26
time (sec)	N/A	0.307	0.805	0.414	0.663	0.532	0.000	2.071	4.448

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	95	144	97	208	0	144	102
normalized size	1	1.00	0.88	1.33	0.90	1.93	0.00	1.33	0.94
time (sec)	N/A	0.104	0.494	0.392	0.494	0.461	0.000	2.926	3.843

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	150	273	168	385	0	251	167
normalized size	1	1.00	0.89	1.62	0.99	2.28	0.00	1.49	0.99
time (sec)	N/A	0.151	2.102	0.409	0.540	0.469	0.000	2.932	4.329

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	603	1211	799	619	0	735	757
normalized size	1	1.00	2.03	4.08	2.69	2.08	0.00	2.47	2.55
time (sec)	N/A	0.914	6.553	0.497	0.962	0.540	0.000	1.530	5.543

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	373	724	507	444	0	513	481
normalized size	1	1.00	1.72	3.34	2.34	2.05	0.00	2.36	2.22
time (sec)	N/A	0.562	3.985	0.484	0.923	0.522	0.000	1.453	4.820

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	246	352	293	292	0	263	255
normalized size	1	1.00	1.66	2.38	1.98	1.97	0.00	1.78	1.72
time (sec)	N/A	0.295	3.387	0.456	0.815	0.469	0.000	0.886	4.056

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	109	75	74	293	0	74	79
normalized size	1	1.00	1.51	1.04	1.03	4.07	0.00	1.03	1.10
time (sec)	N/A	0.065	0.401	0.467	0.571	0.463	0.000	0.892	3.842

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	244	189	144	442	0	203	150
normalized size	1	1.00	1.74	1.35	1.03	3.16	0.00	1.45	1.07
time (sec)	N/A	0.122	2.662	0.513	0.424	0.503	0.000	0.796	3.915

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	589	343	225	787	0	332	237
normalized size	1	1.00	2.69	1.57	1.03	3.59	0.00	1.52	1.08
time (sec)	N/A	0.198	6.249	0.518	0.602	0.527	0.000	1.009	5.015

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	683	1449	1088	932	0	923	1068
normalized size	1	1.00	1.79	3.79	2.85	2.44	0.00	2.42	2.80
time (sec)	N/A	1.432	6.688	0.503	0.556	0.626	0.000	6.864	5.842

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	501	882	744	705	0	588	717
normalized size	1	1.00	1.76	3.09	2.61	2.47	0.00	2.06	2.52
time (sec)	N/A	0.850	6.493	0.493	0.701	0.569	0.000	3.869	5.335

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	316	542	463	526	0	482	433
normalized size	1	1.00	1.53	2.63	2.25	2.55	0.00	2.34	2.10
time (sec)	N/A	0.396	4.005	0.488	0.909	0.520	0.000	1.226	4.623

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	241	96	108	565	0	113	99
normalized size	1	1.00	2.54	1.01	1.14	5.95	0.00	1.19	1.04
time (sec)	N/A	0.077	2.690	0.516	0.484	0.538	0.000	1.041	3.869

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	456	234	192	811	0	237	200
normalized size	1	1.00	2.56	1.31	1.08	4.56	0.00	1.33	1.12
time (sec)	N/A	0.153	3.406	0.552	0.404	0.556	0.000	2.066	4.367

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	494	410	281	1018	0	382	297
normalized size	1	1.00	1.86	1.55	1.06	3.84	0.00	1.44	1.12
time (sec)	N/A	0.239	4.755	0.566	0.776	0.564	0.000	1.356	5.321

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	564	1215	997	1053	0	902	962
normalized size	1	1.00	1.54	3.32	2.72	2.88	0.00	2.46	2.63
time (sec)	N/A	1.370	5.661	0.539	0.935	0.666	0.000	6.916	5.662

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	395	668	662	802	0	642	597
normalized size	1	1.00	1.50	2.53	2.51	3.04	0.00	2.43	2.26
time (sec)	N/A	0.571	3.771	0.549	0.596	0.564	0.000	3.891	5.088

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	259	117	140	874	0	129	131
normalized size	1	1.00	2.23	1.01	1.21	7.53	0.00	1.11	1.13
time (sec)	N/A	0.088	2.217	0.531	0.380	0.544	0.000	2.575	3.976

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	528	278	228	1235	0	222	232
normalized size	1	1.00	2.58	1.36	1.11	6.02	0.00	1.08	1.13
time (sec)	N/A	0.173	2.079	0.588	0.558	0.601	0.000	5.001	5.058

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	673	476	325	1536	0	428	337
normalized size	1	1.00	2.24	1.59	1.08	5.12	0.00	1.43	1.12
time (sec)	N/A	0.271	1.760	0.599	0.615	0.652	0.000	1.947	5.647

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	41	26	50	55	0	44	38
normalized size	1	1.00	1.58	1.00	1.92	2.12	0.00	1.69	1.46
time (sec)	N/A	0.074	0.042	0.121	0.853	0.433	0.000	2.970	3.881

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	205	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.450	2.564	1.202	0.000	0.435	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	166	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.268	1.208	0.941	0.000	0.434	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	109	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.273	1.399	0.000	0.430	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	765	765	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.173	13.216	1.083	0.000	0.437	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	2.463	3.497	1.551	0.000	0.529	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	435	910	0	0	0	0	0	-1
normalized size	1	1.00	2.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.803	6.598	3.519	0.000	0.454	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	270	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.367	1.150	1.999	0.000	0.437	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.938	0.873	0.000	0.467	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	78	0	0	0	0	0	-1
normalized size	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	1.307	0.741	0.000	0.483	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	1.979	3.215	1.869	0.000	0.443	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.855	2.247	0.460	0.000	0.442	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.487	1.590	0.867	0.000	0.438	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	1.700	15.430	0.752	0.000	0.445	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [5] had the largest ratio of [.6364]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.00	13	0.308
2	A	9	7	1.00	13	0.538
3	A	5	4	1.00	13	0.308
4	A	8	6	1.00	11	0.546
5	A	8	7	1.00	11	0.636
6	A	3	2	1.00	13	0.154
7	A	8	7	1.00	13	0.538
8	A	4	3	1.00	13	0.231
9	A	9	8	1.00	13	0.615
10	A	4	3	1.00	13	0.231
11	A	8	5	1.00	19	0.263
12	A	6	4	1.00	19	0.210
13	A	8	5	1.00	19	0.263
14	A	5	4	1.00	19	0.210

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
15	A	6	5	1.00	17	0.294
16	A	4	2	1.00	17	0.118
17	A	3	1	1.00	19	0.053
18	A	7	6	1.00	19	0.316
19	A	3	1	1.00	19	0.053
20	A	9	6	1.00	19	0.316
21	A	3	1	1.00	19	0.053
22	A	8	6	1.00	21	0.286
23	A	11	7	1.00	21	0.333
24	A	6	6	1.00	21	0.286
25	A	9	7	1.00	19	0.368
26	A	6	4	1.00	19	0.210
27	A	3	2	1.00	21	0.095
28	A	10	7	1.00	21	0.333
29	A	3	2	1.00	21	0.095
30	A	13	9	1.00	21	0.429
31	A	3	2	1.00	21	0.095
32	A	16	8	1.00	21	0.381
33	A	7	6	1.00	21	0.286
34	A	13	8	1.00	19	0.421
35	A	8	5	1.00	19	0.263
36	A	3	2	1.00	21	0.095
37	A	12	7	1.00	21	0.333
38	A	3	2	1.00	21	0.095
39	A	17	9	1.00	21	0.429
40	A	3	2	1.00	21	0.095
41	A	19	8	1.00	21	0.381
42	A	7	6	1.00	21	0.286
43	A	16	9	1.00	19	0.474
44	A	10	5	1.00	19	0.263
45	A	3	2	1.00	21	0.095
46	A	14	9	1.00	21	0.429
47	A	3	2	1.00	21	0.095
48	A	21	9	1.00	21	0.429
49	A	3	2	1.00	21	0.095
50	A	25	9	1.00	21	0.429

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
51	A	13	9	1.00	21	0.429
52	A	8	6	1.00	21	0.286
53	A	10	9	1.00	21	0.429
54	A	7	6	1.00	21	0.286
55	A	6	6	1.00	19	0.316
56	A	6	5	1.00	19	0.263
57	A	3	2	1.00	21	0.095
58	A	15	11	1.00	21	0.524
59	A	3	2	1.00	21	0.095
60	A	3	2	1.00	21	0.095
61	A	9	6	1.00	21	0.286
62	A	8	6	1.00	21	0.286
63	A	7	6	1.00	21	0.286
64	A	3	2	1.00	21	0.095
65	A	3	2	1.00	21	0.095
66	A	3	2	1.00	21	0.095
67	A	9	6	1.00	21	0.286
68	A	8	6	1.00	21	0.286
69	A	7	6	1.00	21	0.286
70	A	3	2	1.00	21	0.095
71	A	3	2	1.00	21	0.095
72	A	3	2	1.00	21	0.095
73	A	8	6	1.00	21	0.286
74	A	7	6	1.00	21	0.286
75	A	3	2	1.00	21	0.095
76	A	3	2	1.00	21	0.095
77	A	3	2	1.00	21	0.095
78	A	6	5	1.00	9	0.556
79	A	8	5	1.00	21	0.238
80	A	6	5	1.00	21	0.238
81	A	5	4	1.00	19	0.210
82	A	14	6	1.00	21	0.286
83	A	0	0	0.00	0	0.000
84	A	7	4	1.00	21	0.190
85	A	6	4	1.00	21	0.190
86	A	2	2	1.00	21	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
87	A	4	4	1.00	21	0.190
88	A	0	0	0.00	0	0.000
89	A	0	0	0.00	0	0.000
90	A	0	0	0.00	0	0.000
91	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int \frac{\sin^4(x)}{i+\tan(x)}\,dx$

Optimal. Leaf size=78

$$-\frac{ix}{16}-\frac{i}{8(-\tan(x)+i)}-\frac{3i}{16(\tan(x)+i)}-\frac{1}{32(-\tan(x)+i)^2}-\frac{5}{32(\tan(x)+i)^2}+\frac{i}{24(\tan(x)+i)^3}$$

```
[Out] -1/16*I*x-1/32/(I-tan(x))^2-1/8*I/(I-tan(x))+1/24*I/(I+tan(x))^3-5/32/(I+ta
n(x))^2-3/16*I/(I+tan(x))
```

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3516, 848, 88, 203}

$$-\frac{ix}{16}-\frac{i}{8(-\tan(x)+i)}-\frac{3i}{16(\tan(x)+i)}-\frac{1}{32(-\tan(x)+i)^2}-\frac{5}{32(\tan(x)+i)^2}+\frac{i}{24(\tan(x)+i)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(I + Tan[x]),x]
```

```
[Out] (-I/16)*x - 1/(32*(I - Tan[x])^2) - (I/8)/(I - Tan[x]) + (I/24)/(I + Tan[x]
)^3 - 5/(32*(I + Tan[x])^2) - ((3*I)/16)/(I + Tan[x])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(x)}{i+\tan(x)}\,dx = \text{Subst}\left(\int \frac{x^4}{(i+x)\left(1+x^2\right)^3}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \frac{x^4}{(-i+x)^3(i+x)^4}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{16(-i+x)^3} - \frac{i}{8(-i+x)^2} - \frac{i}{8(i+x)^4} + \frac{5}{16(i+x)^3} + \frac{3i}{16(i+x)^2} - \frac{i}{16\left(1+x^2\right)}\right) dx,$$
$$= -\frac{1}{32(i-\tan(x))^2} - \frac{i}{8(i-\tan(x))} + \frac{i}{24(i+\tan(x))^3} - \frac{5}{32(i+\tan(x))^2} - \frac{3i}{16(i+\tan(x))} - \frac{1}{16}i\,\text{S}$$
$$= -\frac{ix}{16} - \frac{1}{32(i-\tan(x))^2} - \frac{i}{8(i-\tan(x))} + \frac{i}{24(i+\tan(x))^3} - \frac{5}{32(i+\tan(x))^2} - \frac{3i}{16(i+\tan(x))}$$

Mathematica [A] time = 0.09, size = 67, normalized size = 0.86

$$\frac{\sec(x)\left(-32\sin(x) - 27\sin(3x) + 5\sin(5x) - 56i\cos(x) - 9i\cos(3x) + i\cos(5x) + 24\tan^{-1}(\tan(x))(\cos(x) - i\sin\right.}{384(\tan(x)+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(I + Tan[x]),x]
```

```
[Out] (Sec[x]*((-56*I)*Cos[x] - (9*I)*Cos[3*x] + I*Cos[5*x] + 24*ArcTan[Tan[x]]*(
Cos[x] - I*Sin[x]) - 32*Sin[x] - 27*Sin[3*x] + 5*Sin[5*x]))/(384*(I + Tan[x
]))
```

fricas [A] time = 0.53, size = 39, normalized size = 0.50

$$\frac{1}{384}\left(-24i\,xe^{(4i\,x)} - 2\,e^{(10i\,x)} + 9\,e^{(8i\,x)} - 12\,e^{(6i\,x)} - 18\,e^{(2i\,x)} + 3\right)e^{(-4i\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] 1/384*(-24*I*x*e^(4*I*x) - 2*e^(10*I*x) + 9*e^(8*I*x) - 12*e^(6*I*x) - 18*e
^(2*I*x) + 3)*e^(-4*I*x)
```

giac [A] time = 0.36, size = 53, normalized size = 0.68

$$-\frac{3i\,\tan(x)^4 + 21\,\tan(x)^3 + 13i\,\tan(x)^2 + 11\,\tan(x) + 8i}{48\left(\tan(x)+i\right)^3\left(\tan(x)-i\right)^2} + \frac{1}{32}\log\left(\tan(x)+i\right) - \frac{1}{32}\log\left(\tan(x)-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/48*(3*I*tan(x)^4 + 21*tan(x)^3 + 13*I*tan(x)^2 + 11*tan(x) + 8*I)/((tan(
x) + I)^3*(tan(x) - I)^2) + 1/32*log(tan(x) + I) - 1/32*log(tan(x) - I)
```

maple [A] time = 0.18, size = 66, normalized size = 0.85

$$\frac{i}{24\left(i+\tan(x)\right)^3} - \frac{3i}{16\left(i+\tan(x)\right)} - \frac{5}{32\left(i+\tan(x)\right)^2} + \frac{\ln\left(i+\tan(x)\right)}{32} + \frac{i}{8\tan(x)-8i} - \frac{1}{32\left(\tan(x)-i\right)^2} - \frac{\ln\left(\tan(x)-i\right.}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(I+tan(x)),x)
```

```
[Out] 1/24*I/(I+tan(x))^3-3/16*I/(I+tan(x))-5/32/(I+tan(x))^2+1/32*ln(I+tan(x))+1
/8*I/(tan(x)-I)-1/32/(tan(x)-I)^2-1/32*ln(tan(x)-I)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 3.71, size = 49, normalized size = 0.63

$$-\frac{x\,1\mathrm{i}}{16}+\frac{\frac{\tan(x)^4\,1\mathrm{i}}{16}+\frac{7\tan(x)^3}{16}+\frac{\tan(x)^2\,13\mathrm{i}}{48}+\frac{11\tan(x)}{48}+\frac{1}{6}\mathrm{i}}{(\tan(x)+1\mathrm{i})^3\,(1+\tan(x)\,1\mathrm{i})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(tan(x) + 1i),x)
```

```
[Out] ((11*tan(x))/48 + (tan(x)^2*13i)/48 + (7*tan(x)^3)/16 + (tan(x)^4*1i)/16 +
1i/6)/((tan(x) + 1i)^3*(tan(x)*1i + 1)^2) - (x*1i)/16
```

sympy [A] time = 0.20, size = 51, normalized size = 0.65

$$-\frac{ix}{16}-\frac{e^{6ix}}{192}+\frac{3e^{4ix}}{128}-\frac{e^{2ix}}{32}-\frac{3e^{-2ix}}{64}+\frac{e^{-4ix}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(I+tan(x)),x)
```

```
[Out] -I*x/16 - exp(6*I*x)/192 + 3*exp(4*I*x)/128 - exp(2*I*x)/32 - 3*exp(-2*I*x)
/64 + exp(-4*I*x)/128
```

3.2 $\int \frac{\sin^3(x)}{i+\tan(x)}\,dx$

Optimal. Leaf size=29

$$\frac{\sin^5(x)}{5}-\frac{1}{5}i\cos^5(x)+\frac{1}{3}i\cos^3(x)$$

```
[Out] 1/3*I*cos(x)^3-1/5*I*cos(x)^5+1/5*sin(x)^5
```

Rubi [A] time = 0.14, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3518, 3108, 3107, 2565, 14, 2564, 30}

$$\frac{\sin^5(x)}{5}-\frac{1}{5}i\cos^5(x)+\frac{1}{3}i\cos^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(I + Tan[x]),x]
```

```
[Out] (I/3)*Cos[x]^3 - (I/5)*Cos[x]^5 + Sin[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
```

```
Q[p, 0]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^3(x)}{i+\tan(x)}\,dx &= \int \frac{\cos(x)\sin^3(x)}{i\cos(x)+\sin(x)}\,dx \\
&= -\left(i\int \cos(x)(\cos(x)+i\sin(x))\sin^3(x)\,dx\right) \\
&= -\left(i\int \left(\cos^2(x)\sin^3(x)+i\cos(x)\sin^4(x)\right)\,dx\right) \\
&= -\left(i\int \cos^2(x)\sin^3(x)\,dx\right)+\int \cos(x)\sin^4(x)\,dx \\
&= i\,\mathrm{Subst}\left(\int x^2\left(1-x^2\right)\,dx,x,\cos(x)\right)+\mathrm{Subst}\left(\int x^4\,dx,x,\sin(x)\right) \\
&= \frac{\sin^5(x)}{5}+i\,\mathrm{Subst}\left(\int \left(x^2-x^4\right)\,dx,x,\cos(x)\right) \\
&= \frac{1}{3}i\cos^3(x)-\frac{1}{5}i\cos^5(x)+\frac{\sin^5(x)}{5}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.76

$$\frac{\sin(x)}{8}-\frac{1}{16}\sin(3x)+\frac{1}{80}\sin(5x)+\frac{1}{8}i\cos(x)+\frac{1}{48}i\cos(3x)-\frac{1}{80}i\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(I + Tan[x]),x]
```

```
[Out] (I/8)*Cos[x] + (I/48)*Cos[3*x] - (I/80)*Cos[5*x] + Sin[x]/8 - Sin[3*x]/16 +
 Sin[5*x]/80
```

fricas [A] time = 0.46, size = 26, normalized size = 0.90

$$\frac{1}{240}\left(-3i\,e^{(8i\,x)}+10i\,e^{(6i\,x)}+30i\,e^{(2i\,x)}-5i\right)e^{(-3i\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] 1/240*(-3*I*e^(8*I*x) + 10*I*e^(6*I*x) + 30*I*e^(2*I*x) - 5*I)*e^(-3*I*x)
```

giac [B] time = 0.34, size = 71, normalized size = 2.45

$$-\frac{-3i\,\tan\left(\frac{1}{2}\,x\right)^2-12\,\tan\left(\frac{1}{2}\,x\right)+5i}{24\left(-i\,\tan\left(\frac{1}{2}\,x\right)-1\right)^3}-\frac{15\,\tan\left(\frac{1}{2}\,x\right)^4+60i\,\tan\left(\frac{1}{2}\,x\right)^3-10\,\tan\left(\frac{1}{2}\,x\right)^2-20i\,\tan\left(\frac{1}{2}\,x\right)+7}{120\left(\tan\left(\frac{1}{2}\,x\right)+i\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/24*(-3*I*tan(1/2*x)^2 - 12*tan(1/2*x) + 5*I)/(-I*tan(1/2*x) - 1)^3 - 1/1
20*(15*tan(1/2*x)^4 + 60*I*tan(1/2*x)^3 - 10*tan(1/2*x)^2 - 20*I*tan(1/2*x)
 + 7)/(tan(1/2*x) + I)^5
```

maple [B] time = 0.17, size = 81, normalized size = 2.79

$$\frac{i}{\left(\tan\left(\frac{x}{2}\right)+i\right)^4}+\frac{2}{5\left(\tan\left(\frac{x}{2}\right)+i\right)^5}-\frac{2}{3\left(\tan\left(\frac{x}{2}\right)+i\right)^3}-\frac{1}{8\left(\tan\left(\frac{x}{2}\right)+i\right)}-\frac{i}{4\left(\tan\left(\frac{x}{2}\right)-i\right)^2}+\frac{1}{6\left(\tan\left(\frac{x}{2}\right)-i\right)^3}+\frac{1}{8\tan\left(\frac{x}{2}\right)-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(I+tan(x)),x)
```

```
[Out] I/(tan(1/2*x)+I)^4+2/5/(tan(1/2*x)+I)^5-2/3/(tan(1/2*x)+I)^3-1/8/(tan(1/2*x
)+I)-1/4*I/(tan(1/2*x)-I)^2+1/6/(tan(1/2*x)-I)^3+1/8/(tan(1/2*x)-I)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 3.87, size = 59, normalized size = 2.03

$$-\frac{4\left(-\tan\left(\frac{x}{2}\right)^4 15\mathrm{i}+6\tan\left(\frac{x}{2}\right)^3+\tan\left(\frac{x}{2}\right)^2 2\mathrm{i}+2\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)}{15\left(-1+\tan\left(\frac{x}{2}\right)1\mathrm{i}\right)^5\left(1+\tan\left(\frac{x}{2}\right)1\mathrm{i}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(tan(x) + 1i),x)
```

```
[Out] -(4*(2*tan(x/2) + tan(x/2)^2*2i + 6*tan(x/2)^3 - tan(x/2)^4*15i + 1i))/(15*
(tan(x/2)*1i - 1)^5*(tan(x/2)*1i + 1)^3)
```

sympy [A] time = 0.19, size = 37, normalized size = 1.28

$$-\frac{ie^{5ix}}{80}+\frac{ie^{3ix}}{24}+\frac{ie^{-ix}}{8}-\frac{ie^{-3ix}}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(I+tan(x)),x)
```

```
[Out] -I*exp(5*I*x)/80 + I*exp(3*I*x)/24 + I*exp(-I*x)/8 - I*exp(-3*I*x)/48
```

3.3 $\int \frac{\sin^2(x)}{i+\tan(x)}\,dx$

Optimal. Leaf size=50

$$-\frac{ix}{8}-\frac{i}{8(-\tan(x)+i)}-\frac{i}{4(\tan(x)+i)}-\frac{1}{8(\tan(x)+i)^2}$$

```
[Out] -1/8*I*x-1/8*I/(I-tan(x))-1/8/(I+tan(x))^2-1/4*I/(I+tan(x))
```

Rubi [A] time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3516, 848, 88, 203}

$$-\frac{ix}{8}-\frac{i}{8(-\tan(x)+i)}-\frac{i}{4(\tan(x)+i)}-\frac{1}{8(\tan(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(I + Tan[x]),x]
```

```
[Out] (-I/8)*x - (I/8)/(I - Tan[x]) - 1/(8*(I + Tan[x])^2) - (I/4)/(I + Tan[x])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(x)}{i+\tan(x)}\,dx &= \text{Subst}\left(\int \frac{x^2}{(i+x)\left(1+x^2\right)^2}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \frac{x^2}{(-i+x)^2(i+x)^3}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{i}{8(-i+x)^2}+\frac{1}{4(i+x)^3}+\frac{i}{4(i+x)^2}-\frac{i}{8\left(1+x^2\right)}\right)dx, x, \tan(x)\right)\\
&= -\frac{i}{8(i-\tan(x))}-\frac{1}{8(i+\tan(x))^2}-\frac{i}{4(i+\tan(x))}-\frac{1}{8}i\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)\\
&= -\frac{ix}{8}-\frac{i}{8(i-\tan(x))}-\frac{1}{8(i+\tan(x))^2}-\frac{i}{4(i+\tan(x))}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 39, normalized size = 0.78

$$-\frac{i\left(-3i\sin(2x)+\cos(2x)+2\tan^{-1}(\tan(x))(\tan(x)+i)+3\right)}{16(\tan(x)+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(I + Tan[x]),x]
```

```
[Out] ((-1/16*I)*(3 + Cos[2*x] - (3*I)*Sin[2*x] + 2*ArcTan[Tan[x]]*(I + Tan[x])))
/(I + Tan[x])
```

fricas [A] time = 0.48, size = 25, normalized size = 0.50

$$\frac{1}{32}\left(-4i\,xe^{(2i\,x)}+e^{(6i\,x)}-2\,e^{(4i\,x)}-2\right)e^{(-2i\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] 1/32*(-4*I*x*e^(2*I*x) + e^(6*I*x) - 2*e^(4*I*x) - 2)*e^(-2*I*x)
```

giac [A] time = 0.45, size = 41, normalized size = 0.82

$$-\frac{i\,\tan(x)^2+3\,\tan(x)+2i}{8\,(\tan(x)+i)^2(\tan(x)-i)}+\frac{1}{16}\,\log\left(\tan(x)+i\right)-\frac{1}{16}\,\log\left(\tan(x)-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/8*(I*tan(x)^2 + 3*tan(x) + 2*I)/((tan(x) + I)^2*(tan(x) - I)) + 1/16*log
(tan(x) + I) - 1/16*log(tan(x) - I)
```

maple [A] time = 0.17, size = 47, normalized size = 0.94

$$-\frac{i}{4\,(i+\tan(x))}-\frac{1}{8\,(i+\tan(x))^2}+\frac{\ln\left(i+\tan(x)\right)}{16}+\frac{i}{8\tan(x)-8i}-\frac{\ln\left(\tan(x)-i\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(I+tan(x)),x)
```

```
[Out] -1/4*I/(I+tan(x))-1/8/(I+tan(x))^2+1/16*ln(I+tan(x))+1/8*I/(tan(x)-I)-1/16*
ln(tan(x)-I)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [**B**] time = 3.75, size = 35, normalized size = 0.70

$$-\frac{x\,1\mathrm{i}}{8}+\frac{\frac{\tan(x)^2}{8}-\frac{\tan(x)\,3\mathrm{i}}{8}+\frac{1}{4}}{(\tan(x)+1\mathrm{i})^2\,(1+\tan(x)\,1\mathrm{i})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(tan(x) + 1i),x)
```

```
[Out] (tan(x)^2/8 - (tan(x)*3i)/8 + 1/4)/((tan(x) + 1i)^2*(tan(x)*1i + 1)) - (x*1
i)/8
```

sympy [**A**] time = 0.15, size = 31, normalized size = 0.62

$$-\frac{ix}{8}+\frac{e^{4ix}}{32}-\frac{e^{2ix}}{16}-\frac{e^{-2ix}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(I+tan(x)),x)
```

```
[Out] -I*x/8 + exp(4*I*x)/32 - exp(2*I*x)/16 - exp(-2*I*x)/16
```

3.4 $\int \frac{\sin(x)}{i+\tan(x)}\,dx$

Optimal. Leaf size=19

$$\frac{\sin^3(x)}{3}+\frac{1}{3}i\cos^3(x)$$

[Out] 1/3*I*cos(x)^3+1/3*sin(x)^3

Rubi [A] time = 0.09, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {3518, 3108, 3107, 2565, 30, 2564}

$$\frac{\sin^3(x)}{3}+\frac{1}{3}i\cos^3(x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(I + Tan[x]),x]

[Out] (I/3)*Cos[x]^3 + Sin[x]^3/3

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
```

```
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(x)}{i+\tan(x)}\,dx &= \int \frac{\cos(x)\sin(x)}{i\cos(x)+\sin(x)}\,dx \\
&= -(i\int \cos(x)(\cos(x)+i\sin(x))\sin(x)\,dx) \\
&= -\left(i\int \left(\cos^2(x)\sin(x)+i\cos(x)\sin^2(x)\right)\,dx\right) \\
&= -\left(i\int \cos^2(x)\sin(x)\,dx\right)+\int \cos(x)\sin^2(x)\,dx \\
&= i\,\text{Subst}\left(\int x^2\,dx, x, \cos(x)\right)+\text{Subst}\left(\int x^2\,dx, x, \sin(x)\right) \\
&= \frac{1}{3}i\cos^3(x)+\frac{\sin^3(x)}{3}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.74

$$\frac{\sin(x)}{4}-\frac{1}{12}\sin(3x)+\frac{1}{4}i\cos(x)+\frac{1}{12}i\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(I + Tan[x]),x]
[Out] (I/4)*Cos[x] + (I/12)*Cos[3*x] + Sin[x]/4 - Sin[3*x]/12
```

fricas [A] time = 0.47, size = 14, normalized size = 0.74

$$\frac{1}{12}\left(ie^{(4ix)}+3i\right)e^{(-ix)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(I+tan(x)),x, algorithm="fricas")
[Out] 1/12*(I*e^(4*I*x) + 3*I)*e^(-I*x)
```

giac [B] time = 0.76, size = 33, normalized size = 1.74

$$-\frac{i}{2\left(-i\tan\left(\frac{1}{2}x\right)-1\right)}-\frac{3\tan\left(\frac{1}{2}x\right)^2-1}{6\left(\tan\left(\frac{1}{2}x\right)+i\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(I+tan(x)),x, algorithm="giac")
[Out] -1/2*I/(-I*tan(1/2*x) - 1) - 1/6*(3*tan(1/2*x)^2 - 1)/(tan(1/2*x) + I)^3
```

maple [B] time = 0.16, size = 47, normalized size = 2.47

$$\frac{i}{\left(\tan\left(\frac{x}{2}\right)+i\right)^2}+\frac{2}{3\left(\tan\left(\frac{x}{2}\right)+i\right)^3}-\frac{1}{2\left(\tan\left(\frac{x}{2}\right)+i\right)}+\frac{1}{2\tan\left(\frac{x}{2}\right)-2i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(I+tan(x)),x)
```

```
[Out] I/(tan(1/2*x)+I)^2+2/3/(tan(1/2*x)+I)^3-1/2/(tan(1/2*x)+I)+1/2/(tan(1/2*x)-
I)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [**B**] time = 3.83, size = 39, normalized size = 2.05

$$-\frac{2\left(3\tan\left(\frac{x}{2}\right)^2+\tan\left(\frac{x}{2}\right)\,2\mathrm{i}-1\right)}{3\left(1+\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(tan(x) + 1i),x)
```

```
[Out] -(2*(tan(x/2)*2i + 3*tan(x/2)^2 - 1))/(3*(tan(x/2)*1i + 1)*(tan(x/2) + 1i)^
3)
```

sympy [**A**] time = 0.13, size = 17, normalized size = 0.89

$$\frac{ie^{3ix}}{12}+\frac{ie^{-ix}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(I+tan(x)),x)
```

```
[Out] I*exp(3*I*x)/12 + I*exp(-I*x)/4
```

3.5 $\int \frac{\csc(x)}{i+\tan(x)}\, dx$

Optimal. Leaf size=16

$$\sin(x) - i\cos(x) + i\tanh^{-1}(\cos(x))$$

```
[Out] I*arctanh(cos(x))-I*cos(x)+sin(x)
```

Rubi [A] time = 0.09, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3518, 3108, 3107, 2637, 2592, 321, 206}

$$\sin(x) - i\cos(x) + i\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(I + Tan[x]),x]
```

```
[Out] I*ArcTanh[Cos[x]] - I*Cos[x] + Sin[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
```

```
Q[p, 0]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(x)}{i+\tan(x)}\,dx &= \int \frac{\cot(x)}{i\cos(x)+\sin(x)}\,dx \\
&= -(i\int \cot(x)(\cos(x)+i\sin(x))\,dx) \\
&= -(i\int (i\cos(x)+\cos(x)\cot(x))\,dx) \\
&= -(i\int \cos(x)\cot(x)\,dx) + \int \cos(x)\,dx \\
&= \sin(x) + i\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(x)\right) \\
&= -i\cos(x)+\sin(x)+i\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(x)\right) \\
&= i\tanh^{-1}(\cos(x)) - i\cos(x)+\sin(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 1.94

$$\sin(x) - i\cos(x) - i\log\left(\sin\left(\frac{x}{2}\right)\right) + i\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(I + Tan[x]),x]
[Out] (-I)*Cos[x] + I*Log[Cos[x/2]] - I*Log[Sin[x/2]] + Sin[x]
```

fricas [B] time = 0.47, size = 25, normalized size = 1.56

$$-i\,e^{(i\,x)} + i\,\log\left(e^{(i\,x)}+1\right) - i\,\log\left(e^{(i\,x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(I+tan(x)),x, algorithm="fricas")
[Out] -I*e^(I*x) + I*log(e^(I*x) + 1) - I*log(e^(I*x) - 1)
```

giac [A] time = 0.31, size = 22, normalized size = 1.38

$$-\frac{2i}{-i\,\tan\left(\frac{1}{2}\,x\right)+1} - i\,\log\left(-i\,\tan\left(\frac{1}{2}\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(I+tan(x)),x, algorithm="giac")
[Out] -2*I/(-I*tan(1/2*x) + 1) - I*log(-I*tan(1/2*x))
```

maple [A] time = 0.16, size = 21, normalized size = 1.31

$$\frac{2}{\tan\left(\frac{x}{2}\right)+i}-i\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(I+tan(x)),x)
```

```
[Out] 2/(tan(1/2*x)+I)-I*ln(tan(1/2*x))
```

maxima [B] time = 0.31, size = 28, normalized size = 1.75

$$\frac{2}{\frac{\sin(x)}{\cos(x)+1}+i}-i\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] 2/(sin(x)/(cos(x) + 1) + I) - I*log(sin(x)/(cos(x) + 1))
```

mupad [B] time = 3.80, size = 20, normalized size = 1.25

$$-\ln\left(\tan\left(\frac{x}{2}\right)\right)1\mathrm{i}+\frac{2}{\tan\left(\frac{x}{2}\right)+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(tan(x) + 1i)),x)
```

```
[Out] 2/(tan(x/2) + 1i) - log(tan(x/2))*1i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc(x)}{\tan(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(I+tan(x)),x)
```

```
[Out] Integral(csc(x)/(tan(x) + I), x)
```

3.6 $\int \frac{\csc^2(x)}{i+\tan(x)}\,dx$

Optimal. Leaf size=18

$$ix + i\cot(x) + \log(\tan(x)) + \log(\cos(x))$$

```
[Out] I*x+I*cot(x)+ln(cos(x))+ln(tan(x))
```

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3516, 44}

$$ix + i\cot(x) + \log(\tan(x)) + \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(I + Tan[x]),x]
```

```
[Out] I*x + I*Cot[x] + Log[Cos[x]] + Log[Tan[x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(x)}{i+\tan(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x^2(i+x)}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{-i-x}-\frac{i}{x^2}+\frac{1}{x}\right)dx, x, \tan(x)\right)\\ &= ix + i\cot(x) + \log(\cos(x)) + \log(\tan(x))\end{aligned}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 0.83

$$ix + i\cot(x) + \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(I + Tan[x]),x]
```

```
[Out] I*x + I*Cot[x] + Log[Sin[x]]
```

fricas [A] time = 0.46, size = 25, normalized size = 1.39

$$\frac{\left(e^{(2ix)}-1\right)\log\left(e^{(2ix)}-1\right)-2}{e^{(2ix)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(I+tan(x)),x, algorithm="fricas")
[Out] ((e^(2*I*x) - 1)*log(e^(2*I*x) - 1) - 2)/(e^(2*I*x) - 1)
```

giac [A] time = 0.32, size = 18, normalized size = 1.00

$$\frac{i}{\tan(x)} - \log\left(\tan(x) + i\right) + \log\left(|\tan(x)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(I+tan(x)),x, algorithm="giac")
[Out] I/tan(x) - log(tan(x) + I) + log(abs(tan(x)))
```

maple [A] time = 0.17, size = 20, normalized size = 1.11

$$-\ln\left(i + \tan(x)\right) + \ln\left(\tan(x)\right) + \frac{i}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(I+tan(x)),x)
[Out] -ln(I+tan(x))+ln(tan(x))+I/tan(x)
```

maxima [A] time = 0.32, size = 17, normalized size = 0.94

$$\frac{i}{\tan(x)} - \log\left(\tan(x) + i\right) + \log\left(\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(I+tan(x)),x, algorithm="maxima")
[Out] I/tan(x) - log(tan(x) + I) + log(tan(x))
```

mupad [B] time = 3.74, size = 19, normalized size = 1.06

$$\mathrm{atan}\left(2\tan(x) + 1\mathrm{i}\right)\,2\mathrm{i} + \frac{1\mathrm{i}}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(tan(x) + 1i)),x)
[Out] atan(2*tan(x) + 1i)*2i + 1i/tan(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)}{\tan(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(I+tan(x)),x)
[Out] Integral(csc(x)**2/(tan(x) + I), x)
```

3.7 $\int \frac{\csc^3(x)}{i+\tan(x)} \, dx$

Optimal. Leaf size=24

$$-\csc(x)-\frac{1}{2} i \tanh^{-1}(\cos(x))+\frac{1}{2} i \cot(x) \csc(x)$$

```
[Out] -1/2*I*arctanh(cos(x))-csc(x)+1/2*I*cot(x)*csc(x)
```

Rubi [A] time = 0.13, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3518, 3108, 3107, 2606, 8, 2611, 3770}

$$-\csc(x)-\frac{1}{2} i \tanh^{-1}(\cos(x))+\frac{1}{2} i \cot(x) \csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(I + Tan[x]),x]
```

```
[Out] (-I/2)*ArcTanh[Cos[x]] - Csc[x] + (I/2)*Cot[x]*Csc[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
```

```
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(x)}{i+\tan(x)}\,dx &= \int \frac{\cot(x)\csc^2(x)}{i\cos(x)+\sin(x)}\,dx\\ &= -\left(i\int \cot(x)\csc^2(x)(\cos(x)+i\sin(x))\,dx\right)\\ &= -\left(i\int \left(i\cot(x)\csc(x)+\cot^2(x)\csc(x)\right)\,dx\right)\\ &= -\left(i\int \cot^2(x)\csc(x)\,dx\right)+\int \cot(x)\csc(x)\,dx\\ &= \frac{1}{2}i\cot(x)\csc(x)+\frac{1}{2}i\int \csc(x)\,dx-\text{Subst}(\int 1\,dx,x,\csc(x))\\ &= -\frac{1}{2}i\tanh^{-1}(\cos(x))-\csc(x)+\frac{1}{2}i\cot(x)\csc(x)\end{aligned}$$

Mathematica [B] time = 0.02, size = 75, normalized size = 3.12

$$-\frac{1}{2}\tan\left(\frac{x}{2}\right)-\frac{1}{2}\cot\left(\frac{x}{2}\right)+\frac{1}{8}i\csc^2\left(\frac{x}{2}\right)-\frac{1}{8}i\sec^2\left(\frac{x}{2}\right)+\frac{1}{2}i\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{2}i\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(I + Tan[x]),x]
[Out] -1/2*Cot[x/2] + (I/8)*Csc[x/2]^2 - (I/2)*Log[Cos[x/2]] + (I/2)*Log[Sin[x/2]
] - (I/8)*Sec[x/2]^2 - Tan[x/2]/2
```

fricas [B] time = 0.47, size = 73, normalized size = 3.04

$$\frac{\left(-i\,e^{(4ix)}+2i\,e^{(2ix)}-i\right)\log\left(e^{(ix)}+1\right)+\left(i\,e^{(4ix)}-2i\,e^{(2ix)}+i\right)\log\left(e^{(ix)}-1\right)-6i\,e^{(3ix)}+2i\,e^{(ix)}}{2\left(e^{(4ix)}-2\,e^{(2ix)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(I+tan(x)),x, algorithm="fricas")
[Out] 1/2*((-I*e^(4*I*x) + 2*I*e^(2*I*x) - I)*log(e^(I*x) + 1) + (I*e^(4*I*x) - 2
*I*e^(2*I*x) + I)*log(e^(I*x) - 1) - 6*I*e^(3*I*x) + 2*I*e^(I*x))/(e^(4*I*x
) - 2*e^(2*I*x) + 1)
```

giac [B] time = 0.30, size = 46, normalized size = 1.92

$$-\frac{1}{8}i\tan\left(\frac{1}{2}x\right)^2-\frac{6i\tan\left(\frac{1}{2}x\right)^2+4\tan\left(\frac{1}{2}x\right)-i}{8\tan\left(\frac{1}{2}x\right)^2}+\frac{1}{2}i\log\left(\tan\left(\frac{1}{2}x\right)\right)-\frac{1}{2}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/8*I*tan(1/2*x)^2 - 1/8*(6*I*tan(1/2*x)^2 + 4*tan(1/2*x) - I)/tan(1/2*x)^
2 + 1/2*I*log(tan(1/2*x)) - 1/2*tan(1/2*x)
```

maple [B] time = 0.18, size = 42, normalized size = 1.75

$$-\frac{\tan\left(\frac{x}{2}\right)}{2}-\frac{i\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{i}{8\tan\left(\frac{x}{2}\right)^2}+\frac{i\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2}-\frac{1}{2\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(I+tan(x)),x)
```

```
[Out] -1/2*tan(1/2*x)-1/8*I*tan(1/2*x)^2+1/8*I/tan(1/2*x)^2+1/2*I*ln(tan(1/2*x))-
1/2/tan(1/2*x)
```

maxima [B] time = 0.32, size = 59, normalized size = 2.46

$$-\frac{\left(\frac{4\sin(x)}{\cos(x)+1}-i\right)(\cos(x)+1)^2}{8\sin(x)^2}-\frac{\sin(x)}{2(\cos(x)+1)}-\frac{i\sin(x)^2}{8(\cos(x)+1)^2}+\frac{1}{2}i\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*sin(x)/(cos(x) + 1) - I)*(cos(x) + 1)^2/sin(x)^2 - 1/2*sin(x)/(cos(
x) + 1) - 1/8*I*sin(x)^2/(cos(x) + 1)^2 + 1/2*I*log(sin(x)/(cos(x) + 1))
```

mupad [B] time = 3.73, size = 41, normalized size = 1.71

$$-\frac{\tan\left(\frac{x}{2}\right)}{2}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)1i}{2}-\frac{2\tan\left(\frac{x}{2}\right)-\frac{1}{2}i}{4\tan\left(\frac{x}{2}\right)^2}-\frac{\tan\left(\frac{x}{2}\right)^2 1i}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(tan(x) + 1i)),x)
```

```
[Out] (log(tan(x/2))*1i)/2 - tan(x/2)/2 - (2*tan(x/2) - 1i/2)/(4*tan(x/2)^2) - (t
an(x/2)^2*1i)/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^3(x)}{\tan(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(I+tan(x)),x)
```

```
[Out] Integral(csc(x)**3/(tan(x) + I), x)
```

3.8 $\int \frac{\csc^4(x)}{i+\tan(x)} \, dx$

Optimal. Leaf size=19

$$-\frac{\cot^2(x)}{2}+\frac{1}{3} i \cot^3(x)$$

```
[Out] -1/2*cot(x)^2+1/3*I*cot(x)^3
```

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3516, 848, 43}

$$-\frac{\cot^2(x)}{2}+\frac{1}{3} i \cot^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4/(I + Tan[x]),x]
```

```
[Out] -Cot[x]^2/2 + (I/3)*Cot[x]^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^4(x)}{i+\tan(x)} \, dx &= \text{Subst}\left(\int \frac{1+x^2}{x^4(i+x)} \, dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \frac{-i+x}{x^4} \, dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{i}{x^4}+\frac{1}{x^3}\right) dx, x, \tan(x)\right)\\
&= -\frac{1}{2} \cot^2(x)+\frac{1}{3} i \cot^3(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.53

$$-\frac{1}{3} i \cot(x)-\frac{\csc^2(x)}{2}+\frac{1}{3} i \cot(x) \csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(I + Tan[x]),x]
```

```
[Out] (-1/3*I)*Cot[x] - Csc[x]^2/2 + (I/3)*Cot[x]*Csc[x]^2
```

fricas [B] time = 0.46, size = 36, normalized size = 1.89

$$\frac{2\left(6\,e^{(4i\,x)}-3\,e^{(2i\,x)}+1\right)}{3\left(e^{(6i\,x)}-3\,e^{(4i\,x)}+3\,e^{(2i\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] 2/3*(6*e^(4*I*x) - 3*e^(2*I*x) + 1)/(e^(6*I*x) - 3*e^(4*I*x) + 3*e^(2*I*x)
- 1)
```

giac [A] time = 0.29, size = 12, normalized size = 0.63

$$-\frac{3\,\tan(x)-2i}{6\,\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/6*(3*tan(x) - 2*I)/tan(x)^3
```

maple [A] time = 0.18, size = 15, normalized size = 0.79

$$-\frac{1}{2\tan(x)^2}+\frac{i}{3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^4/(I+tan(x)),x)
```

```
[Out] -1/2/tan(x)^2+1/3*I/tan(x)^3
```

maxima [A] time = 0.57, size = 12, normalized size = 0.63

$$-\frac{i\,(-3i\,\tan(x)-2)}{6\,\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] -1/6*I*(-3*I*tan(x) - 2)/tan(x)^3
```

mupad [B] time = 3.62, size = 13, normalized size = 0.68

$$\frac{\cot(x)^2\,\left(-3+\cot(x)\,2\mathrm{i}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^4*(tan(x) + 1i)),x)
```

```
[Out] (cot(x)^2*(cot(x)*2i - 3))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^4(x)}{\tan(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(I+tan(x)),x)
```

```
[Out] Integral(csc(x)**4/(tan(x) + I), x)
```

3.9 $\int \frac{\csc^5(x)}{i+\tan(x)}\,dx$

Optimal. Leaf size=40

$$-\frac{\csc^3(x)}{3}-\frac{1}{8}i\tanh^{-1}(\cos(x))+\frac{1}{4}i\cot(x)\csc^3(x)-\frac{1}{8}i\cot(x)\csc(x)$$

```
[Out] -1/8*I*arctanh(cos(x))-1/8*I*cot(x)*csc(x)-1/3*csc(x)^3+1/4*I*cot(x)*csc(x)
^3
```

Rubi [A] time = 0.15, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3518, 3108, 3107, 2606, 30, 2611, 3768, 3770}

$$-\frac{\csc^3(x)}{3}-\frac{1}{8}i\tanh^{-1}(\cos(x))+\frac{1}{4}i\cot(x)\csc^3(x)-\frac{1}{8}i\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^5/(I + Tan[x]),x]
```

```
[Out] (-I/8)*ArcTanh[Cos[x]] - (I/8)*Cot[x]*Csc[x] - Csc[x]^3/3 + (I/4)*Cot[x]*Cs
c[x]^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^5(x)}{i+\tan(x)}\,dx &= \int \frac{\cot(x)\csc^4(x)}{i\cos(x)+\sin(x)}\,dx \\
&= -\left(i\int \cot(x)\csc^4(x)(\cos(x)+i\sin(x))\,dx\right) \\
&= -\left(i\int \left(i\cot(x)\csc^3(x)+\cot^2(x)\csc^3(x)\right)\,dx\right) \\
&= -\left(i\int \cot^2(x)\csc^3(x)\,dx\right)+\int \cot(x)\csc^3(x)\,dx \\
&= \frac{1}{4}i\cot(x)\csc^3(x)+\frac{1}{4}i\int \csc^3(x)\,dx-\text{Subst}\left(\int x^2\,dx,x,\csc(x)\right) \\
&= -\frac{1}{8}i\cot(x)\csc(x)-\frac{\csc^3(x)}{3}+\frac{1}{4}i\cot(x)\csc^3(x)+\frac{1}{8}i\int \csc(x)\,dx \\
&= -\frac{1}{8}i\tanh^{-1}(\cos(x))-\frac{1}{8}i\cot(x)\csc(x)-\frac{\csc^3(x)}{3}+\frac{1}{4}i\cot(x)\csc^3(x)
\end{aligned}$$

Mathematica [B] time = 0.02, size = 139, normalized size = 3.48

$$-\frac{1}{12}\tan\left(\frac{x}{2}\right)-\frac{1}{12}\cot\left(\frac{x}{2}\right)+\frac{1}{64}i\csc^4\left(\frac{x}{2}\right)-\frac{1}{32}i\csc^2\left(\frac{x}{2}\right)-\frac{1}{64}i\sec^4\left(\frac{x}{2}\right)+\frac{1}{32}i\sec^2\left(\frac{x}{2}\right)+\frac{1}{8}i\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{8}i\log\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^5/(I + Tan[x]),x]
```

```
[Out] -1/12*Cot[x/2] - (I/32)*Csc[x/2]^2 - (Cot[x/2]*Csc[x/2]^2)/24 + (I/64)*Csc[
x/2]^4 - (I/8)*Log[Cos[x/2]] + (I/8)*Log[Sin[x/2]] + (I/32)*Sec[x/2]^2 - (I
/64)*Sec[x/2]^4 - Tan[x/2]/12 - (Sec[x/2]^2*Tan[x/2])/24
```

fricas [B] time = 0.47, size = 121, normalized size = 3.02

$$\frac{\left(-3i\,e^{(8i\,x)}+12i\,e^{(6i\,x)}-18i\,e^{(4i\,x)}+12i\,e^{(2i\,x)}-3i\right)\log\left(e^{(i\,x)}+1\right)+\left(3i\,e^{(8i\,x)}-12i\,e^{(6i\,x)}+18i\,e^{(4i\,x)}-12i\,e^{(2i\,x)}+\right.}{24\left(e^{(8i\,x)}-4\,e^{(6i\,x)}+6\,e^{(4i\,x)}-4\,e^{(2i\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] 1/24*((-3*I*e^(8*I*x) + 12*I*e^(6*I*x) - 18*I*e^(4*I*x) + 12*I*e^(2*I*x) -
3*I)*log(e^(I*x) + 1) + (3*I*e^(8*I*x) - 12*I*e^(6*I*x) + 18*I*e^(4*I*x) -
```

```
12*I*e^(2*I*x) + 3*I)*log(e^(I*x) - 1) + 6*I*e^(7*I*x) + 106*I*e^(5*I*x) -
22*I*e^(3*I*x) + 6*I*e^(I*x))/(e^(8*I*x) - 4*e^(6*I*x) + 6*e^(4*I*x) - 4*e^
(2*I*x) + 1)
```

giac [B] time = 0.20, size = 62, normalized size = 1.55

$$-\frac{1}{64} i \tan\left(\frac{1}{2} x\right)^{4} - \frac{1}{24} \tan\left(\frac{1}{2} x\right)^{3} - \frac{50 i \tan\left(\frac{1}{2} x\right)^{4} + 24 \tan\left(\frac{1}{2} x\right)^{3} + 8 \tan\left(\frac{1}{2} x\right) - 3 i}{192 \tan\left(\frac{1}{2} x\right)^{4}} + \frac{1}{8} i \log\left(\tan\left(\frac{1}{2} x\right)\right) - \frac{1}{8} \tan\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/64*I*tan(1/2*x)^4 - 1/24*tan(1/2*x)^3 - 1/192*(50*I*tan(1/2*x)^4 + 24*ta
n(1/2*x)^3 + 8*tan(1/2*x) - 3*I)/tan(1/2*x)^4 + 1/8*I*log(tan(1/2*x)) - 1/8
*tan(1/2*x)
```

maple [A] time = 0.19, size = 58, normalized size = 1.45

$$-\frac{\tan\left(\frac{x}{2}\right)}{8} - \frac{i\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{64} - \frac{\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{24} - \frac{1}{24 \tan\left(\frac{x}{2}\right)^{3}} + \frac{i}{64 \tan\left(\frac{x}{2}\right)^{4}} + \frac{i \ln\left(\tan\left(\frac{x}{2}\right)\right)}{8} - \frac{1}{8 \tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^5/(I+tan(x)),x)
```

```
[Out] -1/8*tan(1/2*x)-1/64*I*tan(1/2*x)^4-1/24*tan(1/2*x)^3-1/24/tan(1/2*x)^3+1/6
4*I/tan(1/2*x)^4+1/8*I*ln(tan(1/2*x))-1/8/tan(1/2*x)
```

maxima [B] time = 0.51, size = 83, normalized size = 2.08

$$-\frac{\left(\frac{8 \sin(x)}{\cos(x)+1} + \frac{24 \sin(x)^{3}}{(\cos(x)+1)^{3}} - 3i\right)(\cos(x)+1)^{4}}{192 \sin(x)^{4}} - \frac{\sin(x)}{8(\cos(x)+1)} - \frac{\sin(x)^{3}}{24(\cos(x)+1)^{3}} - \frac{i \sin(x)^{4}}{64(\cos(x)+1)^{4}} + \frac{1}{8} i \log\left(\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*sin(x)/(cos(x) + 1) + 24*sin(x)^3/(cos(x) + 1)^3 - 3*I)*(cos(x) +
 1)^4/sin(x)^4 - 1/8*sin(x)/(cos(x) + 1) - 1/24*sin(x)^3/(cos(x) + 1)^3 - 1
/64*I*sin(x)^4/(cos(x) + 1)^4 + 1/8*I*log(sin(x)/(cos(x) + 1))
```

mupad [B] time = 3.70, size = 57, normalized size = 1.42

$$-\frac{\tan\left(\frac{x}{2}\right)}{8} + \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right) 1i}{8} - \frac{2 \tan\left(\frac{x}{2}\right)^{3} + \frac{2 \tan\left(\frac{x}{2}\right)}{3} - \frac{1}{4} i}{16 \tan\left(\frac{x}{2}\right)^{4}} - \frac{\tan\left(\frac{x}{2}\right)^{3}}{24} - \frac{\tan\left(\frac{x}{2}\right)^{4} 1i}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^5*(tan(x) + 1i)),x)
```

```
[Out] (log(tan(x/2))*1i)/8 - tan(x/2)/8 - ((2*tan(x/2))/3 + 2*tan(x/2)^3 - 1i/4)/
(16*tan(x/2)^4) - tan(x/2)^3/24 - (tan(x/2)^4*1i)/64
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{5}(x)}{\tan(x) + i}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**5/(I+tan(x)),x)
```

```
[Out] Integral(csc(x)**5/(tan(x) + I), x)
```

3.10 $\int \frac{\csc^6(x)}{i+\tan(x)}\,dx$

Optimal. Leaf size=37

$$\frac{1}{5}i\cot^5(x) - \frac{\cot^4(x)}{4} + \frac{1}{3}i\cot^3(x) - \frac{\cot^2(x)}{2}$$

```
[Out] -1/2*cot(x)^2+1/3*I*cot(x)^3-1/4*cot(x)^4+1/5*I*cot(x)^5
```

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3516, 848, 75}

$$\frac{1}{5}i\cot^5(x) - \frac{\cot^4(x)}{4} + \frac{1}{3}i\cot^3(x) - \frac{\cot^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^6/(I + Tan[x]),x]
```

```
[Out] -Cot[x]^2/2 + (I/3)*Cot[x]^3 - Cot[x]^4/4 + (I/5)*Cot[x]^5
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^6(x)}{i+\tan(x)}\,dx &= \text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6(i+x)}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \frac{(-i+x)^2(i+x)}{x^6}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{i}{x^6}+\frac{1}{x^5}-\frac{i}{x^4}+\frac{1}{x^3}\right)dx, x, \tan(x)\right)\\
&= -\frac{1}{2}\cot^2(x)+\frac{1}{3}i\cot^3(x)-\frac{\cot^4(x)}{4}+\frac{1}{5}i\cot^5(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.11

$$-\frac{2}{15}i\cot(x) - \frac{\csc^4(x)}{4} + \frac{1}{5}i\cot(x)\csc^4(x) - \frac{1}{15}i\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^6/(I + Tan[x]),x]
```

```
[Out] ((-2*I)/15)*Cot[x] - (I/15)*Cot[x]*Csc[x]^2 - Csc[x]^4/4 + (I/5)*Cot[x]*Csc
[x]^4
```

fricas [B] time = 0.45, size = 54, normalized size = 1.46

$$-\frac{4\left(30\,e^{(6ix)}-10\,e^{(4ix)}+5\,e^{(2ix)}-1\right)}{15\left(e^{(10ix)}-5\,e^{(8ix)}+10\,e^{(6ix)}-10\,e^{(4ix)}+5\,e^{(2ix)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^6/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] -4/15*(30*e^(6*I*x) - 10*e^(4*I*x) + 5*e^(2*I*x) - 1)/(e^(10*I*x) - 5*e^(8*
I*x) + 10*e^(6*I*x) - 10*e^(4*I*x) + 5*e^(2*I*x) - 1)
```

giac [A] time = 0.32, size = 24, normalized size = 0.65

$$-\frac{30\,\tan(x)^3-20i\,\tan(x)^2+15\,\tan(x)-12i}{60\,\tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^6/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/60*(30*tan(x)^3 - 20*I*tan(x)^2 + 15*tan(x) - 12*I)/tan(x)^5
```

maple [A] time = 0.19, size = 28, normalized size = 0.76

$$-\frac{1}{2\tan(x)^2}+\frac{i}{3\tan(x)^3}-\frac{1}{4\tan(x)^4}+\frac{i}{5\tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^6/(I+tan(x)),x)
```

```
[Out] -1/2/tan(x)^2+1/3*I/tan(x)^3-1/4/tan(x)^4+1/5*I/tan(x)^5
```

maxima [A] time = 0.41, size = 24, normalized size = 0.65

$$\frac{i\left(30i\,\tan(x)^3+20\,\tan(x)^2+15i\,\tan(x)+12\right)}{60\,\tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^6/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] 1/60*I*(30*I*tan(x)^3 + 20*tan(x)^2 + 15*I*tan(x) + 12)/tan(x)^5
```

mupad [B] time = 3.60, size = 27, normalized size = 0.73

$$\frac{\cot(x)^5\,1i}{5}-\frac{\cot(x)^4}{4}+\frac{\cot(x)^3\,1i}{3}-\frac{\cot(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^6*(tan(x) + 1i)),x)
```

```
[Out] (cot(x)^3*1i)/3 - cot(x)^2/2 - cot(x)^4/4 + (cot(x)^5*1i)/5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^6(x)}{\tan(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**6/(I+tan(x)),x)
[Out] Integral(csc(x)**6/(tan(x) + I), x)
```

3.11 $\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=101

$$-\frac{a\cos^5(c+dx)}{5d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}-\frac{b\sin^5(c+dx)}{5d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin}{d}$$

```
[Out] b*arctanh(sin(d*x+c))/d-a*cos(d*x+c)/d+2/3*a*cos(d*x+c)^3/d-1/5*a*cos(d*x+c
)^5/d-b*sin(d*x+c)/d-1/3*b*sin(d*x+c)^3/d-1/5*b*sin(d*x+c)^5/d
```

Rubi [A] time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3517, 2633, 2592, 302, 206}

$$-\frac{a\cos^5(c+dx)}{5d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}-\frac{b\sin^5(c+dx)}{5d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d + (2*a*Cos[c + d*x]^3)/(3*
d) - (a*Cos[c + d*x]^5)/(5*d) - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*
d) - (b*Sin[c + d*x]^5)/(5*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx = \int \left(a\sin^5(c+dx)+b\sin^5(c+dx)\tan(c+dx)\right)\,dx$$
$$= a\int \sin^5(c+dx)\,dx + b\int \sin^5(c+dx)\tan(c+dx)\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\cos(c+dx)\right)}{d} + \frac{b\,\text{Subst}\left(\int \frac{x^6}{1-x^2}\,dx,x,\text{si}\right.}{d}$$
$$= -\frac{a\cos(c+dx)}{d} + \frac{2a\cos^3(c+dx)}{3d} - \frac{a\cos^5(c+dx)}{5d} + \frac{b\,\text{Subst}\left(\int \left(-1-x^2\right.\right.}{}$$
$$= -\frac{a\cos(c+dx)}{d} + \frac{2a\cos^3(c+dx)}{3d} - \frac{a\cos^5(c+dx)}{5d} - \frac{b\sin(c+dx)}{d} - \frac{b\,\text{si}}{}$$
$$= \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cos(c+dx)}{d} + \frac{2a\cos^3(c+dx)}{3d} - \frac{a\cos^5(c+dx}{5d}$$

Mathematica [A] time = 0.05, size = 103, normalized size = 1.02

$$-\frac{5a\cos(c+dx)}{8d}+\frac{5a\cos(3(c+dx))}{48d}-\frac{a\cos(5(c+dx))}{80d}-\frac{b\sin^5(c+dx)}{5d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (5*a*Cos[c + d*x])/(8*d) + (5*a*Cos[3*(c + d*
x)])/(48*d) - (a*Cos[5*(c + d*x)])/(80*d) - (b*Sin[c + d*x])/d - (b*Sin[c +
 d*x]^3)/(3*d) - (b*Sin[c + d*x]^5)/(5*d)
```

fricas [A] time = 0.51, size = 97, normalized size = 0.96

$$-\frac{6\,a\cos\left(dx+c\right)^5-20\,a\cos\left(dx+c\right)^3+30\,a\cos\left(dx+c\right)-15\,b\log\left(\sin\left(dx+c\right)+1\right)+15\,b\log\left(-\sin\left(dx+c\right)+\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/30*(6*a*cos(d*x + c)^5 - 20*a*cos(d*x + c)^3 + 30*a*cos(d*x + c) - 15*b*
log(sin(d*x + c) + 1) + 15*b*log(-sin(d*x + c) + 1) + 2*(3*b*cos(d*x + c)^4
 - 11*b*cos(d*x + c)^2 + 23*b)*sin(d*x + c))/d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.32, size = 113, normalized size = 1.12

$$-\frac{8a\cos\left(dx+c\right)}{15d}-\frac{a\cos\left(dx+c\right)\left(\sin^4\left(dx+c\right)\right)}{5d}-\frac{4a\cos\left(dx+c\right)\left(\sin^2\left(dx+c\right)\right)}{15d}-\frac{b\left(\sin^5\left(dx+c\right)\right)}{5d}-\frac{b\left(\sin^3\left(dx+c\right.\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5*(a+b*tan(d*x+c)),x)
```

```
[Out] -8/15*a*cos(d*x+c)/d-1/5/d*a*cos(d*x+c)*sin(d*x+c)^4-4/15/d*a*cos(d*x+c)*si
n(d*x+c)^2-1/5*b*sin(d*x+c)^5/d-1/3*b*sin(d*x+c)^3/d-b*sin(d*x+c)/d+1/d*b*l
n(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.37, size = 91, normalized size = 0.90

$$-\frac{2\left(3\cos\left(dx+c\right)^{5}-10\cos\left(dx+c\right)^{3}+15\cos\left(dx+c\right)\right)a+\left(6\sin\left(dx+c\right)^{5}+10\sin\left(dx+c\right)^{3}-15\log\left(\sin\left(\right.\right.\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*(2*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*a + (6*si
n(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*
x + c) - 1) + 30*sin(d*x + c))*b)/d
```

mupad [B] time = 3.80, size = 121, normalized size = 1.20

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,a\cos\left(c+d\,x\right)^3}{3\,d}-\frac{a\cos\left(c+d\,x\right)^5}{5\,d}-\frac{a\,\cos\left(c+d\,x\right)}{d}-\frac{23\,b\,\sin\left(c+d\,x\right)}{15\,d}+\frac{11\,b\cos\left(c+d\,x\right.}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5*(a + b*tan(c + d*x)),x)
```

```
[Out] (2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*a*cos(c + d*x)^3)
/(3*d) - (a*cos(c + d*x)^5)/(5*d) - (a*cos(c + d*x))/d - (23*b*sin(c + d*x)
)/(15*d) + (11*b*cos(c + d*x)^2*sin(c + d*x))/(15*d) - (b*cos(c + d*x)^4*si
n(c + d*x))/(5*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)\sin^{5}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**5, x)
```

3.12 $\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=83

$$-\frac{\sin^3(c+dx)\cos(c+dx)(a+b\tan(c+dx))}{4d}-\frac{\sin(c+dx)\cos(c+dx)(3a+4b\tan(c+dx))}{8d}+\frac{3ax}{8}-\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] 3/8*a*x-b*ln(cos(d*x+c))/d-1/4*cos(d*x+c)*sin(d*x+c)^3*(a+b*tan(d*x+c))/d-1
/8*cos(d*x+c)*sin(d*x+c)*(3*a+4*b*tan(d*x+c))/d
```

Rubi [A] time = 0.17, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {819, 635, 203, 260}

$$-\frac{\sin^3(c+dx)\cos(c+dx)(a+b\tan(c+dx))}{4d}-\frac{\sin(c+dx)\cos(c+dx)(3a+4b\tan(c+dx))}{8d}+\frac{3ax}{8}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - (b*Log[Cos[c + d*x]])/d - (Cos[c + d*x]*Sin[c + d*x]^3*(a + b*T
an[c + d*x]))/(4*d) - (Cos[c + d*x]*Sin[c + d*x]*(3*a + 4*b*Tan[c + d*x]))/
(8*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+bx)}{(1+x^2)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sin^3(c+dx)(a+b\tan(c+dx))}{4d} + \frac{\text{Subst}\left(\int \frac{x^2(3a+4bx)}{(1+x^2)^2}\,dx,\right.}{4d}$$

$$= -\frac{\cos(c+dx)\sin^3(c+dx)(a+b\tan(c+dx))}{4d} - \frac{\cos(c+dx)\sin(c+dx}{8d}$$

$$= -\frac{\cos(c+dx)\sin^3(c+dx)(a+b\tan(c+dx))}{4d} - \frac{\cos(c+dx)\sin(c+dx}{8d}$$

$$= \frac{3ax}{8} - \frac{b\log(\cos(c+dx))}{d} - \frac{\cos(c+dx)\sin^3(c+dx)(a+b\tan(c+dx)}{4d}$$

Mathematica [A] time = 0.08, size = 82, normalized size = 0.99

$$\frac{3a(c+dx)}{8d} - \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d} - \frac{b\left(\frac{1}{4}\cos^4(c+dx) - \cos^2(c+dx) + \log(\cos(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) - (b*(-Cos[c + d*x]^2 + Cos[c + d*x]^4/4 + Log[Cos[c
+ d*x]]))/d - (a*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

fricas [A] time = 0.46, size = 74, normalized size = 0.89

$$-\frac{2\,b\cos\left(dx+c\right)^4 - 3\,adx - 8\,b\cos\left(dx+c\right)^2 + 8\,b\log\left(-\cos\left(dx+c\right)\right) - \left(2\,a\cos\left(dx+c\right)^3 - 5\,a\cos\left(dx+c\right)\right)\text{s}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*cos(d*x + c)^4 - 3*a*d*x - 8*b*cos(d*x + c)^2 + 8*b*log(-cos(d*x
+ c)) - (2*a*cos(d*x + c)^3 - 5*a*cos(d*x + c))*sin(d*x + c))/d
```

giac [B] time = 0.88, size = 1066, normalized size = 12.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(12*a*d*x*tan(d*x)^4*tan(c)^4 - 16*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*ta
n(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)
/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 + 24*a*d*x*tan(d*x)^4*tan(c)^2 + 24*a*
d*x*tan(d*x)^2*tan(c)^4 + 11*b*tan(d*x)^4*tan(c)^4 - 32*b*log(4*(tan(d*x)^4
*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(
d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^2 + 12*a*tan(d*x)^4*tan(
c)^3 - 32*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*t
an(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*ta
n(c)^4 + 12*a*tan(d*x)^3*tan(c)^4 + 12*a*d*x*tan(d*x)^4 + 48*a*d*x*tan(d*x)
^2*tan(c)^2 + 6*b*tan(d*x)^4*tan(c)^2 - 32*b*tan(d*x)^3*tan(c)^3 + 12*a*d*x
```

```
*tan(c)^4 + 6*b*tan(d*x)^2*tan(c)^4 - 16*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*t
an(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1
)/(tan(c)^2 + 1))*tan(d*x)^4 + 20*a*tan(d*x)^4*tan(c) - 64*b*log(4*(tan(d*x
)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*t
an(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 24*a*tan(d*x)^3*t
an(c)^2 + 24*a*tan(d*x)^2*tan(c)^3 - 16*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*ta
n(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)
/(tan(c)^2 + 1))*tan(c)^4 + 20*a*tan(d*x)*tan(c)^4 + 24*a*d*x*tan(d*x)^2 -
13*b*tan(d*x)^4 - 64*b*tan(d*x)^3*tan(c) + 24*a*d*x*tan(c)^2 - 36*b*tan(d*x
)^2*tan(c)^2 - 64*b*tan(d*x)*tan(c)^3 - 13*b*tan(c)^4 - 32*b*log(4*(tan(d*x
)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*t
an(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2 - 20*a*tan(d*x)^3 - 24*a*tan
(d*x)^2*tan(c) - 32*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + ta
n(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan
(c)^2 - 24*a*tan(d*x)*tan(c)^2 - 20*a*tan(c)^3 + 12*a*d*x + 6*b*tan(d*x)^2
- 32*b*tan(d*x)*tan(c) + 6*b*tan(c)^2 - 16*b*log(4*(tan(d*x)^4*tan(c)^2 - 2
*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
 1)/(tan(c)^2 + 1)) - 12*a*tan(d*x) - 12*a*tan(c) + 11*b)/(d*tan(d*x)^4*tan
(c)^4 + 2*d*tan(d*x)^4*tan(c)^2 + 2*d*tan(d*x)^2*tan(c)^4 + d*tan(d*x)^4 +
4*d*tan(d*x)^2*tan(c)^2 + d*tan(c)^4 + 2*d*tan(d*x)^2 + 2*d*tan(c)^2 + d)
```

maple [A] time = 0.31, size = 92, normalized size = 1.11

$$-\frac{a\cos\left(dx+c\right)\left(\sin^{3}\left(dx+c\right)\right)}{4d}-\frac{3a\cos\left(dx+c\right)\sin\left(dx+c\right)}{8d}+\frac{3ax}{8}+\frac{3ca}{8d}-\frac{b\left(\sin^{4}\left(dx+c\right)\right)}{4d}-\frac{b\left(\sin^{2}\left(dx+c\right)\right)}{2d}-\frac{b\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4*(a+b*tan(d*x+c)),x)
```

```
[Out] -1/4/d*a*cos(d*x+c)*sin(d*x+c)^3-3/8*a*cos(d*x+c)*sin(d*x+c)/d+3/8*a*x+3/8/
d*c*a-1/4/d*b*sin(d*x+c)^4-1/2/d*b*sin(d*x+c)^2-b*ln(cos(d*x+c))/d
```

maxima [A] time = 0.53, size = 87, normalized size = 1.05

$$\frac{3\,(dx+c)a+4\,b\log\left(\tan\left(dx+c\right)^{2}+1\right)-\frac{5\,a\tan(dx+c)^{3}-8\,b\tan(dx+c)^{2}+3\,a\tan(dx+c)-6\,b}{\tan(dx+c)^{4}+2\,\tan(dx+c)^{2}+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*(3*(d*x + c)*a + 4*b*log(tan(d*x + c)^2 + 1) - (5*a*tan(d*x + c)^3 - 8*
b*tan(d*x + c)^2 + 3*a*tan(d*x + c) - 6*b)/(tan(d*x + c)^4 + 2*tan(d*x + c)
^2 + 1))/d
```

mupad [B] time = 3.79, size = 155, normalized size = 1.87

$$\frac{3\,a\,x}{8}+\frac{b\,\ln\left(\tan\left(c+d\,x\right)^{2}+1\right)}{2\,d}+\frac{3\,b}{4\,d\,\left(\tan\left(c+d\,x\right)^{4}+2\,\tan\left(c+d\,x\right)^{2}+1\right)}-\frac{5\,a\,\tan\left(c+d\,x\right)^{3}}{8\,d\,\left(\tan\left(c+d\,x\right)^{4}+2\,\tan\left(c+d\,x\right)^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + b*tan(c + d*x)),x)
```

```
[Out] (3*a*x)/8 + (b*log(tan(c + d*x)^2 + 1))/(2*d) + (3*b)/(4*d*(2*tan(c + d*x)^
2 + tan(c + d*x)^4 + 1)) - (5*a*tan(c + d*x)^3)/(8*d*(2*tan(c + d*x)^2 + ta
n(c + d*x)^4 + 1)) + (b*tan(c + d*x)^2)/(d*(2*tan(c + d*x)^2 + tan(c + d*x)
^4 + 1)) - (3*a*tan(c + d*x))/(8*d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan{(c + dx)})\sin^4{(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**4, x)
```

3.13 $\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=69

$$\frac{a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] b*arctanh(sin(d*x+c))/d-a*cos(d*x+c)/d+1/3*a*cos(d*x+c)^3/d-b*sin(d*x+c)/d-
1/3*b*sin(d*x+c)^3/d
```

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = $\{3517, 2633, 2592, 302, 206\}$

$$\frac{a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d + (a*Cos[c + d*x]^3)/(3*d)
 - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx = \int \left(a\sin^3(c+dx)+b\sin^3(c+dx)\tan(c+dx)\right)\,dx$$
$$= a\int \sin^3(c+dx)\,dx + b\int \sin^3(c+dx)\tan(c+dx)\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos(c+dx)\right)}{d} + \frac{b\,\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx,x,\sin(c$$
$$= -\frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} + \frac{b\,\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)\,dx,x,\right.}{d}$$
$$= -\frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} - \frac{b\sin(c+dx)}{d} - \frac{b\sin^3(c+dx)}{3d} + \frac{b}{}$$
$$= \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} - \frac{b\sin(c+dx}{d}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 1.03

$$-\frac{3a\cos(c+dx)}{4d} + \frac{a\cos(3(c+dx))}{12d} - \frac{b\sin^3(c+dx)}{3d} - \frac{b\sin(c+dx)}{d} + \frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (3*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)
])/(12*d) - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.51, size = 74, normalized size = 1.07

$$\frac{2\,a\cos\left(dx+c\right)^3 - 6\,a\cos\left(dx+c\right) + 3\,b\log\left(\sin\left(dx+c\right)+1\right) - 3\,b\log\left(-\sin\left(dx+c\right)+1\right) + 2\left(b\cos\left(dx+c\right)^2\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*cos(d*x + c)^3 - 6*a*cos(d*x + c) + 3*b*log(sin(d*x + c) + 1) - 3*
b*log(-sin(d*x + c) + 1) + 2*(b*cos(d*x + c)^2 - 4*b)*sin(d*x + c))/d
```

giac [B] time = 31.09, size = 5350, normalized size = 77.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c)
+ 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2
*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
 tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(
1/2*d*x)^6*tan(1/2*c)^6 - 3*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/
2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2
+ 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan
(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 4*a*tan(1/2*d*x)^6*tan(1/2*c)
^6 + 9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*
```

```
c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1
/2*d*x)^6*tan(1/2*c)^4 - 9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2
*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(
1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 12*b*tan(1/2*d*x)^6*tan(1/2*c)
^5 + 9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1
/2*d*x)^4*tan(1/2*c)^6 - 9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2
*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(
1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 12*b*tan(1/2*d*x)^5*tan(1/2*c)
^6 + 12*a*tan(1/2*d*x)^6*tan(1/2*c)^4 + 12*a*tan(1/2*d*x)^4*tan(1/2*c)^6 +
9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*ta
n(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1
/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*
x)^6*tan(1/2*c)^2 - 9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)
^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c
)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 40*b*tan(1/2*d*x)^6*tan(1/2*c)^3 +
27*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*t
an(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(
1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d
*x)^4*tan(1/2*c)^4 - 27*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 60*b*tan(1/2*d*x)^5*tan(1/2*c)^4
- 60*b*tan(1/2*d*x)^4*tan(1/2*c)^5 + 9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*
x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c
) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 9*b*log(2*(tan(1/2
*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1
/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3
 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6
- 40*b*tan(1/2*d*x)^3*tan(1/2*c)^6 - 12*a*tan(1/2*d*x)^6*tan(1/2*c)^2 - 96*
a*tan(1/2*d*x)^5*tan(1/2*c)^3 - 108*a*tan(1/2*d*x)^4*tan(1/2*c)^4 - 96*a*ta
n(1/2*d*x)^3*tan(1/2*c)^5 - 12*a*tan(1/2*d*x)^2*tan(1/2*c)^6 + 3*b*log(2*(t
an(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3
*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*
tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6 - 3*b*l
og(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2
*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6
- 12*b*tan(1/2*d*x)^6*tan(1/2*c) + 27*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2
+ 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x
)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c)
```

+ 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 - 27*b*log(2*(tan(1/2
*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1
/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3
 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2
+ 60*b*tan(1/2*d*x)^5*tan(1/2*c)^2 + 120*b*tan(1/2*d*x)^4*tan(1/2*c)^3 + 27
*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan
(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/
2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x
)^2*tan(1/2*c)^4 - 27*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)
^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c
)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 120*b*tan(1/2*d*x)^3*tan(1/2*c)^4 +
 60*b*tan(1/2*d*x)^2*tan(1/2*c)^5 + 3*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2
+ 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x
)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c)
 + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^6 - 3*b*log(2*(tan(1/2*d*x)^4*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/
2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x
)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1
/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^6 - 12*b*tan(1/2*d*x)*tan(1/2*c)^
6 - 4*a*tan(1/2*d*x)^6 + 108*a*tan(1/2*d*x)^4*tan(1/2*c)^2 + 128*a*tan(1/2*
d*x)^3*tan(1/2*c)^3 + 108*a*tan(1/2*d*x)^2*tan(1/2*c)^4 - 4*a*tan(1/2*c)^6
+ 9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*
tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan
(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*
d*x)^4 - 9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*
c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*t
an(1/2*d*x)^4 + 12*b*tan(1/2*d*x)^5 - 60*b*tan(1/2*d*x)^4*tan(1/2*c) + 27*b
log(2(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1
/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^
2*tan(1/2*c)^2 - 27*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4
*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*
x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^
2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 120*b*tan(1/2*d*x)^3*tan(1/2*c)^2 - 1
20*b*tan(1/2*d*x)^2*tan(1/2*c)^3 + 9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)
^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c)
+ 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^4 - 9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c
)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2
*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/
2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^4 - 60*b*tan(1/2*d*x)*tan(1/2*c)^4
 + 12*b*tan(1/2*c)^5 - 12*a*tan(1/2*d*x)^4 - 96*a*tan(1/2*d*x)^3*tan(1/2*c)
 - 108*a*tan(1/2*d*x)^2*tan(1/2*c)^2 - 96*a*tan(1/2*d*x)*tan(1/2*c)^3 - 12*
a*tan(1/2*c)^4 + 9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*
tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2

```
 + 1))*tan(1/2*d*x)^2 - 9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(
1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 +
2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1
/2*c)^2 + 1))*tan(1/2*d*x)^2 + 40*b*tan(1/2*d*x)^3 + 60*b*tan(1/2*d*x)^2*ta
n(1/2*c) + 9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/
2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))
*tan(1/2*c)^2 - 9*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*t
an(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2
+ 1))*tan(1/2*c)^2 + 60*b*tan(1/2*d*x)*tan(1/2*c)^2 + 40*b*tan(1/2*c)^3 + 1
2*a*tan(1/2*d*x)^2 + 12*a*tan(1/2*c)^2 + 3*b*log(2*(tan(1/2*d*x)^4*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/
2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x
)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1
/2*c) + 1)/(tan(1/2*c)^2 + 1)) - 3*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4
 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) +
1)/(tan(1/2*c)^2 + 1)) + 12*b*tan(1/2*d*x) + 12*b*tan(1/2*c) + 4*a)/(d*tan(
1/2*d*x)^6*tan(1/2*c)^6 + 3*d*tan(1/2*d*x)^6*tan(1/2*c)^4 + 3*d*tan(1/2*d*x
)^4*tan(1/2*c)^6 + 3*d*tan(1/2*d*x)^6*tan(1/2*c)^2 + 9*d*tan(1/2*d*x)^4*tan
(1/2*c)^4 + 3*d*tan(1/2*d*x)^2*tan(1/2*c)^6 + d*tan(1/2*d*x)^6 + 9*d*tan(1/
2*d*x)^4*tan(1/2*c)^2 + 9*d*tan(1/2*d*x)^2*tan(1/2*c)^4 + d*tan(1/2*c)^6 +
3*d*tan(1/2*d*x)^4 + 9*d*tan(1/2*d*x)^2*tan(1/2*c)^2 + 3*d*tan(1/2*c)^4 + 3
*d*tan(1/2*d*x)^2 + 3*d*tan(1/2*c)^2 + d)
```

maple [A] time = 0.30, size = 79, normalized size = 1.14

$$-\frac{a\cos(dx+c)\left(\sin^2(dx+c)\right)}{3d}-\frac{2a\cos(dx+c)}{3d}-\frac{b\left(\sin^3(dx+c)\right)}{3d}-\frac{b\sin(dx+c)}{d}+\frac{b\ln(\sec(dx+c)+\tan(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c)),x)
```

```
[Out] -1/3/d*a*cos(d*x+c)*sin(d*x+c)^2-2/3*a*cos(d*x+c)/d-1/3*b*sin(d*x+c)^3/d-b*
sin(d*x+c)/d+1/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.58, size = 70, normalized size = 1.01

$$\frac{2\left(\cos(dx+c)^3-3\cos(dx+c)\right)a-\left(2\sin(dx+c)^3-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)+6\sin\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(cos(d*x + c)^3 - 3*cos(d*x + c))*a - (2*sin(d*x + c)^3 - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*b)/d
```

mupad [B] time = 3.83, size = 87, normalized size = 1.26

$$\frac{2b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}+\frac{a\cos(c+dx)^3}{3d}-\frac{a\cos(c+dx)}{d}-\frac{4b\sin(c+dx)}{3d}+\frac{b\cos(c+dx)^2\sin(c+dx)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b*tan(c + d*x)),x)
```

```
[Out] (2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a*cos(c + d*x)^3)/(
3*d) - (a*cos(c + d*x))/d - (4*b*sin(c + d*x))/(3*d) + (b*cos(c + d*x)^2*si
n(c + d*x))/(3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan (c + dx)) \sin^3 (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**3, x)
```

3.14 $\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=49

$$-\frac{\sin(c+dx)\cos(c+dx)(a+b\tan(c+dx))}{2d}+\frac{ax}{2}-\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] 1/2*a*x-b*ln(cos(d*x+c))/d-1/2*cos(d*x+c)*sin(d*x+c)*(a+b*tan(d*x+c))/d
```

Rubi [A] time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {819, 635, 203, 260}

$$-\frac{\sin(c+dx)\cos(c+dx)(a+b\tan(c+dx))}{2d}+\frac{ax}{2}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*x)/2 - (b*Log[Cos[c + d*x]])/d - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c
 + d*x]))/(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx)}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))}{2d} + \frac{\text{Subst}\left(\int \frac{a+2bx}{1+x^2}\,dx, x, \text{ta}\right.}{2d}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))}{2d} + \frac{a\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \text{ta}\right.}{2d}$$

$$= \frac{ax}{2} - \frac{b\log(\cos(c+dx))}{d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))}{2d}$$

Mathematica [A] time = 0.05, size = 56, normalized size = 1.14

$$\frac{a(c+dx)}{2d} - \frac{a\sin(2(c+dx))}{4d} - \frac{b\left(\log(\cos(c+dx)) - \frac{1}{2}\cos^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x]),x]
[Out] (a*(c + d*x))/(2*d) - (b*(-1/2*Cos[c + d*x]^2 + Log[Cos[c + d*x]]))/d - (a*
Sin[2*(c + d*x)])/(4*d)
```

fricas [A] time = 0.54, size = 47, normalized size = 0.96

$$\frac{adx + b\cos(dx+c)^2 - a\cos(dx+c)\sin(dx+c) - 2\,b\log(-\cos(dx+c))}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(a*d*x + b*cos(d*x + c)^2 - a*cos(d*x + c)*sin(d*x + c) - 2*b*log(-cos(
d*x + c)))/d
```

giac [B] time = 0.54, size = 413, normalized size = 8.43

$$2\,adx\tan(dx)^2\tan(c)^2 - 2\,b\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2 - 2\tan(dx)^3\tan(c) + \tan(dx)^2\tan(c)^2 + \tan(dx)^2 - 2\tan(dx)\tan(c) + 1\right)}{\tan(c)^2+1}\right)\tan(dx)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] 1/4*(2*a*d*x*tan(d*x)^2*tan(c)^2 - 2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d
*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(t
an(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 2*a*d*x*tan(d*x)^2 + 2*a*d*x*tan(c)^2 +
 b*tan(d*x)^2*tan(c)^2 - 2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(
c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 +
1))*tan(d*x)^2 + 2*a*tan(d*x)^2*tan(c) - 2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2
*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
 1)/(tan(c)^2 + 1))*tan(c)^2 + 2*a*tan(d*x)*tan(c)^2 + 2*a*d*x - b*tan(d*x)
^2 - 4*b*tan(d*x)*tan(c) - b*tan(c)^2 - 2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*
tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
1)/(tan(c)^2 + 1)) - 2*a*tan(d*x) - 2*a*tan(c) + b)/(d*tan(d*x)^2*tan(c)^2
+ d*tan(d*x)^2 + d*tan(c)^2 + d)
```

maple [A] time = 0.18, size = 58, normalized size = 1.18

$$-\frac{a\cos(dx+c)\sin(dx+c)}{2d}+\frac{ax}{2}+\frac{ca}{2d}-\frac{b\left(\sin^2(dx+c)\right)}{2d}-\frac{b\ln(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c)),x)
```

```
[Out] -1/2*a*cos(d*x+c)*sin(d*x+c)/d+1/2*a*x+1/2/d*c*a-1/2/d*b*sin(d*x+c)^2-b*ln(
cos(d*x+c))/d
```

maxima [A] time = 0.57, size = 52, normalized size = 1.06

$$\frac{(dx+c)a+b\log\left(\tan(dx+c)^2+1\right)-\frac{a\tan(dx+c)-b}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((d*x + c)*a + b*log(tan(d*x + c)^2 + 1) - (a*tan(d*x + c) - b)/(tan(d*
x + c)^2 + 1))/d
```

mupad [B] time = 3.82, size = 50, normalized size = 1.02

$$\frac{\frac{b\cos(c+d\,x)^2}{2}-\frac{a\,\sin(c+d\,x)\,\cos(c+d\,x)}{2}+\frac{b\,\ln\left(\tan(c+d\,x)^2+1\right)}{2}+\frac{a\,d\,x}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b*tan(c + d*x)),x)
```

```
[Out] ((b*log(tan(c + d*x)^2 + 1))/2 + (b*cos(c + d*x)^2)/2 - (a*cos(c + d*x)*sin
(c + d*x))/2 + (a*d*x)/2)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\tan(c+dx))\sin^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**2, x)
```

3.15 $\int \sin(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=37

$$-\frac{a\cos(c+dx)}{d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] b*arctanh(sin(d*x+c))/d-a*cos(d*x+c)/d-b*sin(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3517, 2638, 2592, 321, 206}

$$-\frac{a\cos(c+dx)}{d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d - (b*Sin[c + d*x])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\int \sin(c+dx)(a+b\tan(c+dx))\,dx = \int (a\sin(c+dx)+b\sin(c+dx)\tan(c+dx))\,dx$$
$$= a\int \sin(c+dx)\,dx + b\int \sin(c+dx)\tan(c+dx)\,dx$$
$$= -\frac{a\cos(c+dx)}{d} + \frac{b\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= -\frac{a\cos(c+dx)}{d} - \frac{b\sin(c+dx)}{d} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cos(c+dx)}{d} - \frac{b\sin(c+dx)}{d}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 1.30

$$\frac{a\sin(c)\sin(dx)}{d} - \frac{a\cos(c)\cos(dx)}{d} - \frac{b\sin(c+dx)}{d} + \frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x]),x]
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c]*Cos[d*x])/d + (a*Sin[c]*Sin[d*x])/d
 - (b*Sin[c + d*x])/d
```

fricas [A] time = 0.45, size = 49, normalized size = 1.32

$$-\frac{2\,a\cos(dx+c) - b\log(\sin(dx+c)+1) + b\log(-\sin(dx+c)+1) + 2\,b\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
[Out] -1/2*(2*a*cos(d*x + c) - b*log(sin(d*x + c) + 1) + b*log(-sin(d*x + c) + 1)
 + 2*b*sin(d*x + c))/d
```

giac [B] time = 0.80, size = 1236, normalized size = 33.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] -1/2*(b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + t
an(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^2*tan(1/2*c)^2 - b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan
(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/
2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x
)^2 - b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) -
```

```
2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^2 - 4*b*tan(1/2*d*x)^2*tan(1/2*c) + b*log(2*(tan(1/2*d*x)^4*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/
2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x
)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1
/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^2 - b*log(2*(tan(1/2*d*x)^4*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan
(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*
d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*ta
n(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^2 - 4*b*tan(1/2*d*x)*tan(1/2*c
)^2 - 2*a*tan(1/2*d*x)^2 - 8*a*tan(1/2*d*x)*tan(1/2*c) - 2*a*tan(1/2*c)^2 +
 b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan
(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/
2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1)) - b*log(2*(
tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^
3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2
*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1)) + 4*b*tan(1/2*d*x) +
4*b*tan(1/2*c) + 2*a)/(d*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(1/2*d*x)^2 + d
*tan(1/2*c)^2 + d)
```

maple [A] time = 0.18, size = 45, normalized size = 1.22

$$-\frac{a\cos\left(dx+c\right)}{d}-\frac{b\sin\left(dx+c\right)}{d}+\frac{b\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*tan(d*x+c)),x)
```

```
[Out] -a*cos(d*x+c)/d-b*sin(d*x+c)/d+1/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.41, size = 46, normalized size = 1.24

$$\frac{b\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\sin\left(dx+c\right)\right)-2\,a\cos\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) - 2
*a*cos(d*x + c))/d
```

mupad [B] time = 3.82, size = 53, normalized size = 1.43

$$\frac{2\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*tan(c + d*x)),x)
```

```
[Out] (2*b*atanh(tan(c/2 + (d*x)/2)))/d - (2*a + 2*b*tan(c/2 + (d*x)/2))/(d*(tan(
c/2 + (d*x)/2)^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)\sin\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x), x)
```

3.16 $\int \csc(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=26

$$\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -a*arctanh(cos(d*x+c))/d+b*arctanh(sin(d*x+c))/d
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3517, 3770}

$$\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (b*ArcTanh[Sin[c + d*x]])/d
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)(a+b\tan(c+dx))\,dx &= \int (a\csc(c+dx)+b\sec(c+dx))\,dx\\ &= a\int \csc(c+dx)\,dx+b\int \sec(c+dx)\,dx\\ &= -\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 2.00

$$\frac{a\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Log[Cos[c/2 + (d*x)/2]])/d + (a*Log[Sin[c/
2 + (d*x)/2]])/d
```

fricas [B] time = 0.52, size = 58, normalized size = 2.23

$$-\frac{a\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-a\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-b\log\left(\sin\left(dx+c\right)+1\right)+b\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*log(1/2*cos(d*x + c) + 1/2) - a*log(-1/2*cos(d*x + c) + 1/2) - b*lo
g(sin(d*x + c) + 1) + b*log(-sin(d*x + c) + 1))/d
```

giac [A] time = 0.49, size = 49, normalized size = 1.88

$$\frac{b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) + a*log(abs(tan(1/2*d*x + 1/2*c))))/d
```

maple [A] time = 0.28, size = 42, normalized size = 1.62

$$\frac{b\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{a\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.54, size = 46, normalized size = 1.77

$$\frac{b\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-2\,a\log\left(\cot\left(dx+c\right)+\csc\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 2*a*log(cot(d*x +
c) + csc(d*x + c)))/d
```

mupad [B] time = 3.75, size = 86, normalized size = 3.31

$$\frac{a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{2\,b\,\mathrm{atanh}\left(\frac{b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/sin(c + d*x),x)
```

```
[Out] (a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (2*b*atanh((b*cos(c/2 +
(d*x)/2) - a*sin(c/2 + (d*x)/2))/(a*cos(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/
2))))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x), x)
```

3.17 $\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=25

$$\frac{b\log(\tan(c+dx))}{d}-\frac{a\cot(c+dx)}{d}$$

```
[Out] -a*cot(d*x+c)/d+b*ln(tan(d*x+c))/d
```

Rubi [A] time = 0.08, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{b\log(\tan(c+dx))}{d}-\frac{a\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) + (b*Log[Tan[c + d*x]])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{a+bx}{x^2}\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a}{x^2}+\frac{b}{x}\right)dx,x,\tan(c+dx)\right)}{d}\\ &= -\frac{a\cot(c+dx)}{d}+\frac{b\log(\tan(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 36, normalized size = 1.44

$$-\frac{a\cot(c+dx)}{d}-\frac{b(\log(\cos(c+dx))-\log(\sin(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (b*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]]))/d
```

fricas [B] time = 0.55, size = 62, normalized size = 2.48

$$-\frac{b\log\left(\cos(dx+c)^2\right)\sin(dx+c)-b\log\left(-\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\right)\sin(dx+c)+2\,a\cos(dx+c)}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*log(cos(d*x + c)^2)*sin(d*x + c) - b*log(-1/4*cos(d*x + c)^2 + 1/4)
*sin(d*x + c) + 2*a*cos(d*x + c))/(d*sin(d*x + c))
```

giac [A] time = 0.39, size = 35, normalized size = 1.40

$$\frac{b\log\left(|\tan\left(dx+c\right)|\right)-\frac{b\tan(dx+c)+a}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (b*log(abs(tan(d*x + c))) - (b*tan(d*x + c) + a)/tan(d*x + c))/d
```

maple [A] time = 0.37, size = 26, normalized size = 1.04

$$-\frac{a\cot\left(dx+c\right)}{d}+\frac{b\ln\left(\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c)),x)
```

```
[Out] -a*cot(d*x+c)/d+b*ln(tan(d*x+c))/d
```

maxima [A] time = 0.32, size = 25, normalized size = 1.00

$$\frac{b\log\left(\tan\left(dx+c\right)\right)-\frac{a}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b*log(tan(d*x + c)) - a/tan(d*x + c))/d
```

mupad [B] time = 3.63, size = 25, normalized size = 1.00

$$\frac{b\,\ln\left(\tan\left(c+d\,x\right)\right)}{d}-\frac{a\cot\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/sin(c + d*x)^2,x)
```

```
[Out] (b*log(tan(c + d*x)))/d - (a*cot(c + d*x))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)\csc^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x)**2, x)
```

3.18 $\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=60

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-\frac{b\csc(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] -1/2*a*arctanh(cos(d*x+c))/d+b*arctanh(sin(d*x+c))/d-b*csc(d*x+c)/d-1/2*a*c
ot(d*x+c)*csc(d*x+c)/d
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3517, 3768, 3770, 2621, 321, 207}

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-\frac{b\csc(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] -(a*ArcTanh[Cos[c + d*x]])/(2*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Csc[c +
 d*x])/d - (a*Cot[c + d*x]*Csc[c + d*x])/(2*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx = \int \left(a\csc^3(c+dx)+b\csc^2(c+dx)\sec(c+dx)\right)\,dx$$
$$= a\int \csc^3(c+dx)\,dx + b\int \csc^2(c+dx)\sec(c+dx)\,dx$$
$$= -\frac{a\cot(c+dx)\csc(c+dx)}{2d}+\frac{1}{2}a\int \csc(c+dx)\,dx-\frac{b\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,\right.}{d}$$
$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b\csc(c+dx)}{d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-\frac{b\,\text{S}}{}$$
$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{2d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{b\csc(c+dx)}{d}-\frac{a\cot(}{}$$

Mathematica [C] time = 0.03, size = 107, normalized size = 1.78

$$-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{b\csc(c+dx)\,{}_2F_1\left(-\frac{1}{2}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/8*(a*Csc[(c + d*x)/2]^2)/d - (b*Csc[c + d*x]*Hypergeometric2F1[-1/2, 1,
1/2, Sin[c + d*x]^2])/d - (a*Log[Cos[(c + d*x)/2]])/(2*d) + (a*Log[Sin[(c +
 d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

fricas [B] time = 0.46, size = 142, normalized size = 2.37

$$\frac{2\,a\cos(dx+c)-\left(a\cos(dx+c)^2-a\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+\left(a\cos(dx+c)^2-a\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{4\left(d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*cos(d*x + c) - (a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2)
+ (a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2) + 2*(b*cos(d*x + c)^2
 - b)*log(sin(d*x + c) + 1) - 2*(b*cos(d*x + c)^2 - b)*log(-sin(d*x + c) +
1) + 4*b*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

giac [B] time = 0.51, size = 118, normalized size = 1.97

$$\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+8\,b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-8\,b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+4\,a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 + 8*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*
b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a*log(abs(tan(1/2*d*x + 1/2*c))) -
```

```
4*b*tan(1/2*d*x + 1/2*c) - (6*a*tan(1/2*d*x + 1/2*c)^2 + 4*b*tan(1/2*d*x +
1/2*c) + a)/tan(1/2*d*x + 1/2*c)^2)/d
```

maple [A] time = 0.39, size = 75, normalized size = 1.25

$$-\frac{a\cot(dx+c)\csc(dx+c)}{2d}+\frac{a\ln(\csc(dx+c)-\cot(dx+c))}{2d}-\frac{b}{d\sin(dx+c)}+\frac{b\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c)),x)
```

```
[Out] -1/2*a*cot(d*x+c)*csc(d*x+c)/d+1/2/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/d*b/sin(
d*x+c)+1/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.54, size = 83, normalized size = 1.38

$$\frac{a\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-2b\left(\frac{2}{\sin(dx+c)}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + log(c
os(d*x + c) - 1)) - 2*b*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d
*x + c) - 1)))/d
```

mupad [B] time = 3.72, size = 149, normalized size = 2.48

$$\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8d}-\frac{\frac{a}{2}+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}-\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d}-\frac{2b\,\mathrm{atanh}\left(\frac{4b^2}{2ab-4b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{2ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ab-4b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}+\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/sin(c + d*x)^3,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (a/2 + 2*b*tan(c/2 + (d*x)/2))/(4*d*tan(c/
2 + (d*x)/2)^2) - (b*tan(c/2 + (d*x)/2))/(2*d) - (2*b*atanh((4*b^2)/(2*a*b
- 4*b^2*tan(c/2 + (d*x)/2)) - (2*a*b*tan(c/2 + (d*x)/2))/(2*a*b - 4*b^2*tan
(c/2 + (d*x)/2))))/d + (a*log(tan(c/2 + (d*x)/2)))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\tan(c+dx))\csc^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x)**3, x)
```

3.19 $\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=57

$$-\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{b\cot^2(c+dx)}{2d}+\frac{b\log(\tan(c+dx))}{d}$$

```
[Out] -a*cot(d*x+c)/d-1/2*b*cot(d*x+c)^2/d-1/3*a*cot(d*x+c)^3/d+b*ln(tan(d*x+c))/
d
```

Rubi [A] time = 0.09, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {766}

$$-\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{b\cot^2(c+dx)}{2d}+\frac{b\log(\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (b*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(3*d)
 + (b*Log[Tan[c + d*x]])/d
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)(1+x^2)}{x^4}\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a}{x^4}+\frac{b}{x^3}+\frac{a}{x^2}+\frac{b}{x}\right)dx, x, \tan(c+dx)\right)}{d}\\ &= -\frac{a\cot(c+dx)}{d}-\frac{b\cot^2(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+\frac{b\log(\tan(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.26, size = 72, normalized size = 1.26

$$-\frac{2a\cot(c+dx)}{3d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}-\frac{b\left(\csc^2(c+dx)-2\log(\sin(c+dx))+2\log(\cos(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] (-2*a*Cot[c + d*x])/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d) - (b*(Csc
[c + d*x]^2 + 2*Log[Cos[c + d*x]] - 2*Log[Sin[c + d*x]]))/(2*d)
```

fricas [B] time = 0.52, size = 122, normalized size = 2.14

$$-\frac{4\,a\cos(dx+c)^3+3\left(b\cos(dx+c)^2-b\right)\log\left(\cos(dx+c)^2\right)\sin(dx+c)-3\left(b\cos(dx+c)^2-b\right)\log\left(-\frac{1}{4}\cos(\ldots\right.}{6\left(d\cos(dx+c)^2-d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(4*a*cos(d*x + c)^3 + 3*(b*cos(d*x + c)^2 - b)*log(cos(d*x + c)^2)*sin
(d*x + c) - 3*(b*cos(d*x + c)^2 - b)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x
 + c) - 6*a*cos(d*x + c) - 3*b*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*
x + c))
```

giac [A] time = 0.50, size = 62, normalized size = 1.09

$$\frac{6\,b\log\left(|\tan\left(dx+c\right)|\right)-\frac{11\,b\tan\left(dx+c\right)^{3}+6\,a\tan\left(dx+c\right)^{2}+3\,b\tan\left(dx+c\right)+2\,a}{\tan\left(dx+c\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*b*log(abs(tan(d*x + c))) - (11*b*tan(d*x + c)^3 + 6*a*tan(d*x + c)^2
 + 3*b*tan(d*x + c) + 2*a)/tan(d*x + c)^3)/d
```

maple [A] time = 0.38, size = 60, normalized size = 1.05

$$-\frac{2a\cot\left(dx+c\right)}{3d}-\frac{a\cot\left(dx+c\right)\left(\csc^{2}\left(dx+c\right)\right)}{3d}-\frac{b}{2d\sin\left(dx+c\right)^{2}}+\frac{b\ln\left(\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c)),x)
```

```
[Out] -2/3*a*cot(d*x+c)/d-1/3/d*a*cot(d*x+c)*csc(d*x+c)^2-1/2/d*b/sin(d*x+c)^2+b*
ln(tan(d*x+c))/d
```

maxima [A] time = 0.52, size = 50, normalized size = 0.88

$$\frac{6\,b\log\left(\tan\left(dx+c\right)\right)-\frac{6\,a\tan\left(dx+c\right)^{2}+3\,b\tan\left(dx+c\right)+2\,a}{\tan\left(dx+c\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*b*log(tan(d*x + c)) - (6*a*tan(d*x + c)^2 + 3*b*tan(d*x + c) + 2*a)/
tan(d*x + c)^3)/d
```

mupad [B] time = 3.67, size = 49, normalized size = 0.86

$$\frac{b\,\ln\left(\tan\left(c+d\,x\right)\right)}{d}-\frac{a\,{\tan\left(c+d\,x\right)}^2+\frac{b\,\tan\left(c+d\,x\right)}{2}+\frac{a}{3}}{d\,{\tan\left(c+d\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/sin(c + d*x)^4,x)
```

```
[Out] (b*log(tan(c + d*x)))/d - (a/3 + (b*tan(c + d*x))/2 + a*tan(c + d*x)^2)/(d*
tan(c + d*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)\csc^{4}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x)**4, x)
```

3.20 $\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=98

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx}{d}$$

```
[Out] -3/8*a*arctanh(cos(d*x+c))/d+b*arctanh(sin(d*x+c))/d-b*csc(d*x+c)/d-3/8*a*c
ot(d*x+c)*csc(d*x+c)/d-1/3*b*csc(d*x+c)^3/d-1/4*a*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A] time = 0.09, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3517, 3768, 3770, 2621, 302, 207}

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(8*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Csc[c
 + d*x])/d - (3*a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (b*Csc[c + d*x]^3)/(3*
d) - (a*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx = \int \left(a\csc^5(c+dx)+b\csc^4(c+dx)\sec(c+dx)\right)dx$$
$$= a\int \csc^5(c+dx)\,dx + b\int \csc^4(c+dx)\sec(c+dx)\,dx$$
$$= -\frac{a\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{1}{4}(3a)\int \csc^3(c+dx)\,dx - \frac{b\,\mathrm{Subst}\left(\int \frac{x^4}{-1+x}\right.}{}$$
$$= -\frac{3a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{1}{8}(3a)\int \csc(c$$
$$= -\frac{3a\tanh^{-1}(\cos(c+dx))}{8d} - \frac{b\csc(c+dx)}{d} - \frac{3a\cot(c+dx)\csc(c+dx)}{8d} - b$$
$$= -\frac{3a\tanh^{-1}(\cos(c+dx))}{8d} + \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{b\csc(c+dx)}{d} - 3a\,c$$

Mathematica [C] time = 0.04, size = 151, normalized size = 1.54

$$-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} - 3a$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] (-3*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (b*Csc[c
 + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[c + d*x]^2])/(3*d) - (3*a*Lo
g[Cos[(c + d*x)/2]])/(8*d) + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) + (3*a*Sec[(
c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

fricas [B] time = 0.51, size = 213, normalized size = 2.17

$$18\,a\cos(dx+c)^3 - 30\,a\cos(dx+c) - 9\left(a\cos(dx+c)^4 - 2\,a\cos(dx+c)^2 + a\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) + 9\left(a\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(18*a*cos(d*x + c)^3 - 30*a*cos(d*x + c) - 9*(a*cos(d*x + c)^4 - 2*a*c
os(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2) + 9*(a*cos(d*x + c)^4 - 2*a*
cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2) + 24*(b*cos(d*x + c)^4 - 2
*b*cos(d*x + c)^2 + b)*log(sin(d*x + c) + 1) - 24*(b*cos(d*x + c)^4 - 2*b*c
os(d*x + c)^2 + b)*log(-sin(d*x + c) + 1) + 16*(3*b*cos(d*x + c)^2 - 4*b)*s
in(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 2.93, size = 177, normalized size = 1.81

$$3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 8\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 192\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 192\,b\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 - 8*b*tan(1/2*d*x + 1/2*c)^3 + 24*a*tan(1
/2*d*x + 1/2*c)^2 + 192*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 192*b*log(ab
s(tan(1/2*d*x + 1/2*c) - 1)) + 72*a*log(abs(tan(1/2*d*x + 1/2*c))) - 120*b*
tan(1/2*d*x + 1/2*c) - (150*a*tan(1/2*d*x + 1/2*c)^4 + 120*b*tan(1/2*d*x +
1/2*c)^3 + 24*a*tan(1/2*d*x + 1/2*c)^2 + 8*b*tan(1/2*d*x + 1/2*c) + 3*a)/ta
n(1/2*d*x + 1/2*c)^4)/d
```

maple [A] time = 0.39, size = 109, normalized size = 1.11

$$-\frac{a\cot(dx+c)\left(\csc^3(dx+c)\right)}{4d}-\frac{3a\cot(dx+c)\csc(dx+c)}{8d}+\frac{3a\ln(\csc(dx+c)-\cot(dx+c))}{8d}-\frac{b}{3d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+b*tan(d*x+c)),x)
```

```
[Out] -1/4*a*cot(d*x+c)*csc(d*x+c)^3/d-3/8*a*cot(d*x+c)*csc(d*x+c)/d+3/8/d*a*ln(c
sc(d*x+c)-cot(d*x+c))-1/3/d*b/sin(d*x+c)^3-1/d*b/sin(d*x+c)+1/d*b*ln(sec(d*
x+c)+tan(d*x+c))
```

maxima [A] time = 0.51, size = 123, normalized size = 1.26

$$\frac{3a\left(\frac{2\left(3\cos(dx+c)^3-5\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)-8b\left(\frac{2\left(3\sin(dx+c)^2+1\right)}{\sin(dx+c)^3}-3\log\right.}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(3*a*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*
x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 8*b*(2
*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(si
n(d*x + c) - 1)))/d
```

mupad [B] time = 3.85, size = 211, normalized size = 2.15

$$\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8d}-\frac{5b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d}-\frac{2b\,\mathrm{atanh}\left(\frac{4b^2}{\frac{3ab}{2}-4b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{3ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\left(\frac{3ab}{2}-4b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}\right)}{d}+\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64d}-\frac{b\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/sin(c + d*x)^5,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (5*b*tan(c/2 + (d*x)/2))/(8*d) - (2*b*atan
h((4*b^2)/((3*a*b)/2 - 4*b^2*tan(c/2 + (d*x)/2)) - (3*a*b*tan(c/2 + (d*x)/2
))/(2*((3*a*b)/2 - 4*b^2*tan(c/2 + (d*x)/2)))))/d + (a*tan(c/2 + (d*x)/2)^4
)/(64*d) - (b*tan(c/2 + (d*x)/2)^3)/(24*d) + (3*a*log(tan(c/2 + (d*x)/2)))/
(8*d) - (a/4 + (2*b*tan(c/2 + (d*x)/2))/3 + 2*a*tan(c/2 + (d*x)/2)^2 + 10*b
*tan(c/2 + (d*x)/2)^3)/(16*d*tan(c/2 + (d*x)/2)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\tan(c+dx))\csc^5(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x)**5, x)
```

3.21 $\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=87

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{b\cot^4(c+dx)}{4d}-\frac{b\cot^2(c+dx)}{d}+\frac{b\log(\tan(c+dx))}{d}$$

```
[Out] -a*cot(d*x+c)/d-b*cot(d*x+c)^2/d-2/3*a*cot(d*x+c)^3/d-1/4*b*cot(d*x+c)^4/d-
1/5*a*cot(d*x+c)^5/d+b*ln(tan(d*x+c))/d
```

Rubi [A] time = 0.11, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {766}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{b\cot^4(c+dx)}{4d}-\frac{b\cot^2(c+dx)}{d}+\frac{b\log(\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (b*Cot[c + d*x]^2)/d - (2*a*Cot[c + d*x]^3)/(3*d) -
 (b*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5*d) + (b*Log[Tan[c + d*x]]
)/d
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)\left(1+x^2\right)^2}{x^6}\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a}{x^6}+\frac{b}{x^5}+\frac{2a}{x^4}+\frac{2b}{x^3}+\frac{a}{x^2}+\frac{b}{x}\right)dx,x,\tan(c+dx)\right)}{d}\\ &= -\frac{a\cot(c+dx)}{d}-\frac{b\cot^2(c+dx)}{d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{b\cot^4(c+dx)}{4d}-\frac{a\,\text{cc}}{}\end{aligned}$$

Mathematica [A] time = 0.56, size = 104, normalized size = 1.20

$$-\frac{8a\cot(c+dx)}{15d}-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}-\frac{4a\cot(c+dx)\csc^2(c+dx)}{15d}-\frac{b\left(\csc^4(c+dx)+2\csc^2(c+dx)-4\log\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

```
[Out] (-8*a*Cot[c + d*x])/(15*d) - (4*a*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) - (a*
Cot[c + d*x]*Csc[c + d*x]^4)/(5*d) - (b*(2*Csc[c + d*x]^2 + Csc[c + d*x]^4
+ 4*Log[Cos[c + d*x]] - 4*Log[Sin[c + d*x]]))/(4*d)
```

fricas [B] time = 0.49, size = 174, normalized size = 2.00

$$-\frac{32\,a\cos(dx+c)^5-80\,a\cos(dx+c)^3+30\left(b\cos(dx+c)^4-2\,b\cos(dx+c)^2+b\right)\log\left(\cos(dx+c)^2\right)\sin(dx+}{60\left(d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(32*a*cos(d*x + c)^5 - 80*a*cos(d*x + c)^3 + 30*(b*cos(d*x + c)^4 - 2
*b*cos(d*x + c)^2 + b)*log(cos(d*x + c)^2)*sin(d*x + c) - 30*(b*cos(d*x + c
)^4 - 2*b*cos(d*x + c)^2 + b)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) +
 60*a*cos(d*x + c) - 15*(2*b*cos(d*x + c)^2 - 3*b)*sin(d*x + c))/((d*cos(d*
x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [A] time = 1.80, size = 84, normalized size = 0.97

$$\frac{60\,b\log\left(\left|\tan\left(dx+c\right)\right|\right)-\frac{137\,b\tan\left(dx+c\right)^{5}+60\,a\tan\left(dx+c\right)^{4}+60\,b\tan\left(dx+c\right)^{3}+40\,a\tan\left(dx+c\right)^{2}+15\,b\tan\left(dx+c\right)+12\,a}{\tan\left(dx+c\right)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(60*b*log(abs(tan(d*x + c))) - (137*b*tan(d*x + c)^5 + 60*a*tan(d*x +
c)^4 + 60*b*tan(d*x + c)^3 + 40*a*tan(d*x + c)^2 + 15*b*tan(d*x + c) + 12*a
)/tan(d*x + c)^5)/d
```

maple [A] time = 0.40, size = 94, normalized size = 1.08

$$-\frac{8a\cot\left(dx+c\right)}{15d}-\frac{a\cot\left(dx+c\right)\left(\csc^{4}\left(dx+c\right)\right)}{5d}-\frac{4a\cot\left(dx+c\right)\left(\csc^{2}\left(dx+c\right)\right)}{15d}-\frac{b}{4d\sin\left(dx+c\right)^{4}}-\frac{b}{2d\sin\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*tan(d*x+c)),x)
```

```
[Out] -8/15*a*cot(d*x+c)/d-1/5/d*a*cot(d*x+c)*csc(d*x+c)^4-4/15/d*a*cot(d*x+c)*cs
c(d*x+c)^2-1/4/d*b/sin(d*x+c)^4-1/2/d*b/sin(d*x+c)^2+b*ln(tan(d*x+c))/d
```

maxima [A] time = 0.59, size = 72, normalized size = 0.83

$$\frac{60\,b\log\left(\tan\left(dx+c\right)\right)-\frac{60\,a\tan\left(dx+c\right)^{4}+60\,b\tan\left(dx+c\right)^{3}+40\,a\tan\left(dx+c\right)^{2}+15\,b\tan\left(dx+c\right)+12\,a}{\tan\left(dx+c\right)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*b*log(tan(d*x + c)) - (60*a*tan(d*x + c)^4 + 60*b*tan(d*x + c)^3 +
 40*a*tan(d*x + c)^2 + 15*b*tan(d*x + c) + 12*a)/tan(d*x + c)^5)/d
```

mupad [B] time = 3.93, size = 70, normalized size = 0.80

$$\frac{b\,\ln\left(\tan\left(c+d\,x\right)\right)}{d}-\frac{a\,{\tan\left(c+d\,x\right)}^4+b\,{\tan\left(c+d\,x\right)}^3+\frac{2\,a\,{\tan\left(c+d\,x\right)}^2}{3}+\frac{b\,\tan\left(c+d\,x\right)}{4}+\frac{a}{5}}{d\,{\tan\left(c+d\,x\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/sin(c + d*x)^6,x)
```

```
[Out] (b*log(tan(c + d*x)))/d - (a/5 + (b*tan(c + d*x))/4 + (2*a*tan(c + d*x)^2)/
3 + a*tan(c + d*x)^4 + b*tan(c + d*x)^3)/(d*tan(c + d*x)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)\csc^{6}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x)**6, x)
```

3.22 $\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=113

$$\frac{3}{8}x\left(a^2-5b^2\right)+\frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{\sin(c+dx)\cos^3(c}{}$$

```
[Out] 3/8*(a^2-5*b^2)*x-2*a*b*ln(cos(d*x+c))/d+b^2*tan(d*x+c)/d+1/8*cos(d*x+c)^2*
(7*b-5*a*tan(d*x+c))*(a+b*tan(d*x+c))/d+1/4*cos(d*x+c)^3*sin(d*x+c)*(a+b*ta
n(d*x+c))^2/d
```

Rubi [A] time = 0.19, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1645, 1810, 635, 203, 260}

$$\frac{3}{8}x\left(a^2-5b^2\right)+\frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{\sin(c+dx)\cos^3(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (3*(a^2 - 5*b^2)*x)/8 - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Tan[c + d*x])/d
+ (Cos[c + d*x]^2*(7*b - 5*a*Tan[c + d*x])*(a + b*Tan[c + d*x]))/(8*d) + (C
os[c + d*x]^3*Sin[c + d*x]*(a + b*Tan[c + d*x])^2)/(4*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^4(a+x)^2}{\left(b^2+x^2\right)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{4d} - \frac{\text{Subst}\left(\int \frac{(a+x)\left(ab^4+3b^4x-4a\right.}{\left(b^2+x^2\right)}\right.}{}$$

$$= \frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d} + \frac{\cos^3(c+dx)\sin(}{}$$

$$= \frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d} + \frac{\cos^3(c+dx)\sin(}{}$$

$$= \frac{b^2\tan(c+dx)}{d} + \frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d} +$$

$$= \frac{b^2\tan(c+dx)}{d} + \frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d} +$$

$$= \frac{3}{8}\left(a^2-5b^2\right)x - \frac{2ab\log(\cos(c+dx))}{d} + \frac{b^2\tan(c+dx)}{d} + \frac{\cos^2(c+dx)(7}{}$$

Mathematica [B] time = 3.51, size = 240, normalized size = 2.12

$$b\left(\frac{2\left(3b^2-2a^2\right)\sin(2(c+dx))}{b} + \frac{4\left(3b^2-2a^2\right)\tan^{-1}(\tan(c+dx))}{b} + 4\left(\frac{a^2-3b^2}{\sqrt{-b^2}}+2a\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right) + 4\left(\frac{3b^2-a^2}{\sqrt{-b^2}}+2a\right)\text{lo}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b*((4*(-2*a^2 + 3*b^2)*ArcTan[Tan[c + d*x]])/b + 16*a*Cos[c + d*x]^2 - 4*a
*Cos[c + d*x]^4 + 4*(2*a + (a^2 - 3*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan
[c + d*x]] + 4*(2*a + (-a^2 + 3*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c +
 d*x]] + (2*(a^2 - b^2)*Cos[c + d*x]^3*Sin[c + d*x])/b + (2*(-2*a^2 + 3*b^2
)*Sin[2*(c + d*x)])/b + (3*(a^2 - b^2)*(2*ArcTan[Tan[c + d*x]] + Sin[2*(c +
 d*x)]))/(2*b) + 8*b*Tan[c + d*x]))/(8*d)
```

fricas [A] time = 0.55, size = 137, normalized size = 1.21

$$-\frac{8\,ab\cos(dx+c)^5 - 32\,ab\cos(dx+c)^3 + 32\,ab\cos(dx+c)\log(-\cos(dx+c)) - \left(6\left(a^2-5\,b^2\right)dx - 13\,ab\right)\cos(}{16\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*a*b*cos(d*x + c)^5 - 32*a*b*cos(d*x + c)^3 + 32*a*b*cos(d*x + c)*l
og(-cos(d*x + c)) - (6*(a^2 - 5*b^2)*d*x - 13*a*b)*cos(d*x + c) - 2*(2*(a^2
```

```
 - b^2)*cos(d*x + c)^4 - (5*a^2 - 9*b^2)*cos(d*x + c)^2 + 8*b^2)*sin(d*x +
c))/(d*cos(d*x + c))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)(24*a^2*d*x*tan(c)^5*tan(d*x)^5+48*a^2*d*x*tan(c)^5*tan(d*x)^3
+24*a^2*d*x*tan(c)^5*tan(d*x)-24*a^2*d*x*tan(c)^4*tan(d*x)^4-48*a^2*d*x*tan
(c)^4*tan(d*x)^2-24*a^2*d*x*tan(c)^4+48*a^2*d*x*tan(c)^3*tan(d*x)^5+96*a^2*
d*x*tan(c)^3*tan(d*x)^3+48*a^2*d*x*tan(c)^3*tan(d*x)-48*a^2*d*x*tan(c)^2*ta
n(d*x)^4-96*a^2*d*x*tan(c)^2*tan(d*x)^2-48*a^2*d*x*tan(c)^2+24*a^2*d*x*tan(
c)*tan(d*x)^5+48*a^2*d*x*tan(c)*tan(d*x)^3+24*a^2*d*x*tan(c)*tan(d*x)-24*a^
2*d*x*tan(d*x)^4-48*a^2*d*x*tan(d*x)^2-24*a^2*d*x+24*a^2*tan(c)^5*tan(d*x)^
```

```
4+40*a^2*tan(c)^5*tan(d*x)^2+24*a^2*tan(c)^4*tan(d*x)^5+24*a^2*tan(c)^4*tan
(d*x)^3-80*a^2*tan(c)^4*tan(d*x)+24*a^2*tan(c)^3*tan(d*x)^4-96*a^2*tan(c)^3
*tan(d*x)^2+40*a^2*tan(c)^3+40*a^2*tan(c)^2*tan(d*x)^5-96*a^2*tan(c)^2*tan(
d*x)^3+24*a^2*tan(c)^2*tan(d*x)-80*a^2*tan(c)*tan(d*x)^4+24*a^2*tan(c)*tan(
d*x)^2+24*a^2*tan(c)+40*a^2*tan(d*x)^3+24*a^2*tan(d*x)-64*a*b*ln((4*tan(c)^
2*tan(d*x)^4+4*tan(c)^2*tan(d*x)^2-8*tan(c)*tan(d*x)^3-8*tan(c)*tan(d*x)+4*
tan(d*x)^2+4)/(tan(c)^2+1))*tan(c)^5*tan(d*x)^5-128*a*b*ln((4*tan(c)^2*tan(
d*x)^4+4*tan(c)^2*tan(d*x)^2-8*tan(c)*tan(d*x)^3-8*tan(c)*tan(d*x)+4*tan(d*
x)^2+4)/(tan(c)^2+1))*tan(c)^5*tan(d*x)^3-64*a*b*ln((4*tan(c)^2*tan(d*x)^4+
4*tan(c)^2*tan(d*x)^2-8*tan(c)*tan(d*x)^3-8*tan(c)*tan(d*x)+4*tan(d*x)^2+4)
/(tan(c)^2+1))*tan(c)^5*tan(d*x)+64*a*b*ln((4*tan(c)^2*tan(d*x)^4+4*tan(c)^
2*tan(d*x)^2-8*tan(c)*tan(d*x)^3-8*tan(c)*tan(d*x)+4*tan(d*x)^2+4)/(tan(c)^
2+1))*tan(c)^4*tan(d*x)^4+128*a*b*ln((4*tan(c)^2*tan(d*x)^4+4*tan(c)^2*tan(
d*x)^2-8*tan(c)*tan(d*x)^3-8*tan(c)*tan(d*x)+4*tan(d*x)^2+4)/(tan(c)^2+1))*
tan(c)^4*tan(d*x)^2+64*a*b*ln((4*tan(c)^2*tan(d*x)^4+4*tan(c)^2*tan(d*x)^2-
8*tan(c)*tan(d*x)^3-8*tan(c)*tan(d*x)+4*tan(d*x)^2+4)/(tan(c)^2+1))*tan(c)^
4-128*a*b*ln((4*tan(c)^2*tan(d*x)^4+4*tan(c)^2*tan(d*x)^2-8*tan(c)*tan(d*x)
^3-8*tan(c)*tan(d*x)+4*tan(d*x)^2+4)/(tan(c)^2+1))*tan(c)^3*tan(d*x)^5-256*
a*b*ln((4*tan(c)^2*tan(d*x)^4+4*tan(c)^2*tan(d*x)^2-8*tan(c)*tan(d*x)^3-8*t
an(c)*tan(d*x)+4*tan(d*x)^2+4)/(tan(c)^2+1))*tan(c)^3*tan(d*x)^3-128*a*b*ln
((4*tan(c)^2*tan(d*x)^4+4*tan(c)^2*tan(d*x)^2-8*tan(c)*tan(d*x)^3-8*tan(c)*
tan(d*x)+4*tan(d*x)^2+4)/(tan(c)^2+1))*tan(c)^3*tan(d*x)+128*a*b*ln((4*tan(
c)^2*tan(d*x)^4+4*tan(c)^2*tan(d*x)^2-8*tan(c)*tan(d*x)^3-8*tan(c)*tan(d*x)
+4*tan(d*x)^2+4)/(tan(c)^2+1))*tan(c)^2*tan(d*x)^4+256*a*b*ln((4*tan(c)^2*t
an(d*x)^4+4*tan(c)^2*tan(d*x)^2-8*tan(c)*tan(d*x)^3-8*tan(c)*tan(d*x)+4*tan
(d*x)^2+4)/(tan(c)^2+1))*tan(c)^2*tan(d*x)^2+128*a*b*ln((4*tan(c)^2*tan(d*x
)^4+4*tan(c)^2*tan(d*x)^2-8*tan(c)*tan(d*x)^3-8*tan(c)*tan(d*x)+4*tan(d*x)^
2+4)/(tan(c)^2+1))*tan(c)^2-64*a*b*ln((4*tan(c)^2*tan(d*x)^4+4*tan(c)^2*tan
(d*x)^2-8*tan(c)*tan(d*x)^3-8*tan(c)*tan(d*x)+4*tan(d*x)^2+4)/(tan(c)^2+1))
*tan(c)*tan(d*x)^5-128*a*b*ln((4*tan(c)^2*tan(d*x)^4+4*tan(c)^2*tan(d*x)^2-
8*tan(c)*tan(d*x)^3-8*tan(c)*tan(d*x)+4*tan(d*x)^2+4)/(tan(c)^2+1))*tan(c)*
tan(d*x)^3-64*a*b*ln((4*tan(c)^2*tan(d*x)^4+4*tan(c)^2*tan(d*x)^2-8*tan(c)*
tan(d*x)^3-8*tan(c)*tan(d*x)+4*tan(d*x)^2+4)/(tan(c)^2+1))*tan(c)*tan(d*x)+
64*a*b*ln((4*tan(c)^2*tan(d*x)^4+4*tan(c)^2*tan(d*x)^2-8*tan(c)*tan(d*x)^3-
8*tan(c)*tan(d*x)+4*tan(d*x)^2+4)/(tan(c)^2+1))*tan(d*x)^4+128*a*b*ln((4*ta
n(c)^2*tan(d*x)^4+4*tan(c)^2*tan(d*x)^2-8*tan(c)*tan(d*x)^3-8*tan(c)*tan(d*
x)+4*tan(d*x)^2+4)/(tan(c)^2+1))*tan(d*x)^2+64*a*b*ln((4*tan(c)^2*tan(d*x)^
4+4*tan(c)^2*tan(d*x)^2-8*tan(c)*tan(d*x)^3-8*tan(c)*tan(d*x)+4*tan(d*x)^2+
4)/(tan(c)^2+1))+44*a*b*tan(c)^5*tan(d*x)^5+24*a*b*tan(c)^5*tan(d*x)^3-52*a
*b*tan(c)^5*tan(d*x)-172*a*b*tan(c)^4*tan(d*x)^4-280*a*b*tan(c)^4*tan(d*x)^
2+52*a*b*tan(c)^4+24*a*b*tan(c)^3*tan(d*x)^5-16*a*b*tan(c)^3*tan(d*x)^3+280
*a*b*tan(c)^3*tan(d*x)-280*a*b*tan(c)^2*tan(d*x)^4+16*a*b*tan(c)^2*tan(d*x)
^2-24*a*b*tan(c)^2-52*a*b*tan(c)*tan(d*x)^5+280*a*b*tan(c)*tan(d*x)^3+172*a
*b*tan(c)*tan(d*x)+52*a*b*tan(d*x)^4-24*a*b*tan(d*x)^2-44*a*b-120*b^2*d*x*t
an(c)^5*tan(d*x)^5-240*b^2*d*x*tan(c)^5*tan(d*x)^3-120*b^2*d*x*tan(c)^5*tan
(d*x)+120*b^2*d*x*tan(c)^4*tan(d*x)^4+240*b^2*d*x*tan(c)^4*tan(d*x)^2+120*b
^2*d*x*tan(c)^4-240*b^2*d*x*tan(c)^3*tan(d*x)^5-480*b^2*d*x*tan(c)^3*tan(d*
x)^3-240*b^2*d*x*tan(c)^3*tan(d*x)+240*b^2*d*x*tan(c)^2*tan(d*x)^4+480*b^2*
d*x*tan(c)^2*tan(d*x)^2+240*b^2*d*x*tan(c)^2-120*b^2*d*x*tan(c)*tan(d*x)^5-
240*b^2*d*x*tan(c)*tan(d*x)^3-120*b^2*d*x*tan(c)*tan(d*x)+120*b^2*d*x*tan(d
*x)^4+240*b^2*d*x*tan(d*x)^2+120*b^2*d*x+3*b^2*pi*sign(2*tan(c)^2*tan(d*x)-
2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(
c)^5*tan(d*x)^5+6*b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan
(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(c)^5*tan(d*x)^3+3*b^2*pi*
sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*ta
n(c)^2*tan(d*x)^2-2)*tan(c)^5*tan(d*x)-3*b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*
tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(c)
^4*tan(d*x)^4-6*b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c
)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(c)^4*tan(d*x)^2-3*b^2*pi*si
```

```
gn(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(
c)^2*tan(d*x)^2-2)*tan(c)^4+6*b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(
d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(c)^3*tan(d*x)
^5+12*b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*
x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(c)^3*tan(d*x)^3+6*b^2*pi*sign(2*tan(c
)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d
*x)^2-2)*tan(c)^3*tan(d*x)-6*b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d
*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(c)^2*tan(d*x)^
4-12*b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x
))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(c)^2*tan(d*x)^2-6*b^2*pi*sign(2*tan(c)
^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*
x)^2-2)*tan(c)^2+3*b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*ta
n(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(c)*tan(d*x)^5+6*b^2*pi*s
ign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan
(c)^2*tan(d*x)^2-2)*tan(c)*tan(d*x)^3+3*b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*t
an(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(c)*
tan(d*x)-3*b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*t
an(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(d*x)^4-6*b^2*pi*sign(2*tan(c)^2*
tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^
2-2)*tan(d*x)^2-3*b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan
(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)+3*b^2*pi*sign(2*tan(c)^2*tan(
d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(c)^5*tan(d*x)^5+6*b^2*pi*
sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(c)^5*
tan(d*x)^3+3*b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2
*tan(d*x))*tan(c)^5*tan(d*x)-3*b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan
(d*x)^2-2*tan(c)+2*tan(d*x))*tan(c)^4*tan(d*x)^4-6*b^2*pi*sign(2*tan(c)^2*t
an(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(c)^4*tan(d*x)^2-3*b^2*
pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(c)
^4+6*b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x
))*tan(c)^3*tan(d*x)^5+12*b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)
^2-2*tan(c)+2*tan(d*x))*tan(c)^3*tan(d*x)^3+6*b^2*pi*sign(2*tan(c)^2*tan(d*
x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(c)^3*tan(d*x)-6*b^2*pi*sign
(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(c)^2*tan(
d*x)^4-12*b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*ta
n(d*x))*tan(c)^2*tan(d*x)^2-6*b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(
d*x)^2-2*tan(c)+2*tan(d*x))*tan(c)^2+3*b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*ta
n(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(c)*tan(d*x)^5+6*b^2*pi*sign(2*tan(
c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(c)*tan(d*x)^3+3*
b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*ta
n(c)*tan(d*x)-3*b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c
)+2*tan(d*x))*tan(d*x)^4-6*b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x
)^2-2*tan(c)+2*tan(d*x))*tan(d*x)^2-3*b^2*pi*sign(2*tan(c)^2*tan(d*x)-2*tan
(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))+6*b^2*atan((tan(c)+tan(d*x))/(tan(c)*ta
n(d*x)-1))*tan(c)^5*tan(d*x)^5+12*b^2*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*
x)-1))*tan(c)^5*tan(d*x)^3+6*b^2*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1)
)*tan(c)^5*tan(d*x)-6*b^2*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(c
)^4*tan(d*x)^4-12*b^2*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(c)^4*
tan(d*x)^2-6*b^2*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(c)^4+12*b^
2*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(c)^3*tan(d*x)^5+24*b^2*at
an((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(c)^3*tan(d*x)^3+12*b^2*atan((
tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(c)^3*tan(d*x)-12*b^2*atan((tan(c)
+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(c)^2*tan(d*x)^4-24*b^2*atan((tan(c)+tan
(d*x))/(tan(c)*tan(d*x)-1))*tan(c)^2*tan(d*x)^2-12*b^2*atan((tan(c)+tan(d*x
))/(tan(c)*tan(d*x)-1))*tan(c)^2+6*b^2*atan((tan(c)+tan(d*x))/(tan(c)*tan(d
*x)-1))*tan(c)*tan(d*x)^5+12*b^2*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1)
)*tan(c)*tan(d*x)^3+6*b^2*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(c
)*tan(d*x)-6*b^2*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(d*x)^4-12*
b^2*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(d*x)^2-6*b^2*atan((tan(
c)+tan(d*x))/(tan(c)*tan(d*x)-1))-6*b^2*atan((tan(c)-tan(d*x))/(tan(c)*tan(
```

```
d*x)+1))*tan(c)^5*tan(d*x)^5-12*b^2*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)
+1))*tan(c)^5*tan(d*x)^3-6*b^2*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*
tan(c)^5*tan(d*x)+6*b^2*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*tan(c)^
4*tan(d*x)^4+12*b^2*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*tan(c)^4*ta
n(d*x)^2+6*b^2*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*tan(c)^4-12*b^2*
atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*tan(c)^3*tan(d*x)^5-24*b^2*atan
((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*tan(c)^3*tan(d*x)^3-12*b^2*atan((ta
n(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*tan(c)^3*tan(d*x)+12*b^2*atan((tan(c)-t
an(d*x))/(tan(c)*tan(d*x)+1))*tan(c)^2*tan(d*x)^4+24*b^2*atan((tan(c)-tan(d
*x))/(tan(c)*tan(d*x)+1))*tan(c)^2*tan(d*x)^2+12*b^2*atan((tan(c)-tan(d*x))
/(tan(c)*tan(d*x)+1))*tan(c)^2-6*b^2*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x
)+1))*tan(c)*tan(d*x)^5-12*b^2*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*
tan(c)*tan(d*x)^3-6*b^2*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*tan(c)*
tan(d*x)+6*b^2*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*tan(d*x)^4+12*b^
2*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*tan(d*x)^2+6*b^2*atan((tan(c)
-tan(d*x))/(tan(c)*tan(d*x)+1))-120*b^2*tan(c)^5*tan(d*x)^4-200*b^2*tan(c)^
5*tan(d*x)^2-64*b^2*tan(c)^5-120*b^2*tan(c)^4*tan(d*x)^5-120*b^2*tan(c)^4*t
an(d*x)^3+80*b^2*tan(c)^4*tan(d*x)-120*b^2*tan(c)^3*tan(d*x)^4-160*b^2*tan(
c)^3*tan(d*x)^2-200*b^2*tan(c)^3-200*b^2*tan(c)^2*tan(d*x)^5-160*b^2*tan(c)
^2*tan(d*x)^3-120*b^2*tan(c)^2*tan(d*x)+80*b^2*tan(c)*tan(d*x)^4-120*b^2*ta
n(c)*tan(d*x)^2-120*b^2*tan(c)-64*b^2*tan(d*x)^5-200*b^2*tan(d*x)^3-120*b^2
*tan(d*x))/(64*d*tan(c)^5*tan(d*x)^5+128*d*tan(c)^5*tan(d*x)^3+64*d*tan(c)^
5*tan(d*x)-64*d*tan(c)^4*tan(d*x)^4-128*d*tan(c)^4*tan(d*x)^2-64*d*tan(c)^4
+128*d*tan(c)^3*tan(d*x)^5+256*d*tan(c)^3*tan(d*x)^3+128*d*tan(c)^3*tan(d*x
)-128*d*tan(c)^2*tan(d*x)^4-256*d*tan(c)^2*tan(d*x)^2-128*d*tan(c)^2+64*d*t
an(c)*tan(d*x)^5+128*d*tan(c)*tan(d*x)^3+64*d*tan(c)*tan(d*x)-64*d*tan(d*x)
^4-128*d*tan(d*x)^2-64*d)
```

maple [A] time = 0.42, size = 204, normalized size = 1.81

$$-\frac{a^2\cos(dx+c)\left(\sin^3(dx+c)\right)}{4d}-\frac{3a^2\cos(dx+c)\sin(dx+c)}{8d}+\frac{3a^2x}{8}+\frac{3a^2c}{8d}-\frac{ab\left(\sin^4(dx+c)\right)}{2d}-\frac{ab\left(\sin^2(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x)
```

```
[Out] -1/4/d*a^2*cos(d*x+c)*sin(d*x+c)^3-3/8/d*a^2*cos(d*x+c)*sin(d*x+c)+3/8*a^2*
x+3/8/d*a^2*c-1/2/d*a*b*sin(d*x+c)^4-1/d*a*b*sin(d*x+c)^2-2*a*b*ln(cos(d*x+
c))/d+1/d*b^2*sin(d*x+c)^7/cos(d*x+c)+1/d*b^2*cos(d*x+c)*sin(d*x+c)^5+5/4/d
*b^2*cos(d*x+c)*sin(d*x+c)^3+15/8/d*b^2*cos(d*x+c)*sin(d*x+c)-15/8*b^2*x-15
/8/d*c*b^2
```

maxima [A] time = 0.89, size = 128, normalized size = 1.13

$$\frac{8\,ab\log\left(\tan(dx+c)^2+1\right)+8\,b^2\tan(dx+c)+3\left(a^2-5\,b^2\right)(dx+c)+\frac{16\,ab\tan(dx+c)^2-\left(5\,a^2-9\,b^2\right)\tan(dx+c)^3+12\,ab-\left(3\,a^2\right.}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(8*a*b*log(tan(d*x + c)^2 + 1) + 8*b^2*tan(d*x + c) + 3*(a^2 - 5*b^2)*(
d*x + c) + (16*a*b*tan(d*x + c)^2 - (5*a^2 - 9*b^2)*tan(d*x + c)^3 + 12*a*b
 - (3*a^2 - 7*b^2)*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

mupad [B] time = 3.82, size = 127, normalized size = 1.12

$$x\left(\frac{3\,a^2}{8}-\frac{15\,b^2}{8}\right)+\frac{b^2\tan(c+d\,x)}{d}+\frac{\left(\frac{9\,b^2}{8}-\frac{5\,a^2}{8}\right)\tan(c+d\,x)^3+2\,a\,b\tan(c+d\,x)^2+\left(\frac{7\,b^2}{8}-\frac{3\,a^2}{8}\right)\tan(c+d\,x)+}{d\left(\tan(c+d\,x)^4+2\tan(c+d\,x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + b*tan(c + d*x))^2,x)
```

```
[Out] x*((3*a^2)/8 - (15*b^2)/8) + (b^2*tan(c + d*x))/d + ((3*a*b)/2 - tan(c + d*
x)*((3*a^2)/8 - (7*b^2)/8) - tan(c + d*x)^3*((5*a^2)/8 - (9*b^2)/8) + 2*a*b
*tan(c + d*x)^2)/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1)) + (a*b*log(tan
(c + d*x)^2 + 1))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^2 \sin^4(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sin(c + d*x)**4, x)
```

3.23 $\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=122

$$\frac{a^2\cos^3(c+dx)}{3d}-\frac{a^2\cos(c+dx)}{d}-\frac{2ab\sin^3(c+dx)}{3d}-\frac{2ab\sin(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2\cos^3(c+dx)}{3d}+2$$

```
[Out] 2*a*b*arctanh(sin(d*x+c))/d-a^2*cos(d*x+c)/d+2*b^2*cos(d*x+c)/d+1/3*a^2*cos
(d*x+c)^3/d-1/3*b^2*cos(d*x+c)^3/d+b^2*sec(d*x+c)/d-2*a*b*sin(d*x+c)/d-2/3*
a*b*sin(d*x+c)^3/d
```

Rubi [A] time = 0.13, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3517, 2633, 2592, 302, 206, 2590, 270}

$$\frac{a^2\cos^3(c+dx)}{3d}-\frac{a^2\cos(c+dx)}{d}-\frac{2ab\sin^3(c+dx)}{3d}-\frac{2ab\sin(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2\cos^3(c+dx)}{3d}+2$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - (a^2*Cos[c + d*x])/d + (2*b^2*Cos[c + d*x
])/d + (a^2*Cos[c + d*x]^3)/(3*d) - (b^2*Cos[c + d*x]^3)/(3*d) + (b^2*Sec[c
 + d*x])/d - (2*a*b*Sin[c + d*x])/d - (2*a*b*Sin[c + d*x]^3)/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a^2\sin^3(c+dx)+2ab\sin^3(c+dx)\tan(c+dx)+b^2\sin^3(c+dx)\,\mathrm{ta}\right.$$

$$= a^2\int \sin^3(c+dx)\,dx + (2ab)\int \sin^3(c+dx)\tan(c+dx)\,dx + b^2\int \mathrm{si}$$

$$= -\frac{a^2\,\mathrm{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(c+dx)\right)}{d} + \frac{(2ab)\,\mathrm{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x\right.}{d}$$

$$= -\frac{a^2\cos(c+dx)}{d} + \frac{a^2\cos^3(c+dx)}{3d} + \frac{(2ab)\,\mathrm{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)\right.}{d}$$

$$= -\frac{a^2\cos(c+dx)}{d} + \frac{2b^2\cos(c+dx)}{d} + \frac{a^2\cos^3(c+dx)}{3d} - \frac{b^2\cos^3(c+d}{3d}$$

$$= \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} - \frac{a^2\cos(c+dx)}{d} + \frac{2b^2\cos(c+dx)}{d} + \frac{a^2\cos}{}$$

Mathematica [A] time = 0.99, size = 152, normalized size = 1.25

$$\frac{\sec(c+dx)\left(\left(20b^2-8a^2\right)\cos(2(c+dx))+\left(a^2-b^2\right)\cos(4(c+dx))-9a^2-28ab\sin(2(c+dx))+2ab\sin(4(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(-9*a^2 + 45*b^2 + (-8*a^2 + 20*b^2)*Cos[2*(c + d*x)] + (a^2
- b^2)*Cos[4*(c + d*x)] - 48*a*b*Cos[c + d*x]*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] + 48*a*b*Cos[c + d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
- 28*a*b*Sin[2*(c + d*x)] + 2*a*b*Sin[4*(c + d*x)]))/(24*d)
```

fricas [A] time = 0.47, size = 126, normalized size = 1.03

$$\frac{\left(a^2-b^2\right)\cos\left(dx+c\right)^4+3\,ab\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,ab\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)-3}{3\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((a^2 - b^2)*cos(d*x + c)^4 + 3*a*b*cos(d*x + c)*log(sin(d*x + c) + 1)
- 3*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - 3*(a^2 - 2*b^2)*cos(d*x + c)^
2 + 3*b^2 + 2*(a*b*cos(d*x + c)^3 - 4*a*b*cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.40, size = 167, normalized size = 1.37

$$-\frac{\cos(dx+c)\left(\sin^2(dx+c)\right)a^2}{3d}-\frac{2a^2\cos(dx+c)}{3d}-\frac{2ab\left(\sin^3(dx+c)\right)}{3d}-\frac{2ab\sin(dx+c)}{d}+\frac{2ab\ln(\sec(dx+c)+\mathrm{ta}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x)
```

```
[Out] -1/3/d*cos(d*x+c)*sin(d*x+c)^2*a^2-2/3*a^2*cos(d*x+c)/d-2/3*a*b*sin(d*x+c)^
3/d-2*a*b*sin(d*x+c)/d+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*b^2*sin(d*x+c)
^6/cos(d*x+c)+8/3*b^2*cos(d*x+c)/d+1/d*b^2*cos(d*x+c)*sin(d*x+c)^4+4/3/d*b^
2*cos(d*x+c)*sin(d*x+c)^2
```

maxima [A] time = 0.47, size = 104, normalized size = 0.85

$$\frac{\left(\cos(dx+c)^3-3\cos(dx+c)\right)a^2-\left(2\sin(dx+c)^3-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)+6\sin(\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((cos(d*x + c)^3 - 3*cos(d*x + c))*a^2 - (2*sin(d*x + c)^3 - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*a*b - (cos(d*x +
c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*b^2)/d
```

mupad [B] time = 6.71, size = 174, normalized size = 1.43

$$\frac{4\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{8\,a^2}{3}-\frac{32\,b^2}{3}\right)-4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{4\,a^2}{3}-\frac{16\,b^2}{3}+\frac{28\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-\frac{28\,a}{}}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (4*a*b*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^2*((8*a^2)/3 - (3
2*b^2)/3) - 4*a^2*tan(c/2 + (d*x)/2)^4 + (4*a^2)/3 - (16*b^2)/3 + (28*a*b*t
an(c/2 + (d*x)/2)^3)/3 - (28*a*b*tan(c/2 + (d*x)/2)^5)/3 - 4*a*b*tan(c/2 +
(d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2)^2 - 2*tan(c
/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^2\sin^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sin(c + d*x)**3, x)
```

3.24 $\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=76

$$\frac{1}{2}x\left(a^2-3b^2\right)-\frac{2ab\log(\cos(c+dx))}{d}-\frac{\sin(c+dx)\cos(c+dx)(a+b\tan(c+dx))^2}{2d}+\frac{3b^2\tan(c+dx)}{2d}$$

```
[Out] 1/2*(a^2-3*b^2)*x-2*a*b*ln(cos(d*x+c))/d+3/2*b^2*tan(d*x+c)/d-1/2*cos(d*x+c
)*sin(d*x+c)*(a+b*tan(d*x+c))^2/d
```

Rubi [A] time = 0.11, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3516, 1645, 774, 635, 203, 260}

$$\frac{1}{2}x\left(a^2-3b^2\right)-\frac{2ab\log(\cos(c+dx))}{d}-\frac{\sin(c+dx)\cos(c+dx)(a+b\tan(c+dx))^2}{2d}+\frac{3b^2\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 - 3*b^2)*x)/2 - (2*a*b*Log[Cos[c + d*x]])/d + (3*b^2*Tan[c + d*x])/(2
*d) - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^2(a+x)^2}{\left(b^2+x^2\right)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{2d} - \frac{\text{Subst}\left(\int \frac{(a+x)\left(-ab^2-3b^2x\right)}{b^2+x^2}\right.}{2bd}$$

$$= \frac{3b^2\tan(c+dx)}{2d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{2d} - \frac{\text{Subst}\Big(}{}$$

$$= \frac{3b^2\tan(c+dx)}{2d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{2d} + \frac{(2ab)\,\text{Su}}{}$$

$$= \frac{1}{2}\left(a^2-3b^2\right)x - \frac{2ab\log(\cos(c+dx))}{d} + \frac{3b^2\tan(c+dx)}{2d} - \frac{\cos(c+dx)\,\text{s}}{}$$

Mathematica [B] time = 2.50, size = 162, normalized size = 2.13

$$\frac{b\left(\frac{\left(b^2-a^2\right)\sin(2(c+dx))}{2b} + \frac{\left(b^2-a^2\right)\tan^{-1}(\tan(c+dx))}{b} + \left(\frac{a^2-2b^2}{\sqrt{-b^2}}+2a\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right) + \left(\frac{2b^2-a^2}{\sqrt{-b^2}}+2a\right)\log\left(\sqrt{-b^2}\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b*(((-a^2 + b^2)*ArcTan[Tan[c + d*x]])/b + 2*a*Cos[c + d*x]^2 + (2*a + (a^
2 - 2*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + (2*a + (-a^2 + 2*
b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + ((-a^2 + b^2)*Sin[2*(c
+ d*x)])/(2*b) + 2*b*Tan[c + d*x]))/(2*d)
```

fricas [A] time = 0.50, size = 101, normalized size = 1.33

$$\frac{2\,ab\cos(dx+c)^3 - 4\,ab\cos(dx+c)\log(-\cos(dx+c)) + \left(\left(a^2-3\,b^2\right)dx - ab\right)\cos(dx+c) - \left(\left(a^2-b^2\right)\cos(dx\right.\right.}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*cos(d*x + c)^3 - 4*a*b*cos(d*x + c)*log(-cos(d*x + c)) + ((a^2 -
 3*b^2)*d*x - a*b)*cos(d*x + c) - ((a^2 - b^2)*cos(d*x + c)^2 - 2*b^2)*sin(
d*x + c))/(d*cos(d*x + c))
```

giac [B] time = 2.17, size = 1061, normalized size = 13.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*d*x*tan(d*x)^3*tan(c)^3 - 3*b^2*d*x*tan(d*x)^3*tan(c)^3 - 2*a*b*lo
g(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(
d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + a^2*d
*x*tan(d*x)^3*tan(c) - 3*b^2*d*x*tan(d*x)^3*tan(c) - a^2*d*x*tan(d*x)^2*tan
(c)^2 + 3*b^2*d*x*tan(d*x)^2*tan(c)^2 + a^2*d*x*tan(d*x)*tan(c)^3 - 3*b^2*d
*x*tan(d*x)*tan(c)^3 + a*b*tan(d*x)^3*tan(c)^3 - 2*a*b*log(4*(tan(d*x)^4*ta
n(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x
)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c) + 2*a*b*log(4*(tan(d*x)^4*t
an(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*
x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + a^2*tan(d*x)^3*tan(c)^
2 - 3*b^2*tan(d*x)^3*tan(c)^2 - 2*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*
x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(ta
n(c)^2 + 1))*tan(d*x)*tan(c)^3 + a^2*tan(d*x)^2*tan(c)^3 - 3*b^2*tan(d*x)^2
*tan(c)^3 - a^2*d*x*tan(d*x)^2 + 3*b^2*d*x*tan(d*x)^2 + a^2*d*x*tan(d*x)*ta
n(c) - 3*b^2*d*x*tan(d*x)*tan(c) - a*b*tan(d*x)^3*tan(c) - a^2*d*x*tan(c)^2
 + 3*b^2*d*x*tan(c)^2 - 5*a*b*tan(d*x)^2*tan(c)^2 - a*b*tan(d*x)*tan(c)^3 +
 2*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)
^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2 - 2*b^2
*tan(d*x)^3 - 2*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(
d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d
*x)*tan(c) - 2*a^2*tan(d*x)^2*tan(c) + 2*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2
*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
 1)/(tan(c)^2 + 1))*tan(c)^2 - 2*a^2*tan(d*x)*tan(c)^2 - 2*b^2*tan(c)^3 - a
^2*d*x + 3*b^2*d*x + a*b*tan(d*x)^2 + 5*a*b*tan(d*x)*tan(c) + a*b*tan(c)^2
+ 2*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c
)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + a^2*tan(d*x) -
3*b^2*tan(d*x) + a^2*tan(c) - 3*b^2*tan(c) - a*b)/(d*tan(d*x)^3*tan(c)^3 +
d*tan(d*x)^3*tan(c) - d*tan(d*x)^2*tan(c)^2 + d*tan(d*x)*tan(c)^3 - d*tan(d
*x)^2 + d*tan(d*x)*tan(c) - d*tan(c)^2 - d)
```

maple [B] time = 0.34, size = 145, normalized size = 1.91

$$-\frac{a^2\cos(dx+c)\sin(dx+c)}{2d}+\frac{a^2x}{2}+\frac{a^2c}{2d}-\frac{ab\left(\sin^2(dx+c)\right)}{d}-\frac{2ab\ln(\cos(dx+c))}{d}+\frac{b^2\left(\sin^5(dx+c)\right)}{d\cos(dx+c)}+\frac{b^2\cos(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x)
```

```
[Out] -1/2/d*a^2*cos(d*x+c)*sin(d*x+c)+1/2*a^2*x+1/2/d*a^2*c-1/d*a*b*sin(d*x+c)^2
-2*a*b*ln(cos(d*x+c))/d+1/d*b^2*sin(d*x+c)^5/cos(d*x+c)+1/d*b^2*cos(d*x+c)*
sin(d*x+c)^3+3/2/d*b^2*cos(d*x+c)*sin(d*x+c)-3/2*b^2*x-3/2/d*c*b^2
```

maxima [A] time = 0.43, size = 82, normalized size = 1.08

$$\frac{2\,ab\log\left(\tan(dx+c)^2+1\right)+2\,b^2\tan(dx+c)+\left(a^2-3\,b^2\right)(dx+c)+\frac{2\,ab-\left(a^2-b^2\right)\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*b*log(tan(d*x + c)^2 + 1) + 2*b^2*tan(d*x + c) + (a^2 - 3*b^2)*(d*
x + c) + (2*a*b - (a^2 - b^2)*tan(d*x + c))/(tan(d*x + c)^2 + 1))/d
```

mupad [B] time = 3.69, size = 75, normalized size = 0.99

$$\frac{\cos(c+d\,x)^2\,\left(a\,b-\tan(c+d\,x)\,\left(\frac{a^2}{2}-\frac{b^2}{2}\right)\right)+b^2\tan(c+d\,x)+a\,b\,\ln\left(\tan(c+d\,x)^2+1\right)+d\,x\,\left(\frac{a^2}{2}-\frac{3\,b^2}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (cos(c + d*x)^2*(a*b - tan(c + d*x)*(a^2/2 - b^2/2)) + b^2*tan(c + d*x) + a
*b*log(tan(c + d*x)^2 + 1) + d*x*(a^2/2 - (3*b^2)/2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^2 \sin^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sin(c + d*x)**2, x)
```

3.25 $\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=68

$$-\frac{a^2\cos(c+dx)}{d}-\frac{2ab\sin(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\cos(c+dx)}{d}+\frac{b^2\sec(c+dx)}{d}$$

```
[Out] 2*a*b*arctanh(sin(d*x+c))/d-a^2*cos(d*x+c)/d+b^2*cos(d*x+c)/d+b^2*sec(d*x+c
)/d-2*a*b*sin(d*x+c)/d
```

Rubi [A] time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3517, 2638, 2592, 321, 206, 2590, 14}

$$-\frac{a^2\cos(c+dx)}{d}-\frac{2ab\sin(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\cos(c+dx)}{d}+\frac{b^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - (a^2*Cos[c + d*x])/d + (b^2*Cos[c + d*x])
/d + (b^2*Sec[c + d*x])/d - (2*a*b*Sin[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a^2\sin(c+dx) + 2ab\sin(c+dx)\tan(c+dx) + b^2\sin(c+dx)\tan^2(c+$$

$$= a^2\int \sin(c+dx)\,dx + (2ab)\int \sin(c+dx)\tan(c+dx)\,dx + b^2\int \sin(c+$$

$$= -\frac{a^2\cos(c+dx)}{d} + \frac{(2ab)\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} - \frac{b^2\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{a^2\cos(c+dx)}{d} - \frac{2ab\sin(c+dx)}{d} + \frac{(2ab)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx\right.}{d}$$

$$= \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} - \frac{a^2\cos(c+dx)}{d} + \frac{b^2\cos(c+dx)}{d} + \frac{b^2\sec(c+}{d}$$

Mathematica [A] time = 0.47, size = 111, normalized size = 1.63

$$-\frac{\sec(c+dx)\left(\left(a^2-b^2\right)\cos(2(c+dx)) + a^2 + 2ab\sin(2(c+dx)) + 4ab\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}\right.\right.\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/2*(Sec[c + d*x]*(a^2 - 3*b^2 + (a^2 - b^2)*Cos[2*(c + d*x)] + 4*a*b*Cos[
c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]]) + 2*a*b*Sin[2*(c + d*x)]))/d
```

fricas [A] time = 0.52, size = 90, normalized size = 1.32

$$\frac{ab\cos(dx+c)\log(\sin(dx+c)+1) - ab\cos(dx+c)\log(-\sin(dx+c)+1) - 2\,ab\cos(dx+c)\sin(dx+c) - \left(a\right.}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a*b*cos(d*x + c)*log(sin(d*x + c) + 1) - a*b*cos(d*x + c)*log(-sin(d*x + c
) + 1) - 2*a*b*cos(d*x + c)*sin(d*x + c) - (a^2 - b^2)*cos(d*x + c)^2 + b^2
)/(d*cos(d*x + c))
```

giac [B] time = 11.78, size = 2837, normalized size = 41.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*
tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan
(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*
```

```
d*x)^4*tan(1/2*c)^4 - a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + a^2*tan(1/2*d*x)^4*tan(1/2*c)^4 -
 2*b^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*
d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2
*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 4*a*b*log(2*(tan
(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*t
an(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*ta
n(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c
)^3 - 4*a*b*tan(1/2*d*x)^4*tan(1/2*c)^3 - 4*a*b*tan(1/2*d*x)^3*tan(1/2*c)^4
 - 2*a^2*tan(1/2*d*x)^4*tan(1/2*c)^2 - 8*a^2*tan(1/2*d*x)^3*tan(1/2*c)^3 +
8*b^2*tan(1/2*d*x)^3*tan(1/2*c)^3 - 2*a^2*tan(1/2*d*x)^2*tan(1/2*c)^4 - a*b
*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1
/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^
4 + a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) -
2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^4 - 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(
1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1
))*tan(1/2*d*x)^3*tan(1/2*c) + 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4
 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) +
1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c) + 4*a*b*tan(1/2*d*x)^4*tan
(1/2*c) + 24*a*b*tan(1/2*d*x)^3*tan(1/2*c)^2 - 4*a*b*log(2*(tan(1/2*d*x)^4*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2
+ tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan
(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^3 + 4*a*b*lo
g(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*
d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan
(1/2*c)^3 + 24*a*b*tan(1/2*d*x)^2*tan(1/2*c)^3 - a*b*log(2*(tan(1/2*d*x)^4*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2
+ tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan
(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^4 + a*b*log(2*(tan(1/2*d*
x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*
c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 -
2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d
*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^4 + 4*a*b*tan(1/2*d*
x)*tan(1/2*c)^4 + a^2*tan(1/2*d*x)^4 - 2*b^2*tan(1/2*d*x)^4 + 8*a^2*tan(1/2
*d*x)^3*tan(1/2*c) - 8*b^2*tan(1/2*d*x)^3*tan(1/2*c) + 20*a^2*tan(1/2*d*x)^
2*tan(1/2*c)^2 - 24*b^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a^2*tan(1/2*d*x)*ta
n(1/2*c)^3 - 8*b^2*tan(1/2*d*x)*tan(1/2*c)^3 + a^2*tan(1/2*c)^4 - 2*b^2*tan
(1/2*c)^4 - 4*a*b*tan(1/2*d*x)^3 - 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*
x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*ta
```

```
n(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c
) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c) + 4*a*b*log(2*(tan(1/2*d
*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2
*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 -
 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*
d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c) - 24*a
*b*tan(1/2*d*x)^2*tan(1/2*c) - 24*a*b*tan(1/2*d*x)*tan(1/2*c)^2 - 4*a*b*tan
(1/2*c)^3 - 2*a^2*tan(1/2*d*x)^2 - 8*a^2*tan(1/2*d*x)*tan(1/2*c) + 8*b^2*ta
n(1/2*d*x)*tan(1/2*c) - 2*a^2*tan(1/2*c)^2 + a*b*log(2*(tan(1/2*d*x)^4*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + ta
n(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2
*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*t
an(1/2*c) + 1)/(tan(1/2*c)^2 + 1)) - a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*
x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c
) + 1)/(tan(1/2*c)^2 + 1)) + 4*a*b*tan(1/2*d*x) + 4*a*b*tan(1/2*c) + a^2 -
2*b^2)/(d*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*d*tan(1/2*d*x)^3*tan(1/2*c)^3 - d
*tan(1/2*d*x)^4 - 4*d*tan(1/2*d*x)^3*tan(1/2*c) - 4*d*tan(1/2*d*x)*tan(1/2*
c)^3 - d*tan(1/2*c)^4 - 4*d*tan(1/2*d*x)*tan(1/2*c) + d)
```

maple [A] time = 0.31, size = 108, normalized size = 1.59

$$-\frac{a^2\cos(dx+c)}{d}-\frac{2ab\sin(dx+c)}{d}+\frac{2ab\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^2\left(\sin^4(dx+c)\right)}{d\cos(dx+c)}+\frac{b^2\cos(dx+c)\left(\sin^2\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*tan(d*x+c))^2,x)
```

```
[Out] -a^2*cos(d*x+c)/d-2*a*b*sin(d*x+c)/d+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*
b^2*sin(d*x+c)^4/cos(d*x+c)+1/d*b^2*cos(d*x+c)*sin(d*x+c)^2+2*b^2*cos(d*x+c
)/d
```

maxima [A] time = 0.37, size = 67, normalized size = 0.99

$$\frac{b^2\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)+ab\left(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\sin(dx+c)\right)-a^2\cos(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (b^2*(1/cos(d*x + c) + cos(d*x + c)) + a*b*(log(sin(d*x + c) + 1) - log(sin
(d*x + c) - 1) - 2*sin(d*x + c)) - a^2*cos(d*x + c))/d
```

mupad [B] time = 4.04, size = 93, normalized size = 1.37

$$\frac{4\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-2\,a^2+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4\,b^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (4*a*b*atanh(tan(c/2 + (d*x)/2)))/d - (2*a^2*tan(c/2 + (d*x)/2)^2 - 2*a^2 +
 4*b^2 + 4*a*b*tan(c/2 + (d*x)/2)^3 - 4*a*b*tan(c/2 + (d*x)/2))/(d*(tan(c/2
 + (d*x)/2)^4 - 1))
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^2 \sin(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sin(c + d*x), x)
```

3.26 $\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=43

$$-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

```
[Out] -a^2*arctanh(cos(d*x+c))/d+2*a*b*arctanh(sin(d*x+c))/d+b^2*sec(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3517, 3770, 2606, 8}

$$-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((a^2*ArcTanh[Cos[c + d*x]])/d) + (2*a*b*ArcTanh[Sin[c + d*x]])/d + (b^2*S
ec[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx &= \int \left(a^2\csc(c+dx)+2ab\sec(c+dx)+b^2\sec(c+dx)\tan(c+dx)\right)dx\\ &= a^2\int \csc(c+dx)\,dx+(2ab)\int \sec(c+dx)\,dx+b^2\int \sec(c+dx)\tan(c+\\ &= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\,\text{Subst}(\int 1\,dx,x}{d}\\ &= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}\end{aligned}$$

Mathematica [B] time = 0.25, size = 97, normalized size = 2.26

$$\frac{a\left(a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2b\log\left(\sin\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a*(-(a*Log[Cos[(c + d*x)/2]]) - 2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] + a*Log[Sin[(c + d*x)/2]] + 2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
) + b^2*Sec[c + d*x])/d
```

fricas [B] time = 0.45, size = 102, normalized size = 2.37

$$-\frac{a^2\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-a^2\cos(dx+c)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2\,ab\cos(dx+c)\log(\mathrm{s}}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2) - a^2*cos(d*x + c)*log(-
1/2*cos(d*x + c) + 1/2) - 2*a*b*cos(d*x + c)*log(sin(d*x + c) + 1) + 2*a*b*
cos(d*x + c)*log(-sin(d*x + c) + 1) - 2*b^2)/(d*cos(d*x + c))
```

giac [A] time = 0.93, size = 74, normalized size = 1.72

$$\frac{2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\,b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a*b*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*b^2/(tan(1/2*d*x + 1/2*c
)^2 - 1))/d
```

maple [A] time = 0.33, size = 61, normalized size = 1.42

$$\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{d}+\frac{2ab\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^2}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*a^2*ln(csc(d*x+c)-cot(d*x+c))+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*b^2
/cos(d*x+c)
```

maxima [A] time = 0.55, size = 60, normalized size = 1.40

$$\frac{ab\left(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)-a^2\log(\cot(dx+c)+\csc(dx+c))+\frac{b^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - a^2*log(cot(d*x + c)
 + csc(d*x + c)) + b^2/cos(d*x + c))/d
```

mupad [B] time = 3.72, size = 125, normalized size = 2.91

$$\frac{a^2 \ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,b^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}-\frac{4\,a\,b\,\mathrm{atanh}\left(\frac{16\,a^2\,b^2}{8\,a^3\,b-16\,a^2\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{8\,a^3\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^3\,b-16\,a^2\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/sin(c + d*x),x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d - (2*b^2)/(d*(tan(c/2 + (d*x)/2)^2 - 1)) -
(4*a*b*atanh((16*a^2*b^2)/(8*a^3*b - 16*a^2*b^2*tan(c/2 + (d*x)/2)) - (8*a^
3*b*tan(c/2 + (d*x)/2))/(8*a^3*b - 16*a^2*b^2*tan(c/2 + (d*x)/2))))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^2 \csc(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x), x)
```

3.27 $\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=42

$$-\frac{a^2\cot(c+dx)}{d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] -a^2*cot(d*x+c)/d+2*a*b*ln(tan(d*x+c))/d+b^2*tan(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 43}

$$-\frac{a^2\cot(c+dx)}{d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((a^2*Cot[c + d*x])/d) + (2*a*b*Log[Tan[c + d*x]])/d + (b^2*Tan[c + d*x])/
d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx &= \frac{b\,\text{Subst}\left(\int \frac{(a+x)^2}{x^2}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\,\text{Subst}\left(\int \left(1+\frac{a^2}{x^2}+\frac{2a}{x}\right)dx,x,b\tan(c+dx)\right)}{d}\\ &= -\frac{a^2\cot(c+dx)}{d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}\end{aligned}$$

Mathematica [B] time = 0.58, size = 91, normalized size = 2.17

$$-\frac{\cos(c+dx)(a+b\tan(c+dx))^2\left(a\cos(c+dx)(a\cot(c+dx)+2b(\log(\cos(c+dx))-\log(\sin(c+dx))))-b^2 s\right.}{d(a\cos(c+dx)+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((Cos[c + d*x]*(a*Cos[c + d*x]*(a*Cot[c + d*x] + 2*b*(Log[Cos[c + d*x]] -
Log[Sin[c + d*x]])) - b^2*Sin[c + d*x])*(a + b*Tan[c + d*x])^2)/(d*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^2))
```

fricas [B] time = 0.46, size = 96, normalized size = 2.29

$$-\frac{ab\cos\left(dx+c\right)\log\left(\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)-ab\cos\left(dx+c\right)\log\left(-\frac{1}{4}\cos\left(dx+c\right)^2+\frac{1}{4}\right)\sin\left(dx+c\right)+\left(a^2+\right.}{d\cos\left(dx+c\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a*b*cos(d*x + c)*log(cos(d*x + c)^2)*sin(d*x + c) - a*b*cos(d*x + c)*log(
-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) + (a^2 + b^2)*cos(d*x + c)^2 - b^2)
/(d*cos(d*x + c)*sin(d*x + c))
```

giac [A] time = 0.87, size = 51, normalized size = 1.21

$$\frac{2\,ab\log\left(\left|\tan\left(dx+c\right)\right|\right)+b^2\tan\left(dx+c\right)-\frac{2\,ab\tan\left(dx+c\right)+a^2}{\tan\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a*b*log(abs(tan(d*x + c))) + b^2*tan(d*x + c) - (2*a*b*tan(d*x + c) + a^
2)/tan(d*x + c))/d
```

maple [A] time = 0.47, size = 43, normalized size = 1.02

$$-\frac{a^2\cot\left(dx+c\right)}{d}+\frac{2ab\ln\left(\tan\left(dx+c\right)\right)}{d}+\frac{b^2\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x)
```

```
[Out] -a^2*cot(d*x+c)/d+2*a*b*ln(tan(d*x+c))/d+b^2*tan(d*x+c)/d
```

maxima [A] time = 0.47, size = 39, normalized size = 0.93

$$\frac{2\,ab\log\left(\tan\left(dx+c\right)\right)+b^2\tan\left(dx+c\right)-\frac{a^2}{\tan\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (2*a*b*log(tan(d*x + c)) + b^2*tan(d*x + c) - a^2/tan(d*x + c))/d
```

mupad [B] time = 3.64, size = 44, normalized size = 1.05

$$\frac{b^2\tan\left(c+d\,x\right)}{d}-\frac{a^2}{d\tan\left(c+d\,x\right)}+\frac{2\,a\,b\,\ln\left(\tan\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/sin(c + d*x)^2,x)
```

```
[Out] (b^2*tan(c + d*x))/d - a^2/(d*tan(c + d*x)) + (2*a*b*log(tan(c + d*x)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)^2\csc^2\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**2, x)
```

3.28 $\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=95

$$-\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}-\frac{2ab\csc(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sec(c+d}{d}$$

```
[Out] -1/2*a^2*arctanh(cos(d*x+c))/d-b^2*arctanh(cos(d*x+c))/d+2*a*b*arctanh(sin(
d*x+c))/d-2*a*b*csc(d*x+c)/d-1/2*a^2*cot(d*x+c)*csc(d*x+c)/d+b^2*sec(d*x+c)
/d
```

Rubi [A] time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3517, 3768, 3770, 2621, 321, 207, 2622}

$$-\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}-\frac{2ab\csc(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sec(c+d}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(a^2*ArcTanh[Cos[c + d*x]])/(2*d) - (b^2*ArcTanh[Cos[c + d*x]])/d + (2*a*b
*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Csc[c + d*x])/d - (a^2*Cot[c + d*x]*Csc[
c + d*x])/(2*d) + (b^2*Sec[c + d*x])/d
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a^2\csc^3(c+dx)+2ab\csc^2(c+dx)\sec(c+dx)+b^2\csc(c+dx)\sec^2(\right.$$

$$= a^2\int \csc^3(c+dx)\,dx+(2ab)\int \csc^2(c+dx)\sec(c+dx)\,dx+b^2\int \csc(c$$

$$= -\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}+\frac{1}{2}a^2\int \csc(c+dx)\,dx-\frac{(2ab)\operatorname{Subst}\left(\int \frac{}{-1}\right.}{}$$

$$= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{2ab\csc(c+dx)}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}$$

$$= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\tanh^{-1}(\sin(c}{d}$$

Mathematica [B] time = 1.92, size = 250, normalized size = 2.63

$$4\left(a^2+2b^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-4\left(a^2+2b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-a^2\csc^2\left(\frac{1}{2}(c+dx)\right)+a^2\sec^2\left(\frac{1}{2}(c+dx)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (8*b^2 - 8*a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 - 4*(a^2 + 2*b^2)*
Log[Cos[(c + d*x)/2]] - 16*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4
*(a^2 + 2*b^2)*Log[Sin[(c + d*x)/2]] + 16*a*b*Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]] + a^2*Sec[(c + d*x)/2]^2 + (8*b^2*Sin[(c + d*x)/2])/(Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]) - (8*b^2*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + S
in[(c + d*x)/2]) - 8*a*b*Tan[(c + d*x)/2])/(8*d)
```

fricas [B] time = 0.63, size = 230, normalized size = 2.42

$$8\,ab\cos(dx+c)\sin(dx+c)+2\left(a^2+2\,b^2\right)\cos(dx+c)^2-4\,b^2-\left(\left(a^2+2\,b^2\right)\cos(dx+c)^3-\left(a^2+2\,b^2\right)\cos(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(8*a*b*cos(d*x + c)*sin(d*x + c) + 2*(a^2 + 2*b^2)*cos(d*x + c)^2 - 4*b
^2 - ((a^2 + 2*b^2)*cos(d*x + c)^3 - (a^2 + 2*b^2)*cos(d*x + c))*log(1/2*co
s(d*x + c) + 1/2) + ((a^2 + 2*b^2)*cos(d*x + c)^3 - (a^2 + 2*b^2)*cos(d*x +
 c))*log(-1/2*cos(d*x + c) + 1/2) + 4*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c
```

```
))*log(sin(d*x + c) + 1) - 4*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*log(-s
in(d*x + c) + 1))/(d*cos(d*x + c)^3 - d*cos(d*x + c))
```

giac [A] time = 1.59, size = 172, normalized size = 1.81

$$\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+16\,ab\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-16\,ab\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-8\,ab\tan\left(\frac{1}{2}dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*tan(1/2*d*x + 1/2*c)^2 + 16*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1))
 - 16*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 8*a*b*tan(1/2*d*x + 1/2*c) +
 4*(a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) - 16*b^2/(tan(1/2*d*x + 1/2
*c)^2 - 1) - (6*a^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^2*tan(1/2*d*x + 1/2*c)^2
+ 8*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c)^2)/d
```

maple [A] time = 0.53, size = 120, normalized size = 1.26

$$-\frac{a^2\cot(dx+c)\csc(dx+c)}{2d}+\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{2d}-\frac{2ab}{d\sin(dx+c)}+\frac{2ab\ln(\sec(dx+c)+\tan(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x)
```

```
[Out] -1/2*a^2*cot(d*x+c)*csc(d*x+c)/d+1/2/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/d*a*
b/sin(d*x+c)+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*b^2/cos(d*x+c)+1/d*b^2*l
n(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.35, size = 122, normalized size = 1.28

$$\frac{a^2\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)+2\,b^2\left(\frac{2}{\cos(dx+c)}-\log(\cos(dx+c)+1)+\log\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + log
(cos(d*x + c) - 1)) + 2*b^2*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(c
os(d*x + c) - 1)) - 4*a*b*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin
(d*x + c) - 1)))/d
```

mupad [B] time = 3.84, size = 292, normalized size = 3.07

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(\frac{a^2}{2}+b^2\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{a^2}{2}+8\,b^2\right)-\frac{a^2}{2}+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-4\,a\,b\,\mathrm{ta}}{d\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/sin(c + d*x)^3,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) + (log(tan(c/2 + (d*x)/2))*(a^2/2 + b^2))/
d + (tan(c/2 + (d*x)/2)^2*(a^2/2 + 8*b^2) - a^2/2 + 4*a*b*tan(c/2 + (d*x)/2
)^3 - 4*a*b*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d
```

```
*x)/2)^4)) + (4*a*b*atanh((8*a*b^3*tan(c/2 + (d*x)/2))/(8*a*b^3 + 4*a^3*b -
 16*a^2*b^2*tan(c/2 + (d*x)/2)) - (16*a^2*b^2)/(8*a*b^3 + 4*a^3*b - 16*a^2*
b^2*tan(c/2 + (d*x)/2)) + (4*a^3*b*tan(c/2 + (d*x)/2))/(8*a*b^3 + 4*a^3*b -
 16*a^2*b^2*tan(c/2 + (d*x)/2))))/d - (a*b*tan(c/2 + (d*x)/2))/d
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan (c + dx))^2 \csc^3 (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**3, x)
```

3.29 $\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=79

$$-\frac{(a^2+b^2)\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{ab\cot^2(c+dx)}{d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] -(a^2+b^2)*cot(d*x+c)/d-a*b*cot(d*x+c)^2/d-1/3*a^2*cot(d*x+c)^3/d+2*a*b*ln(
tan(d*x+c))/d+b^2*tan(d*x+c)/d
```

Rubi [A] time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 894}

$$-\frac{(a^2+b^2)\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{ab\cot^2(c+dx)}{d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 + b^2)*Cot[c + d*x])/d) - (a*b*Cot[c + d*x]^2)/d - (a^2*Cot[c + d*x
]^3)/(3*d) + (2*a*b*Log[Tan[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx &= \frac{b\,\text{Subst}\left(\int \frac{(a+x)^2(b^2+x^2)}{x^4}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\,\text{Subst}\left(\int \left(1+\frac{a^2b^2}{x^4}+\frac{2ab^2}{x^3}+\frac{a^2+b^2}{x^2}+\frac{2a}{x}\right)dx,x,b\tan(c+dx)\right)}{d}\\ &= -\frac{(a^2+b^2)\cot(c+dx)}{d}-\frac{ab\cot^2(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{2ab\log}{}\end{aligned}$$

Mathematica [A] time = 1.43, size = 127, normalized size = 1.61

$$-\frac{(a+b\tan(c+dx))^2\left(\cos^2(c+dx)\left((2a^2+3b^2)\cot(c+dx)+6ab(\log(\cos(c+dx))-\log(\sin(c+dx)))\right)+a^2\right.}{3d(a\cos(c+dx)+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/3*((3*a*b*Cot[c + d*x]^2 + a^2*Cot[c + d*x]^3 + Cos[c + d*x]^2*((2*a^2 +
 3*b^2)*Cot[c + d*x] + 6*a*b*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]])) - (3*
b^2*Sin[2*(c + d*x)])/2)*(a + b*Tan[c + d*x])^2)/(d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)
```

fricas [B] time = 0.45, size = 174, normalized size = 2.20

$$-\frac{2\left(a^2+3\,b^2\right)\cos\left(dx+c\right)^4-3\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)-3\left(a^2+3\,b^2\right)\cos\left(dx+c\right)^2+3\left(ab\cos\left(dx+c\right)^3-ab\right.}{3\left(d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^2 + 3*b^2)*cos(d*x + c)^4 - 3*a*b*cos(d*x + c)*sin(d*x + c) - 3*
(a^2 + 3*b^2)*cos(d*x + c)^2 + 3*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*lo
g(cos(d*x + c)^2)*sin(d*x + c) - 3*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*
log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) + 3*b^2)/((d*cos(d*x + c)^3 - d
*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 0.78, size = 91, normalized size = 1.15

$$\frac{6\,ab\log\left(\left|\tan\left(dx+c\right)\right|\right)+3\,b^2\tan\left(dx+c\right)-\frac{11\,ab\tan\left(dx+c\right)^3+3\,a^2\tan\left(dx+c\right)^2+3\,b^2\tan\left(dx+c\right)^2+3\,ab\tan\left(dx+c\right)+a^2}{\tan\left(dx+c\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*a*b*log(abs(tan(d*x + c))) + 3*b^2*tan(d*x + c) - (11*a*b*tan(d*x +
c)^3 + 3*a^2*tan(d*x + c)^2 + 3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) + a
^2)/tan(d*x + c)^3)/d
```

maple [A] time = 0.56, size = 104, normalized size = 1.32

$$-\frac{2a^2\cot\left(dx+c\right)}{3d}-\frac{a^2\cot\left(dx+c\right)\left(\csc^2\left(dx+c\right)\right)}{3d}-\frac{ab}{d\sin\left(dx+c\right)^2}+\frac{2ab\ln\left(\tan\left(dx+c\right)\right)}{d}+\frac{b^2}{d\sin\left(dx+c\right)\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x)
```

```
[Out] -2/3*a^2*cot(d*x+c)/d-1/3/d*a^2*cot(d*x+c)*csc(d*x+c)^2-1/d*a*b/sin(d*x+c)^
2+2*a*b*ln(tan(d*x+c))/d+1/d*b^2/sin(d*x+c)/cos(d*x+c)-2/d*b^2*cot(d*x+c)
```

maxima [A] time = 0.62, size = 69, normalized size = 0.87

$$\frac{6\,ab\log\left(\tan\left(dx+c\right)\right)+3\,b^2\tan\left(dx+c\right)-\frac{3\,ab\tan\left(dx+c\right)+3\left(a^2+b^2\right)\tan\left(dx+c\right)^2+a^2}{\tan\left(dx+c\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(6*a*b*log(tan(d*x + c)) + 3*b^2*tan(d*x + c) - (3*a*b*tan(d*x + c) + 3
*(a^2 + b^2)*tan(d*x + c)^2 + a^2)/tan(d*x + c)^3)/d
```

mupad [B] time = 3.79, size = 72, normalized size = 0.91

$$\frac{b^2 \tan(c + d\,x)}{d} - \frac{\tan(c + d\,x)^2 \left(a^2 + b^2\right) + \frac{a^2}{3} + a\,b\tan(c + d\,x)}{d\tan(c + d\,x)^3} + \frac{2\,a\,b\,\ln(\tan(c + d\,x))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/sin(c + d*x)^4,x)
```

```
[Out] (b^2*tan(c + d*x))/d - (tan(c + d*x)^2*(a^2 + b^2) + a^2/3 + a*b*tan(c + d*
x))/(d*tan(c + d*x)^3) + (2*a*b*log(tan(c + d*x)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^2 \csc^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**4, x)
```

3.30 $\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=165

$$-\frac{3a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^2\cot(c+dx)\csc(c+dx)}{8d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c}{d}$$

```
[Out] -3/8*a^2*arctanh(cos(d*x+c))/d-3/2*b^2*arctanh(cos(d*x+c))/d+2*a*b*arctanh(
sin(d*x+c))/d-2*a*b*csc(d*x+c)/d-3/8*a^2*cot(d*x+c)*csc(d*x+c)/d-2/3*a*b*cs
c(d*x+c)^3/d-1/4*a^2*cot(d*x+c)*csc(d*x+c)^3/d+3/2*b^2*sec(d*x+c)/d-1/2*b^2
*csc(d*x+c)^2*sec(d*x+c)/d
```

Rubi [A] time = 0.16, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3517, 3768, 3770, 2621, 302, 207, 2622, 288, 321}

$$-\frac{3a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^2\cot(c+dx)\csc(c+dx)}{8d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-3*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (3*b^2*ArcTanh[Cos[c + d*x]])/(2*d)
+ (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Csc[c + d*x])/d - (3*a^2*Cot[c +
 d*x]*Csc[c + d*x])/(8*d) - (2*a*b*Csc[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x
]*Csc[c + d*x]^3)/(4*d) + (3*b^2*Sec[c + d*x])/(2*d) - (b^2*Csc[c + d*x]^2*
Sec[c + d*x])/(2*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
```

```
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a^2\csc^5(c+dx)+2ab\csc^4(c+dx)\sec(c+dx)+b^2\csc^3(c+dx)\,\mathrm{se}\right.$$

$$= a^2\int \csc^5(c+dx)\,dx+(2ab)\int \csc^4(c+dx)\sec(c+dx)\,dx+b^2\int \mathrm{cs}$$

$$= -\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{1}{4}\left(3a^2\right)\int \csc^3(c+dx)\,dx-\frac{(2ab)\,\mathrm{Sul}}{}$$

$$= -\frac{3a^2\cot(c+dx)\csc(c+dx)}{8d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{b^2\csc^2(}{}$$

$$= -\frac{3a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{2ab\csc(c+dx)}{d}-\frac{3a^2\cot(c+dx)\csc(c-}{8d}$$

$$= -\frac{3a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{3b^2\tanh^{-1}(\cos(c+dx))}{2d}+\frac{2ab\tanh^{-1}(\mathrm{s}}{d}$$

Mathematica [B] time = 6.22, size = 994, normalized size = 6.02

$$-\frac{a^2\cos^2(c+dx)(a+b\tan(c+dx))^2\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d(a\cos(c+dx)+b\sin(c+dx))^2}+\frac{\left(-3a^2-4b^2\right)\cos^2(c+dx)(a+b\tan(c+dx))^2\csc^2\left(\frac{1}{2}(c\right.}{32d(a\cos(c+dx)+b\sin(c+dx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b^2*Cos[c + d*x]^2*(a + b*Tan[c + d*x])^2)/(d*(a*Cos[c + d*x] + b*Sin[c +
d*x])^2) - (7*a*b*Cos[c + d*x]^2*Cot[(c + d*x)/2]*(a + b*Tan[c + d*x])^2)/(
6*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + ((-3*a^2 - 4*b^2)*Cos[c + d*x]^2
*Csc[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^2)/(32*d*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^2) - (a*b*Cos[c + d*x]^2*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(a +
b*Tan[c + d*x])^2)/(12*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (a^2*Cos[c
+ d*x]^2*Csc[(c + d*x)/2]^4*(a + b*Tan[c + d*x])^2)/(64*d*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^2) - (3*(a^2 + 4*b^2)*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2]]
*(a + b*Tan[c + d*x])^2)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (2*a*b
*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x
])^2)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (3*(a^2 + 4*b^2)*Cos[c + d*
x]^2*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^2)/(8*d*(a*Cos[c + d*x] + b
*Sin[c + d*x])^2) + (2*a*b*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2] + Sin[(c + d
*x)/2]]*(a + b*Tan[c + d*x])^2)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (
(3*a^2 + 4*b^2)*Cos[c + d*x]^2*Sec[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^2)/(
32*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (a^2*Cos[c + d*x]^2*Sec[(c + d*
x)/2]^4*(a + b*Tan[c + d*x])^2)/(64*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
+ (b^2*Cos[c + d*x]^2*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^2)/(d*(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (b^2*Co
s[c + d*x]^2*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^2)/(d*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (7*a*b*Cos[c + d
*x]^2*Tan[(c + d*x)/2]*(a + b*Tan[c + d*x])^2)/(6*d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2) - (a*b*Cos[c + d*x]^2*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]*(a
+ b*Tan[c + d*x])^2)/(12*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
```

fricas [B] time = 0.51, size = 333, normalized size = 2.02

$$18\left(a^{2}+4\,b^{2}\right)\cos\left(dx+c\right)^{4}-30\left(a^{2}+4\,b^{2}\right)\cos\left(dx+c\right)^{2}+48\,b^{2}-9\left(\left(a^{2}+4\,b^{2}\right)\cos\left(dx+c\right)^{5}-2\left(a^{2}+4\,b^{2}\right)\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(18*(a^2 + 4*b^2)*cos(d*x + c)^4 - 30*(a^2 + 4*b^2)*cos(d*x + c)^2 + 4
8*b^2 - 9*((a^2 + 4*b^2)*cos(d*x + c)^5 - 2*(a^2 + 4*b^2)*cos(d*x + c)^3 +
(a^2 + 4*b^2)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 9*((a^2 + 4*b^2)*
cos(d*x + c)^5 - 2*(a^2 + 4*b^2)*cos(d*x + c)^3 + (a^2 + 4*b^2)*cos(d*x + c
))*log(-1/2*cos(d*x + c) + 1/2) + 48*(a*b*cos(d*x + c)^5 - 2*a*b*cos(d*x +
c)^3 + a*b*cos(d*x + c))*log(sin(d*x + c) + 1) - 48*(a*b*cos(d*x + c)^5 - 2
*a*b*cos(d*x + c)^3 + a*b*cos(d*x + c))*log(-sin(d*x + c) + 1) + 32*(3*a*b*
cos(d*x + c)^3 - 4*a*b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5 - 2*d*
cos(d*x + c)^3 + d*cos(d*x + c))
```

giac [A] time = 0.81, size = 269, normalized size = 1.63

$$3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-16\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+24\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+24\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+384\,ab\log\left(\left|\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 - 16*a*b*tan(1/2*d*x + 1/2*c)^3 + 24*a^
2*tan(1/2*d*x + 1/2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 384*a*b*log(abs(
tan(1/2*d*x + 1/2*c) + 1)) - 384*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2
40*a*b*tan(1/2*d*x + 1/2*c) + 72*(a^2 + 4*b^2)*log(abs(tan(1/2*d*x + 1/2*c)
)) - 384*b^2/(tan(1/2*d*x + 1/2*c)^2 - 1) - (150*a^2*tan(1/2*d*x + 1/2*c)^4
```

```
 + 600*b^2*tan(1/2*d*x + 1/2*c)^4 + 240*a*b*tan(1/2*d*x + 1/2*c)^3 + 24*a^2
*tan(1/2*d*x + 1/2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 16*a*b*tan(1/2*d*
x + 1/2*c) + 3*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

maple [A] time = 0.43, size = 183, normalized size = 1.11

$$-\frac{a^2\cot(dx+c)\left(\csc^3(dx+c)\right)}{4d}-\frac{3a^2\cot(dx+c)\csc(dx+c)}{8d}+\frac{3a^2\ln(\csc(dx+c)-\cot(dx+c))}{8d}-\frac{2ab}{3d\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x)
```

```
[Out] -1/4*a^2*cot(d*x+c)*csc(d*x+c)^3/d-3/8*a^2*cot(d*x+c)*csc(d*x+c)/d+3/8/d*a^
2*ln(csc(d*x+c)-cot(d*x+c))-2/3/d*a*b/sin(d*x+c)^3-2/d*a*b/sin(d*x+c)+2/d*a
*b*ln(sec(d*x+c)+tan(d*x+c))-1/2/d*b^2/sin(d*x+c)^2/cos(d*x+c)+3/2/d*b^2/co
s(d*x+c)+3/2/d*b^2*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.47, size = 187, normalized size = 1.13

$$\frac{3\,a^2\left(\frac{2\left(3\,\cos(dx+c)^3-5\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+12\,b^2\left(\frac{2\left(3\,\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*a^2*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(
d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 12*b
^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(cos(d*
x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 16*a*b*(2*(3*sin(d*x + c)^2 + 1)/s
in(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 3.91, size = 378, normalized size = 2.29

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,a^2}{8}+\frac{3\,b^2}{2}\right)}{d}+\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{7\,a^2}{4}+2\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(2\,a^2+34\,b^2\right)}{d\,\left(16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/sin(c + d*x)^5,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))*((3*a^2)/8 + (3*b^2)/2))/d + (a^2*tan(c/2 + (d*x)/
2)^4)/(64*d) - (tan(c/2 + (d*x)/2)^2*((7*a^2)/4 + 2*b^2) - tan(c/2 + (d*x)/
2)^4*(2*a^2 + 34*b^2) + a^2/4 + (56*a*b*tan(c/2 + (d*x)/2)^3)/3 - 20*a*b*ta
n(c/2 + (d*x)/2)^5 + (4*a*b*tan(c/2 + (d*x)/2))/3)/(d*(16*tan(c/2 + (d*x)/2
)^4 - 16*tan(c/2 + (d*x)/2)^6)) + (tan(c/2 + (d*x)/2)^2*(a^2/8 + b^2/8))/d
- (a*b*tan(c/2 + (d*x)/2)^3)/(12*d) + (4*a*b*atanh((12*a*b^3*tan(c/2 + (d*x
)/2))/(12*a*b^3 + 3*a^3*b - 16*a^2*b^2*tan(c/2 + (d*x)/2)) - (16*a^2*b^2)/(
12*a*b^3 + 3*a^3*b - 16*a^2*b^2*tan(c/2 + (d*x)/2)) + (3*a^3*b*tan(c/2 + (d
*x)/2))/(12*a*b^3 + 3*a^3*b - 16*a^2*b^2*tan(c/2 + (d*x)/2))))/d - (5*a*b*t
an(c/2 + (d*x)/2))/(4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^2\csc^5(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+b*tan(d*x+c))**2,x)
[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**5, x)
```

3.31 $\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=122

$$-\frac{(2a^2+b^2)\cot^3(c+dx)}{3d}-\frac{(a^2+2b^2)\cot(c+dx)}{d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{ab\cot^4(c+dx)}{2d}-\frac{2ab\cot^2(c+dx)}{d}+\frac{2ab\log}{}$$

```
[Out] -(a^2+2*b^2)*cot(d*x+c)/d-2*a*b*cot(d*x+c)^2/d-1/3*(2*a^2+b^2)*cot(d*x+c)^3
/d-1/2*a*b*cot(d*x+c)^4/d-1/5*a^2*cot(d*x+c)^5/d+2*a*b*ln(tan(d*x+c))/d+b^2
*tan(d*x+c)/d
```

Rubi [A] time = 0.10, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 948}

$$-\frac{(2a^2+b^2)\cot^3(c+dx)}{3d}-\frac{(a^2+2b^2)\cot(c+dx)}{d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{ab\cot^4(c+dx)}{2d}-\frac{2ab\cot^2(c+dx)}{d}+\frac{2ab\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 + 2*b^2)*Cot[c + d*x])/d) - (2*a*b*Cot[c + d*x]^2)/d - ((2*a^2 + b^
2)*Cot[c + d*x]^3)/(3*d) - (a*b*Cot[c + d*x]^4)/(2*d) - (a^2*Cot[c + d*x]^5
)/(5*d) + (2*a*b*Log[Tan[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx &= \frac{b\,\text{Subst}\left(\int \frac{(a+x)^2(b^2+x^2)^2}{x^6}\,dx, x, b\tan(c+dx)\right)}{d}\\
&= \frac{b\,\text{Subst}\left(\int \left(1+\frac{a^2b^4}{x^6}+\frac{2ab^4}{x^5}+\frac{2a^2b^2+b^4}{x^4}+\frac{4ab^2}{x^3}+\frac{a^2+2b^2}{x^2}+\frac{2a}{x}\right)dx, x, b\,\text{ta}\right.}{d}\\
&= -\frac{(a^2+2b^2)\cot(c+dx)}{d}-\frac{2ab\cot^2(c+dx)}{d}-\frac{(2a^2+b^2)\cot^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 1.51, size = 114, normalized size = 0.93

$$-\frac{2\cot(c+dx)\left(\left((4a^2+5b^2)\csc^2(c+dx)+3a^2\csc^4(c+dx)+8a^2+25b^2\right)+15b\left(a\csc^4(c+dx)+2a\csc^2(c+\right.\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/30*(2*Cot[c + d*x]*(8*a^2 + 25*b^2 + (4*a^2 + 5*b^2)*Csc[c + d*x]^2 + 3*
a^2*Csc[c + d*x]^4) + 15*b*(2*a*Csc[c + d*x]^2 + a*Csc[c + d*x]^4 + 4*a*Log
[Cos[c + d*x]] - 4*a*Log[Sin[c + d*x]] - 2*b*Tan[c + d*x]))/d
```

fricas [B] time = 0.44, size = 240, normalized size = 1.97

$$-\frac{16\left(a^2+5\,b^2\right)\cos\left(dx+c\right)^6-40\left(a^2+5\,b^2\right)\cos\left(dx+c\right)^4+30\left(a^2+5\,b^2\right)\cos\left(dx+c\right)^2+30\left(ab\cos\left(dx+c\right)^5-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/30*(16*(a^2 + 5*b^2)*cos(d*x + c)^6 - 40*(a^2 + 5*b^2)*cos(d*x + c)^4 +
30*(a^2 + 5*b^2)*cos(d*x + c)^2 + 30*(a*b*cos(d*x + c)^5 - 2*a*b*cos(d*x +
c)^3 + a*b*cos(d*x + c))*log(cos(d*x + c)^2)*sin(d*x + c) - 30*(a*b*cos(d*x
 + c)^5 - 2*a*b*cos(d*x + c)^3 + a*b*cos(d*x + c))*log(-1/4*cos(d*x + c)^2
+ 1/4)*sin(d*x + c) - 30*b^2 - 15*(2*a*b*cos(d*x + c)^3 - 3*a*b*cos(d*x + c
))*sin(d*x + c))/((d*cos(d*x + c)^5 - 2*d*cos(d*x + c)^3 + d*cos(d*x + c))*
sin(d*x + c))
```

giac [A] time = 0.71, size = 131, normalized size = 1.07

$$\frac{60\,ab\log\left(\left|\tan\left(dx+c\right)\right|\right)+30\,b^2\tan\left(dx+c\right)-\frac{137\,ab\tan(dx+c)^5+30\,a^2\tan(dx+c)^4+60\,b^2\tan(dx+c)^4+60\,ab\tan(dx+c)^3+20\,a^2\tan(dx-}{\tan(dx+c)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(60*a*b*log(abs(tan(d*x + c))) + 30*b^2*tan(d*x + c) - (137*a*b*tan(d*
x + c)^5 + 30*a^2*tan(d*x + c)^4 + 60*b^2*tan(d*x + c)^4 + 60*a*b*tan(d*x +
 c)^3 + 20*a^2*tan(d*x + c)^2 + 10*b^2*tan(d*x + c)^2 + 15*a*b*tan(d*x + c)
 + 6*a^2)/tan(d*x + c)^5)/d
```

maple [A] time = 0.52, size = 166, normalized size = 1.36

$$-\frac{8a^2\cot\left(dx+c\right)}{15d}-\frac{a^2\cot\left(dx+c\right)\left(\csc^4\left(dx+c\right)\right)}{5d}-\frac{4a^2\cot\left(dx+c\right)\left(\csc^2\left(dx+c\right)\right)}{15d}-\frac{ab}{2d\sin\left(dx+c\right)^4}-\frac{ab}{d\sin\left(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x)
```

```
[Out] -8/15*a^2*cot(d*x+c)/d-1/5/d*a^2*cot(d*x+c)*csc(d*x+c)^4-4/15/d*a^2*cot(d*x
+c)*csc(d*x+c)^2-1/2/d*a*b/sin(d*x+c)^4-1/d*a*b/sin(d*x+c)^2+2*a*b*ln(tan(d
*x+c))/d-1/3/d*b^2/sin(d*x+c)^3/cos(d*x+c)+4/3/d*b^2/sin(d*x+c)/cos(d*x+c)-
8/3/d*b^2*cot(d*x+c)
```

maxima [A] time = 0.74, size = 104, normalized size = 0.85

$$\frac{60\,ab\log\left(\tan\left(dx+c\right)\right)+30\,b^2\tan\left(dx+c\right)-\frac{60\,ab\tan(dx+c)^3+30\left(a^2+2\,b^2\right)\tan(dx+c)^4+15\,ab\tan(dx+c)+10\left(2\,a^2+b^2\right)\tan(dx+c)^2+6}{\tan(dx+c)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(60*a*b*log(tan(d*x + c)) + 30*b^2*tan(d*x + c) - (60*a*b*tan(d*x + c)
^3 + 30*(a^2 + 2*b^2)*tan(d*x + c)^4 + 15*a*b*tan(d*x + c) + 10*(2*a^2 + b^
2)*tan(d*x + c)^2 + 6*a^2)/tan(d*x + c)^5)/d
```

mupad [B] time = 4.06, size = 107, normalized size = 0.88

$$\frac{b^2 \tan(c+d\,x)}{d} - \frac{\tan(c+d\,x)^4\,\left(a^2+2\,b^2\right)+\frac{a^2}{5}+\tan(c+d\,x)^2\,\left(\frac{2\,a^2}{3}+\frac{b^2}{3}\right)+\frac{a\,b\,\tan(c+d\,x)}{2}+2\,a\,b\tan(c+d\,x)^3}{d\tan(c+d\,x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/sin(c + d*x)^6,x)
```

```
[Out] (b^2*tan(c + d*x))/d - (tan(c + d*x)^4*(a^2 + 2*b^2) + a^2/5 + tan(c + d*x)
^2*((2*a^2)/3 + b^2/3) + (a*b*tan(c + d*x))/2 + 2*a*b*tan(c + d*x)^3)/(d*ta
n(c + d*x)^5) + (2*a*b*log(tan(c + d*x)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^2 \csc^6(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**6, x)
```

3.32 $\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=205

$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{a^3\cos(c+dx)}{d}-\frac{a^2b\sin^3(c+dx)}{d}-\frac{3a^2b\sin(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{ab^2\cos^3(c+dx)}{d}$$

```
[Out] 3*a^2*b*arctanh(sin(d*x+c))/d-5/2*b^3*arctanh(sin(d*x+c))/d-a^3*cos(d*x+c)/
d+6*a*b^2*cos(d*x+c)/d+1/3*a^3*cos(d*x+c)^3/d-a*b^2*cos(d*x+c)^3/d+3*a*b^2*
sec(d*x+c)/d-3*a^2*b*sin(d*x+c)/d+5/2*b^3*sin(d*x+c)/d-a^2*b*sin(d*x+c)^3/d
+5/6*b^3*sin(d*x+c)^3/d+1/2*b^3*sin(d*x+c)^3*tan(d*x+c)^2/d
```

Rubi [A] time = 0.19, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3517, 2633, 2592, 302, 206, 2590, 270, 288}

$$-\frac{a^2b\sin^3(c+dx)}{d}-\frac{3a^2b\sin(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{a^3\cos(c+dx)}{d}-\frac{ab^2\cos^3(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (5*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cos[c + d*x])/d + (6*a*b^2*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d)
 - (a*b^2*Cos[c + d*x]^3)/d + (3*a*b^2*Sec[c + d*x])/d - (3*a^2*b*Sin[c + d
*x])/d + (5*b^3*Sin[c + d*x])/(2*d) - (a^2*b*Sin[c + d*x]^3)/d + (5*b^3*Sin
[c + d*x]^3)/(6*d) + (b^3*Sin[c + d*x]^3*Tan[c + d*x]^2)/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a^3\sin^3(c+dx)+3a^2b\sin^3(c+dx)\tan(c+dx)+3ab^2\sin^3(c+d$$

$$= a^3\int \sin^3(c+dx)\,dx+\left(3a^2b\right)\int \sin^3(c+dx)\tan(c+dx)\,dx+\left(3ab^2\right)$$

$$= -\frac{a^3\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos(c+dx)\right)}{d}+\frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx,\right.}{d}$$

$$= -\frac{a^3\cos(c+dx)}{d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{b^3\sin^3(c+dx)\tan^2(c+dx)}{2d}+$$

$$= -\frac{a^3\cos(c+dx)}{d}+\frac{6ab^2\cos(c+dx)}{d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{ab^2\cos^3(c+}{d}$$

$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cos(c+dx)}{d}+\frac{6ab^2\cos(c+dx)}{d}+\frac{a^3 c}{}$$

$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{5b^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\cos(c+dx)}{d}$$

Mathematica [B] time = 6.30, size = 771, normalized size = 3.76

$$\frac{\left(5b^3-6a^2b\right)\cos^3(c+dx)(a+b\tan(c+dx))^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{\left(6a^2b-5b^3\right)\cos^3(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3) - (3*a*(a^2 - 7*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3)/(4
*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (a*(a^2 - 3*b^2)*Cos[c + d*x]^3*C
os[3*(c + d*x)]*(a + b*Tan[c + d*x])^3)/(12*d*(a*Cos[c + d*x] + b*Sin[c + d
*x])^3) + ((-6*a^2*b + 5*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3
) + ((6*a^2*b - 5*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
```

```
2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (b^
3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c + d*x)/2] - Sin[(c +
d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (3*a*b^2*Cos[c + d*x]^3*S
in[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x
)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (b^3*Cos[c + d*x]^3*(a + b*Tan
[c + d*x])^3)/(4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x]
+ b*Sin[c + d*x])^3) - (3*a*b^2*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[
c + d*x])^3)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*S
in[c + d*x])^3) - (3*b*(5*a^2 - 3*b^2)*Cos[c + d*x]^3*Sin[c + d*x]*(a + b*T
an[c + d*x])^3)/(4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (b*(3*a^2 - b^2
)*Cos[c + d*x]^3*Sin[3*(c + d*x)]*(a + b*Tan[c + d*x])^3)/(12*d*(a*Cos[c +
d*x] + b*Sin[c + d*x])^3)
```

fricas [A] time = 0.47, size = 188, normalized size = 0.92

$$\frac{4\left(a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{5}+36\,ab^{2}\cos\left(dx+c\right)-12\left(a^{3}-6\,ab^{2}\right)\cos\left(dx+c\right)^{3}+3\left(6\,a^{2}b-5\,b^{3}\right)\cos\left(dx+c\right)^{2}\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*(a^3 - 3*a*b^2)*cos(d*x + c)^5 + 36*a*b^2*cos(d*x + c) - 12*(a^3 -
6*a*b^2)*cos(d*x + c)^3 + 3*(6*a^2*b - 5*b^3)*cos(d*x + c)^2*log(sin(d*x +
c) + 1) - 3*(6*a^2*b - 5*b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*
(3*a^2*b - b^3)*cos(d*x + c)^4 + 3*b^3 - 2*(12*a^2*b - 7*b^3)*cos(d*x + c)^
2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.44, size = 271, normalized size = 1.32

$$-\frac{\cos\left(dx+c\right)\left(\sin^{2}\left(dx+c\right)\right)a^{3}}{3d}-\frac{2a^{3}\cos\left(dx+c\right)}{3d}-\frac{a^{2}b\left(\sin^{3}\left(dx+c\right)\right)}{d}-\frac{3a^{2}b\sin\left(dx+c\right)}{d}+\frac{3a^{2}b\ln\left(\sec\left(dx+c\right)+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x)
```

```
[Out] -1/3/d*cos(d*x+c)*sin(d*x+c)^2*a^3-2/3*a^3*cos(d*x+c)/d-a^2*b*sin(d*x+c)^3/
d-3*a^2*b*sin(d*x+c)/d+3/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3/d*b^2*a*sin(d*
x+c)^6/cos(d*x+c)+8*a*b^2*cos(d*x+c)/d+3/d*b^2*a*cos(d*x+c)*sin(d*x+c)^4+4/
d*cos(d*x+c)*sin(d*x+c)^2*a*b^2+1/2/d*b^3*sin(d*x+c)^7/cos(d*x+c)^2+1/2/d*b
^3*sin(d*x+c)^5+5/6*b^3*sin(d*x+c)^3/d+5/2*b^3*sin(d*x+c)/d-5/2/d*b^3*ln(se
c(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.71, size = 173, normalized size = 0.84

$$\frac{4\left(\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)a^{3}-6\left(2\,\sin\left(dx+c\right)^{3}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)+6\,\mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^3 - 6*(2*sin(d*x + c)^3 - 3*log
(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*a^2*b - 12*(
cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a*b^2 + (4*sin(d*x + c)^3
 - 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(
sin(d*x + c) - 1) + 24*sin(d*x + c))*b^3)/d
```

mupad [B] time = 6.44, size = 291, normalized size = 1.42

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(6\,a^2\,b-5\,b^3\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a^2\,b-5\,b^3\right)+4\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-16\,a\,b^2-\tan\left(\frac{c}{2}+\frac{d}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(6*a^2*b - 5*b^3))/d - (tan(c/2 + (d*x)/2)*(6*a^
2*b - 5*b^3) + 4*a^3*tan(c/2 + (d*x)/2)^6 - 16*a*b^2 - tan(c/2 + (d*x)/2)^2
*(16*a*b^2 - (4*a^3)/3) + tan(c/2 + (d*x)/2)^4*(32*a*b^2 - (20*a^3)/3) + ta
n(c/2 + (d*x)/2)^9*(6*a^2*b - 5*b^3) + tan(c/2 + (d*x)/2)^3*(8*a^2*b - (20*
b^3)/3) + tan(c/2 + (d*x)/2)^7*(8*a^2*b - (20*b^3)/3) - tan(c/2 + (d*x)/2)^
5*(28*a^2*b - (22*b^3)/3) + (4*a^3)/3)/(d*(tan(c/2 + (d*x)/2)^2 - 2*tan(c/2
 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + tan(c/2 + (
d*x)/2)^10 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^3\sin^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sin(c + d*x)**3, x)
```

3.33 $\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=103

$$-\frac{b(3a^2-2b^2)\log(\cos(c+dx))}{d}+\frac{1}{2}ax(a^2-9b^2)+\frac{9ab^2\tan(c+dx)}{2d}-\frac{\sin(c+dx)\cos(c+dx)(a+b\tan(c+dx))^3}{2d}+$$

```
[Out] 1/2*a*(a^2-9*b^2)*x-b*(3*a^2-2*b^2)*ln(cos(d*x+c))/d+9/2*a*b^2*tan(d*x+c)/d
+b^3*tan(d*x+c)^2/d-1/2*cos(d*x+c)*sin(d*x+c)*(a+b*tan(d*x+c))^3/d
```

Rubi [A] time = 0.14, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1645, 801, 635, 203, 260}

$$-\frac{b(3a^2-2b^2)\log(\cos(c+dx))}{d}+\frac{1}{2}ax(a^2-9b^2)+\frac{9ab^2\tan(c+dx)}{2d}-\frac{\sin(c+dx)\cos(c+dx)(a+b\tan(c+dx))^3}{2d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(a^2 - 9*b^2)*x)/2 - (b*(3*a^2 - 2*b^2)*Log[Cos[c + d*x]])/d + (9*a*b^2*
Tan[c + d*x])/(2*d) + (b^3*Tan[c + d*x]^2)/d - (Cos[c + d*x]*Sin[c + d*x]*(
a + b*Tan[c + d*x])^3)/(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^2(a+x)^3}{\left(b^2+x^2\right)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{2d} - \frac{\text{Subst}\left(\int \frac{(a+x)^2\left(-ab^2-4\right.}{b^2+x^2}\right.}{2}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{2d} - \frac{\text{Subst}\left(\int\left(-9ab^2-4b\right.\right.}{}$$

$$= \frac{9ab^2\tan(c+dx)}{2d} + \frac{b^3\tan^2(c+dx)}{d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\,\text{ta}}{2d}$$

$$= \frac{9ab^2\tan(c+dx)}{2d} + \frac{b^3\tan^2(c+dx)}{d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\,\text{ta}}{2d}$$

$$= \frac{1}{2}a\left(a^2-9b^2\right)x - \frac{b\left(3a^2-2b^2\right)\log(\cos(c+dx))}{d} + \frac{9ab^2\tan(c+dx)}{2d}$$

Mathematica [A] time = 4.52, size = 203, normalized size = 1.97

$$\frac{b\left(-\frac{a\left(a^2-3b^2\right)\sin(2(c+dx))}{2b} + \left(3a^2-b^2\right)\cos^2(c+dx) - \frac{a\left(a^2-3b^2\right)\tan^{-1}(\tan(c+dx))}{b} + \left(\frac{a^3-6ab^2}{\sqrt{-b^2}} + 3a^2-2b^2\right)\log\left(\sqrt{-b^2}\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b*(-((a*(a^2 - 3*b^2)*ArcTan[Tan[c + d*x]])/b) + (3*a^2 - b^2)*Cos[c + d*x
]^2 + (3*a^2 - 2*b^2 + (a^3 - 6*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c
 + d*x]] + (3*a^2 - 2*b^2 + (-a^3 + 6*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b
*Tan[c + d*x]] - (a*(a^2 - 3*b^2)*Sin[2*(c + d*x)])/(2*b) + 6*a*b*Tan[c + d
*x] + b^2*Tan[c + d*x]^2))/(2*d)
```

fricas [A] time = 0.46, size = 149, normalized size = 1.45

$$\frac{2\left(3a^2b-b^3\right)\cos(dx+c)^4 - 4\left(3a^2b-2b^3\right)\cos(dx+c)^2\log(-\cos(dx+c)) + 2b^3 - \left(3a^2b-b^3-2\left(a^3-9\right.\right.}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(3*a^2*b - b^3)*cos(d*x + c)^4 - 4*(3*a^2*b - 2*b^3)*cos(d*x + c)^2*
log(-cos(d*x + c)) + 2*b^3 - (3*a^2*b - b^3 - 2*(a^3 - 9*a*b^2)*d*x)*cos(d*
x + c)^2 + 2*(6*a*b^2*cos(d*x + c) - (a^3 - 3*a*b^2)*cos(d*x + c)^3)*sin(d*
x + c))/(d*cos(d*x + c)^2)
```

giac [B] time = 7.28, size = 2590, normalized size = 25.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*a^3*d*x*tan(d*x)^4*tan(c)^4 - 18*a*b^2*d*x*tan(d*x)^4*tan(c)^4 - 6*a
^2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4
+ 4*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c
)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)
^4 + 2*a^3*d*x*tan(d*x)^4*tan(c)^2 - 18*a*b^2*d*x*tan(d*x)^4*tan(c)^2 - 4*a
^3*d*x*tan(d*x)^3*tan(c)^3 + 36*a*b^2*d*x*tan(d*x)^3*tan(c)^3 + 2*a^3*d*x*t
an(d*x)^2*tan(c)^4 - 18*a*b^2*d*x*tan(d*x)^2*tan(c)^4 + 3*a^2*b*tan(d*x)^4*
tan(c)^4 + b^3*tan(d*x)^4*tan(c)^4 - 6*a^2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2
*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^2 + 4*b^3*log(4*(tan(d*x)^4*tan(c)^2
- 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c
) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^2 + 12*a^2*b*log(4*(tan(d*x)^4*tan
(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)
*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 8*b^3*log(4*(tan(d*x)^4*
tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d
*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 2*a^3*tan(d*x)^4*tan(
c)^3 - 18*a*b^2*tan(d*x)^4*tan(c)^3 - 6*a^2*b*log(4*(tan(d*x)^4*tan(c)^2 -
2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
+ 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^4 + 4*b^3*log(4*(tan(d*x)^4*tan(c)^2
 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(
c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^4 + 2*a^3*tan(d*x)^3*tan(c)^4 - 1
8*a*b^2*tan(d*x)^3*tan(c)^4 - 4*a^3*d*x*tan(d*x)^3*tan(c) + 36*a*b^2*d*x*ta
n(d*x)^3*tan(c) + 4*a^3*d*x*tan(d*x)^2*tan(c)^2 - 36*a*b^2*d*x*tan(d*x)^2*t
an(c)^2 - 3*a^2*b*tan(d*x)^4*tan(c)^2 + 5*b^3*tan(d*x)^4*tan(c)^2 - 4*a^3*d
*x*tan(d*x)*tan(c)^3 + 36*a*b^2*d*x*tan(d*x)*tan(c)^3 - 18*a^2*b*tan(d*x)^3
*tan(c)^3 + 6*b^3*tan(d*x)^3*tan(c)^3 - 3*a^2*b*tan(d*x)^2*tan(c)^4 + 5*b^3
*tan(d*x)^2*tan(c)^4 + 12*a^2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*t
an(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2
 + 1))*tan(d*x)^3*tan(c) - 8*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*
tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^
2 + 1))*tan(d*x)^3*tan(c) - 12*a*b^2*tan(d*x)^4*tan(c) - 12*a^2*b*log(4*(ta
n(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2
- 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 8*b^3*log(4*
(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)
^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 6*a^3*tan
(d*x)^3*tan(c)^2 + 18*a*b^2*tan(d*x)^3*tan(c)^2 + 12*a^2*b*log(4*(tan(d*x)^
4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan
(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c)^3 - 8*b^3*log(4*(tan(d*x)
^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*ta
n(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c)^3 - 6*a^3*tan(d*x)^2*tan
(c)^3 + 18*a*b^2*tan(d*x)^2*tan(c)^3 - 12*a*b^2*tan(d*x)*tan(c)^4 + 2*a^3*d
*x*tan(d*x)^2 - 18*a*b^2*d*x*tan(d*x)^2 + 2*b^3*tan(d*x)^4 - 4*a^3*d*x*tan(
d*x)*tan(c) + 36*a*b^2*d*x*tan(d*x)*tan(c) + 6*a^2*b*tan(d*x)^3*tan(c) - 2*
b^3*tan(d*x)^3*tan(c) + 2*a^3*d*x*tan(c)^2 - 18*a*b^2*d*x*tan(c)^2 + 30*a^2
*b*tan(d*x)^2*tan(c)^2 - 2*b^3*tan(d*x)^2*tan(c)^2 + 6*a^2*b*tan(d*x)*tan(c
)^3 - 2*b^3*tan(d*x)*tan(c)^3 + 2*b^3*tan(c)^4 - 6*a^2*b*log(4*(tan(d*x)^4*
tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d
*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2 + 4*b^3*log(4*(tan(d*x)^4*tan(c)
^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*ta
n(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2 + 12*a*b^2*tan(d*x)^3 + 12*a^2*b*log(4
*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x
```

```
)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) - 8*b^3*log(4*
(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)
^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) + 6*a^3*tan(d*x
)^2*tan(c) - 18*a*b^2*tan(d*x)^2*tan(c) - 6*a^2*b*log(4*(tan(d*x)^4*tan(c)^
2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan
(c) + 1)/(tan(c)^2 + 1))*tan(c)^2 + 4*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*ta
n(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)
/(tan(c)^2 + 1))*tan(c)^2 + 6*a^3*tan(d*x)*tan(c)^2 - 18*a*b^2*tan(d*x)*tan
(c)^2 + 12*a*b^2*tan(c)^3 + 2*a^3*d*x - 18*a*b^2*d*x - 3*a^2*b*tan(d*x)^2 +
 5*b^3*tan(d*x)^2 - 18*a^2*b*tan(d*x)*tan(c) + 6*b^3*tan(d*x)*tan(c) - 3*a^
2*b*tan(c)^2 + 5*b^3*tan(c)^2 - 6*a^2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(
d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(
tan(c)^2 + 1)) + 4*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + t
an(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) -
2*a^3*tan(d*x) + 18*a*b^2*tan(d*x) - 2*a^3*tan(c) + 18*a*b^2*tan(c) + 3*a^2
*b + b^3)/(d*tan(d*x)^4*tan(c)^4 + d*tan(d*x)^4*tan(c)^2 - 2*d*tan(d*x)^3*t
an(c)^3 + d*tan(d*x)^2*tan(c)^4 - 2*d*tan(d*x)^3*tan(c) + 2*d*tan(d*x)^2*ta
n(c)^2 - 2*d*tan(d*x)*tan(c)^3 + d*tan(d*x)^2 - 2*d*tan(d*x)*tan(c) + d*tan
(c)^2 + d)
```

maple [B] time = 0.37, size = 226, normalized size = 2.19

$$-\frac{a^3 \sin(dx+c)\cos(dx+c)}{2d}+\frac{a^3 x}{2}+\frac{a^3 c}{2d}-\frac{3a^2 b\left(\sin^2(dx+c)\right)}{2d}-\frac{3a^2 b\ln(\cos(dx+c))}{d}+\frac{3b^2 a\left(\sin^5(dx+c)\right)}{d\cos(dx+c)}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x)
```

```
[Out] -1/2/d*a^3*sin(d*x+c)*cos(d*x+c)+1/2*a^3*x+1/2/d*a^3*c-3/2/d*a^2*b*sin(d*x+
c)^2-3/d*a^2*b*ln(cos(d*x+c))+3/d*b^2*a*sin(d*x+c)^5/cos(d*x+c)+3/d*b^2*a*c
os(d*x+c)*sin(d*x+c)^3+9/2*a*b^2*cos(d*x+c)*sin(d*x+c)/d-9/2*b^2*a*x-9/2/d*
a*b^2*c+1/2/d*b^3*sin(d*x+c)^6/cos(d*x+c)^2+1/2/d*b^3*sin(d*x+c)^4+1/d*sin(
d*x+c)^2*b^3+2*b^3*ln(cos(d*x+c))/d
```

maxima [A] time = 0.45, size = 113, normalized size = 1.10

$$\frac{b^3\tan(dx+c)^2+6\,ab^2\tan(dx+c)+\left(a^3-9\,ab^2\right)(dx+c)+\left(3\,a^2b-2\,b^3\right)\log\left(\tan(dx+c)^2+1\right)+\frac{3\,a^2b-b^3-(a\cdots}{\tan\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*tan(d*x + c)^2 + 6*a*b^2*tan(d*x + c) + (a^3 - 9*a*b^2)*(d*x + c)
+ (3*a^2*b - 2*b^3)*log(tan(d*x + c)^2 + 1) + (3*a^2*b - b^3 - (a^3 - 3*a*b
^2)*tan(d*x + c))/(tan(d*x + c)^2 + 1))/d
```

mupad [B] time = 3.70, size = 151, normalized size = 1.47

$$\frac{b^3\tan(c+d\,x)^2}{2\,d}+\frac{\cos(c+d\,x)^2\,\left(\frac{3\,a^2\,b}{2}-\frac{b^3}{2}+\tan(c+d\,x)\,\left(\frac{3\,a\,b^2}{2}-\frac{a^3}{2}\right)\right)}{d}+\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{3\,a^2\,b}{2}-b^3\right)}{d}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (b^3*tan(c + d*x)^2)/(2*d) + (cos(c + d*x)^2*((3*a^2*b)/2 - b^3/2 + tan(c +
 d*x)*((3*a*b^2)/2 - a^3/2)))/d + (log(tan(c + d*x)^2 + 1)*((3*a^2*b)/2 - b
```

```
^3))/d + (3*a*b^2*tan(c + d*x))/d - (a*atan((a*tan(c + d*x)*(a - 3*b)*(a +
3*b))/(2*((9*a*b^2)/2 - a^3/2)))*(a - 3*b)*(a + 3*b))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^3 \sin^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sin(c + d*x)**2, x)
```

3.34 $\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=133

$$-\frac{a^3\cos(c+dx)}{d}-\frac{3a^2b\sin(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{3ab^2\cos(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{3b^3\sin(c}{2d}$$

```
[Out] 3*a^2*b*arctanh(sin(d*x+c))/d-3/2*b^3*arctanh(sin(d*x+c))/d-a^3*cos(d*x+c)/
d+3*a*b^2*cos(d*x+c)/d+3*a*b^2*sec(d*x+c)/d-3*a^2*b*sin(d*x+c)/d+3/2*b^3*si
n(d*x+c)/d+1/2*b^3*sin(d*x+c)*tan(d*x+c)^2/d
```

Rubi [A] time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3517, 2638, 2592, 321, 206, 2590, 14, 288}

$$-\frac{3a^2b\sin(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cos(c+dx)}{d}+\frac{3ab^2\cos(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{3b^3\sin(c}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (3*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cos[c + d*x])/d + (3*a*b^2*Cos[c + d*x])/d + (3*a*b^2*Sec[c + d*x])/d -
 (3*a^2*b*Sin[c + d*x])/d + (3*b^3*Sin[c + d*x])/(2*d) + (b^3*Sin[c + d*x]*
Tan[c + d*x]^2)/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a^3\sin(c+dx)+3a^2b\sin(c+dx)\tan(c+dx)+3ab^2\sin(c+dx)\tan^2(\right.$$

$$= a^3\int \sin(c+dx)\,dx+\left(3a^2b\right)\int \sin(c+dx)\tan(c+dx)\,dx+\left(3ab^2\right)\int \text{si}$$

$$= -\frac{a^3\cos(c+dx)}{d}+\frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d}-\frac{\left(3ab^2\right)\text{Subs}}{}$$

$$= -\frac{a^3\cos(c+dx)}{d}-\frac{3a^2b\sin(c+dx)}{d}+\frac{b^3\sin(c+dx)\tan^2(c+dx)}{2d}+\frac{\left(3a^2\right.}{}$$

$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cos(c+dx)}{d}+\frac{3ab^2\cos(c+dx)}{d}+\frac{3ab^2\,\text{se}}{}$$

$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{3b^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\cos(c+dx)}{d}+$$

Mathematica [B] time = 6.17, size = 637, normalized size = 4.79

$$-\frac{3\left(2a^2b-b^3\right)\cos^3(c+dx)(a+b\tan(c+dx))^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{3\left(2a^2b-b^3\right)\cos^3(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3) - (a*(a^2 - 3*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3)/(d*(
a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*(2*a^2*b - b^3)*Cos[c + d*x]^3*Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^3) + (3*(2*a^2*b - b^3)*Cos[c + d*x]^3*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x]
 + b*Sin[c + d*x])^3) + (b^3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
+ (3*a*b^2*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[
(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (b^
3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c + d*x)/2] + Sin[(c +
```

```
d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*a*b^2*Cos[c + d*x]^3*S
in[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (b*(3*a^2 - b^2)*Cos[c + d*x]^
3*Sin[c + d*x]*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])
^3)
```

fricas [A] time = 0.45, size = 144, normalized size = 1.08

$$\frac{12\,ab^2\cos(dx+c) - 4\left(a^3 - 3\,ab^2\right)\cos(dx+c)^3 + 3\left(2\,a^2b - b^3\right)\cos(dx+c)^2\log(\sin(dx+c)+1) - 3\left(2\,a^2b\right.}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(12*a*b^2*cos(d*x + c) - 4*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 3*(2*a^2*b
- b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*(2*a^2*b - b^3)*cos(d*x + c
)^2*log(-sin(d*x + c) + 1) + 2*(b^3 - 2*(3*a^2*b - b^3)*cos(d*x + c)^2)*sin
(d*x + c))/(d*cos(d*x + c)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)(-4*a^3*tan(c/2)^6*tan(d*
x/2)^6+12*a^3*tan(c/2)^6*tan(d*x/2)^4-12*a^3*tan(c/2)^6*tan(d*x/2)^2+4*a^3*
tan(c/2)^6+48*a^3*tan(c/2)^5*tan(d*x/2)^5-96*a^3*tan(c/2)^5*tan(d*x/2)^3+48
*a^3*tan(c/2)^5*tan(d*x/2)+12*a^3*tan(c/2)^4*tan(d*x/2)^6-228*a^3*tan(c/2)^
4*tan(d*x/2)^4+228*a^3*tan(c/2)^4*tan(d*x/2)^2-12*a^3*tan(c/2)^4-96*a^3*tan
(c/2)^3*tan(d*x/2)^5+448*a^3*tan(c/2)^3*tan(d*x/2)^3-96*a^3*tan(c/2)^3*tan(
d*x/2)-12*a^3*tan(c/2)^2*tan(d*x/2)^6+228*a^3*tan(c/2)^2*tan(d*x/2)^4-228*a
^3*tan(c/2)^2*tan(d*x/2)^2+12*a^3*tan(c/2)^2+48*a^3*tan(c/2)*tan(d*x/2)^5-9
6*a^3*tan(c/2)*tan(d*x/2)^3+48*a^3*tan(c/2)*tan(d*x/2)+4*a^3*tan(d*x/2)^6-1
2*a^3*tan(d*x/2)^4+12*a^3*tan(d*x/2)^2-4*a^3-6*a^2*b*ln((2*tan(c/2)^2*tan(d
*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*ta
n(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*t
an(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^6*tan(d
*x/2)^6+6*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*t
an(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(
d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/
2)+2)/(tan(c/2)^2+1))*tan(c/2)^6*tan(d*x/2)^4+6*a^2*b*ln((2*tan(c/2)^2*tan(
d*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*t
an(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*
tan(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^6*tan(
d*x/2)^2-6*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*
```

```
tan(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan
(d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x
/2)+2)/(tan(c/2)^2+1))*tan(c/2)^6+48*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*
tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2
*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^
3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^5*tan(d*x/2)^5-48
*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2
*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-
4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(ta
n(c/2)^2+1))*tan(c/2)^5*tan(d*x/2)+6*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*
tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2
*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^
3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^4*tan(d*x/2)^6-10
2*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^
2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4
-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(t
an(c/2)^2+1))*tan(c/2)^4*tan(d*x/2)^4-102*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)
^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x
/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*
x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^4*tan(d*x/2)
^2+6*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/
2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2
)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)
/(tan(c/2)^2+1))*tan(c/2)^4+6*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2
)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/
2)^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan
(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^2*tan(d*x/2)^6-102*a^2*b
*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d
*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(
c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)
^2+1))*tan(c/2)^2*tan(d*x/2)^4-102*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*ta
n(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*t
an(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+
4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^2*tan(d*x/2)^2+6*a^
2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*ta
n(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*t
an(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c
/2)^2+1))*tan(c/2)^2-48*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*ta
n(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4
*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2
)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)*tan(d*x/2)^5+48*a^2*b*ln((2*ta
n(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4
*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*ta
n(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*ta
n(c/2)*tan(d*x/2)-6*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan(d*
x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*tan
(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)^2-
4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(d*x/2)^6+6*a^2*b*ln((2*tan(c/2)^2*tan(d
*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*ta
n(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*t
an(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(d*x/2)^4+6*a
^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*t
an(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*
tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(
c/2)^2+1))*tan(d*x/2)^2-6*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*
tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2
+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x
/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))+6*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4
-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2
```

)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/
2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^6*tan(d*x/2)^6
-6*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)
^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x/2)^
4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+2)/(
tan(c/2)^2+1))*tan(c/2)^6*tan(d*x/2)^4-6*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^
4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/
2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x
/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^6*tan(d*x/2)^
2+6*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2
)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x/2)
^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+2)/
(tan(c/2)^2+1))*tan(c/2)^6-48*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2
)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/
2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan
(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^5*tan(d*x/2)^5+48*a^2*b*
ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*
x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c
/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^
2+1))*tan(c/2)^5*tan(d*x/2)-6*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2
)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/
2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan
(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^4*tan(d*x/2)^6+102*a^2*b
*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d
*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(
c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)
^2+1))*tan(c/2)^4*tan(d*x/2)^4+102*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*ta
n(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*t
an(c/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+
4*tan(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^4*tan(d*x/2)^2-6*a^
2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*ta
n(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*t
an(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c
/2)^2+1))*tan(c/2)^4-6*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan
(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*
tan(c/2)*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)
^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^2*tan(d*x/2)^6+102*a^2*b*ln((2*
tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2
-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2)+2*
tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1))*
tan(c/2)^2*tan(d*x/2)^4+102*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^
2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)
^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d
*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^2*tan(d*x/2)^2-6*a^2*b*ln(
(2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2
)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2)
+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1
))*tan(c/2)^2+48*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2
)^3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/
2)*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*t
an(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)*tan(d*x/2)^5-48*a^2*b*ln((2*tan(c/2)^
2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/
2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2
)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)*
tan(d*x/2)+6*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+
4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*t
an(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d
*x/2)+2)/(tan(c/2)^2+1))*tan(d*x/2)^6-6*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4
-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2

)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/
2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(d*x/2)^4-6*a^2*b*ln
((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/
2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2
)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+
1))*tan(d*x/2)^2+6*a^2*b*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x
/2)^3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(
c/2)*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4
*tan(d*x/2)+2)/(tan(c/2)^2+1))+24*a^2*b*tan(c/2)^6*tan(d*x/2)^5-48*a^2*b*ta
n(c/2)^6*tan(d*x/2)^3+24*a^2*b*tan(c/2)^6*tan(d*x/2)+24*a^2*b*tan(c/2)^5*ta
n(d*x/2)^6-264*a^2*b*tan(c/2)^5*tan(d*x/2)^4+264*a^2*b*tan(c/2)^5*tan(d*x/2
)^2-24*a^2*b*tan(c/2)^5-264*a^2*b*tan(c/2)^4*tan(d*x/2)^5+912*a^2*b*tan(c/2
)^4*tan(d*x/2)^3-264*a^2*b*tan(c/2)^4*tan(d*x/2)-48*a^2*b*tan(c/2)^3*tan(d*
x/2)^6+912*a^2*b*tan(c/2)^3*tan(d*x/2)^4-912*a^2*b*tan(c/2)^3*tan(d*x/2)^2+
48*a^2*b*tan(c/2)^3+264*a^2*b*tan(c/2)^2*tan(d*x/2)^5-912*a^2*b*tan(c/2)^2*
tan(d*x/2)^3+264*a^2*b*tan(c/2)^2*tan(d*x/2)+24*a^2*b*tan(c/2)*tan(d*x/2)^6
-264*a^2*b*tan(c/2)*tan(d*x/2)^4+264*a^2*b*tan(c/2)*tan(d*x/2)^2-24*a^2*b*t
an(c/2)-24*a^2*b*tan(d*x/2)^5+48*a^2*b*tan(d*x/2)^3-24*a^2*b*tan(d*x/2)+9*a
*b^2*pi*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d
*x/2)^2+1)*tan(c/2)^6*tan(d*x/2)^6-9*a*b^2*pi*sign(tan(c/2)^2*tan(d*x/2)^2-
tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)*tan(c/2)^6*tan(d*x/2)^4-9*
a*b^2*pi*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(
d*x/2)^2+1)*tan(c/2)^6*tan(d*x/2)^2+9*a*b^2*pi*sign(tan(c/2)^2*tan(d*x/2)^2
-tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)*tan(c/2)^6-72*a*b^2*pi*si
gn(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)
*tan(c/2)^5*tan(d*x/2)^5+72*a*b^2*pi*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^
2-4*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)*tan(c/2)^5*tan(d*x/2)-9*a*b^2*pi*si
gn(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)
*tan(c/2)^4*tan(d*x/2)^6+153*a*b^2*pi*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)
^2-4*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)*tan(c/2)^4*tan(d*x/2)^4+153*a*b^2*
pi*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d*x/2)
^2+1)*tan(c/2)^4*tan(d*x/2)^2-9*a*b^2*pi*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c
/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)*tan(c/2)^4-9*a*b^2*pi*sign(tan(
c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)*tan(c/
2)^2*tan(d*x/2)^6+153*a*b^2*pi*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4*ta
n(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)*tan(c/2)^2*tan(d*x/2)^4+153*a*b^2*pi*sign
(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)*t
an(c/2)^2*tan(d*x/2)^2-9*a*b^2*pi*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4
*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)*tan(c/2)^2+72*a*b^2*pi*sign(tan(c/2)^2
*tan(d*x/2)^2-tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)*tan(c/2)*tan
(d*x/2)^5-72*a*b^2*pi*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4*tan(c/2)*ta
n(d*x/2)-tan(d*x/2)^2+1)*tan(c/2)*tan(d*x/2)+9*a*b^2*pi*sign(tan(c/2)^2*tan
(d*x/2)^2-tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)*tan(d*x/2)^6-9*a
*b^2*pi*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d
*x/2)^2+1)*tan(d*x/2)^4-9*a*b^2*pi*sign(tan(c/2)^2*tan(d*x/2)^2-tan(c/2)^2-
4*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)*tan(d*x/2)^2+9*a*b^2*pi*sign(tan(c/2)
^2*tan(d*x/2)^2-tan(c/2)^2-4*tan(c/2)*tan(d*x/2)-tan(d*x/2)^2+1)+24*a*b^2*t
an(c/2)^6*tan(d*x/2)^6-24*a*b^2*tan(c/2)^6*tan(d*x/2)^4+24*a*b^2*tan(c/2)^6
*tan(d*x/2)^2-24*a*b^2*tan(c/2)^6-192*a*b^2*tan(c/2)^5*tan(d*x/2)^5+192*a*b
^2*tan(c/2)^5*tan(d*x/2)^3-192*a*b^2*tan(c/2)^5*tan(d*x/2)-24*a*b^2*tan(c/2
)^4*tan(d*x/2)^6+696*a*b^2*tan(c/2)^4*tan(d*x/2)^4-696*a*b^2*tan(c/2)^4*tan
(d*x/2)^2+24*a*b^2*tan(c/2)^4+192*a*b^2*tan(c/2)^3*tan(d*x/2)^5-1536*a*b^2*
tan(c/2)^3*tan(d*x/2)^3+192*a*b^2*tan(c/2)^3*tan(d*x/2)+24*a*b^2*tan(c/2)^2
*tan(d*x/2)^6-696*a*b^2*tan(c/2)^2*tan(d*x/2)^4+696*a*b^2*tan(c/2)^2*tan(d*
x/2)^2-24*a*b^2*tan(c/2)^2-192*a*b^2*tan(c/2)*tan(d*x/2)^5+192*a*b^2*tan(c/
2)*tan(d*x/2)^3-192*a*b^2*tan(c/2)*tan(d*x/2)-24*a*b^2*tan(d*x/2)^6+24*a*b^
2*tan(d*x/2)^4-24*a*b^2*tan(d*x/2)^2+24*a*b^2+3*b^3*ln((2*tan(c/2)^2*tan(d*
x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan
(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*ta

```
n(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^6*tan(d*
x/2)^6-3*b^3*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(
c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x
/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+
2)/(tan(c/2)^2+1))*tan(c/2)^6*tan(d*x/2)^4-3*b^3*ln((2*tan(c/2)^2*tan(d*x/2
)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*
x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d
*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^6*tan(d*x/2
)^2+3*b^3*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2
)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)
^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/
(tan(c/2)^2+1))*tan(c/2)^6-24*b^3*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^
2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)
^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d
*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^5*tan(d*x/2)^5+24*b^3*ln((
2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)
^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+
2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1)
)*tan(c/2)^5*tan(d*x/2)-3*b^3*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*ta
n(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4
*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2
)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^4*tan(d*x/2)^6+51*b^3*ln((2*ta
n(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4
*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*ta
n(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*ta
n(c/2)^4*tan(d*x/2)^4+51*b^3*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan
(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*
tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)
^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^4*tan(d*x/2)^2-3*b^3*ln((2*tan(
c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*t
an(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(
d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(
c/2)^4-3*b^3*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(
c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x
/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+
2)/(tan(c/2)^2+1))*tan(c/2)^2*tan(d*x/2)^6+51*b^3*ln((2*tan(c/2)^2*tan(d*x/
2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d
*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(
d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^2*tan(d*x/
2)^4+51*b^3*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c
/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/
2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2
)/(tan(c/2)^2+1))*tan(c/2)^2*tan(d*x/2)^2-3*b^3*ln((2*tan(c/2)^2*tan(d*x/2)
^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x
/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*
x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^2+24*b^3*ln(
(2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2
)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)
+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1
))*tan(c/2)*tan(d*x/2)^5-24*b^3*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*
tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2
+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x
/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)*tan(d*x/2)+3*b^3*ln((2*tan(c
/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*ta
n(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d
*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(d
*x/2)^6-3*b^3*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan
(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*
x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)
```

```
+2)/(tan(c/2)^2+1))*tan(d*x/2)^4-3*b^3*ln((2*tan(c/2)^2*tan(d*x/2)^4+4*tan(
c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*tan(c/2)^2*tan(d*x/2)+2*tan
(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d*x/2)^4-4*tan(d*x/2)^3+4*
tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(d*x/2)^2+3*b^3*ln((2*tan(c
/2)^2*tan(d*x/2)^4+4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2+4*ta
n(c/2)^2*tan(d*x/2)+2*tan(c/2)^2+4*tan(c/2)*tan(d*x/2)^4-4*tan(c/2)+2*tan(d
*x/2)^4-4*tan(d*x/2)^3+4*tan(d*x/2)^2-4*tan(d*x/2)+2)/(tan(c/2)^2+1))-3*b^3
*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d
*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(
c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)
^2+1))*tan(c/2)^6*tan(d*x/2)^6+3*b^3*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/
2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c
/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*ta
n(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^6*tan(d*x/2)^4+3*b^3*ln
((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/
2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2
)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+
1))*tan(c/2)^6*tan(d*x/2)^2-3*b^3*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^
2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)
^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d
*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^6+24*b^3*ln((2*tan(c/2)^2*
tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)
^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^
4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^5*
tan(d*x/2)^5-24*b^3*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3
+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*
tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(
d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^5*tan(d*x/2)+3*b^3*ln((2*tan(c/2)^2*tan(
d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*t
an(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*
tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^4*tan(
d*x/2)^6-51*b^3*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*t
an(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(
d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/
2)+2)/(tan(c/2)^2+1))*tan(c/2)^4*tan(d*x/2)^4-51*b^3*ln((2*tan(c/2)^2*tan(d
*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*ta
n(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*t
an(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^4*tan(d
*x/2)^2+3*b^3*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan
(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*
x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)
+2)/(tan(c/2)^2+1))*tan(c/2)^4+3*b^3*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/
2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c
/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*ta
n(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^2*tan(d*x/2)^6-51*b^3*l
n((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x
/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/
2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2
+1))*tan(c/2)^2*tan(d*x/2)^4-51*b^3*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2
)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/
2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan
(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)^2*tan(d*x/2)^2+3*b^3*ln(
(2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2
)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2)
+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1
))*tan(c/2)^2-24*b^3*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^
3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)
*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan
(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)*tan(d*x/2)^5+24*b^3*ln((2*tan(c/2)^2*ta
```

```
n(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2
*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+
4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(c/2)*tan(
d*x/2)-3*b^3*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(
c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x
/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+
2)/(tan(c/2)^2+1))*tan(d*x/2)^6+3*b^3*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c
/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(
c/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*t
an(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(d*x/2)^4+3*b^3*ln((2*tan(c/
2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(c/2)^2*tan(d*x/2)^2-4*tan
(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x/2)^4+4*tan(c/2)+2*tan(d*
x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+2)/(tan(c/2)^2+1))*tan(d*
x/2)^2-3*b^3*ln((2*tan(c/2)^2*tan(d*x/2)^4-4*tan(c/2)^2*tan(d*x/2)^3+4*tan(
c/2)^2*tan(d*x/2)^2-4*tan(c/2)^2*tan(d*x/2)+2*tan(c/2)^2-4*tan(c/2)*tan(d*x
/2)^4+4*tan(c/2)+2*tan(d*x/2)^4+4*tan(d*x/2)^3+4*tan(d*x/2)^2+4*tan(d*x/2)+
2)/(tan(c/2)^2+1))-12*b^3*tan(c/2)^6*tan(d*x/2)^5+8*b^3*tan(c/2)^6*tan(d*x/
2)^3-12*b^3*tan(c/2)^6*tan(d*x/2)-12*b^3*tan(c/2)^5*tan(d*x/2)^6+84*b^3*tan
(c/2)^5*tan(d*x/2)^4-84*b^3*tan(c/2)^5*tan(d*x/2)^2+12*b^3*tan(c/2)^5+84*b^
3*tan(c/2)^4*tan(d*x/2)^5-312*b^3*tan(c/2)^4*tan(d*x/2)^3+84*b^3*tan(c/2)^4
*tan(d*x/2)+8*b^3*tan(c/2)^3*tan(d*x/2)^6-312*b^3*tan(c/2)^3*tan(d*x/2)^4+3
12*b^3*tan(c/2)^3*tan(d*x/2)^2-8*b^3*tan(c/2)^3-84*b^3*tan(c/2)^2*tan(d*x/2
)^5+312*b^3*tan(c/2)^2*tan(d*x/2)^3-84*b^3*tan(c/2)^2*tan(d*x/2)-12*b^3*tan
(c/2)*tan(d*x/2)^6+84*b^3*tan(c/2)*tan(d*x/2)^4-84*b^3*tan(c/2)*tan(d*x/2)^
2+12*b^3*tan(c/2)+12*b^3*tan(d*x/2)^5-8*b^3*tan(d*x/2)^3+12*b^3*tan(d*x/2))
/(4*d*tan(c/2)^6*tan(d*x/2)^6-4*d*tan(c/2)^6*tan(d*x/2)^4-4*d*tan(c/2)^6*ta
n(d*x/2)^2+4*d*tan(c/2)^6-32*d*tan(c/2)^5*tan(d*x/2)^5+32*d*tan(c/2)^5*tan(
d*x/2)-4*d*tan(c/2)^4*tan(d*x/2)^6+68*d*tan(c/2)^4*tan(d*x/2)^4+68*d*tan(c/
2)^4*tan(d*x/2)^2-4*d*tan(c/2)^4-4*d*tan(c/2)^2*tan(d*x/2)^6+68*d*tan(c/2)^
2*tan(d*x/2)^4+68*d*tan(c/2)^2*tan(d*x/2)^2-4*d*tan(c/2)^2+32*d*tan(c/2)*ta
n(d*x/2)^5-32*d*tan(c/2)*tan(d*x/2)+4*d*tan(d*x/2)^6-4*d*tan(d*x/2)^4-4*d*t
an(d*x/2)^2+4*d)
```

maple [A] time = 0.34, size = 193, normalized size = 1.45

$$-\frac{a^3\cos(dx+c)}{d}-\frac{3a^2b\sin(dx+c)}{d}+\frac{3a^2b\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{3b^2a\left(\sin^4(dx+c)\right)}{d\cos(dx+c)}+\frac{3\cos(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*tan(d*x+c))^3,x)
```

```
[Out] -a^3*cos(d*x+c)/d-3*a^2*b*sin(d*x+c)/d+3/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+
3/d*b^2*a*sin(d*x+c)^4/cos(d*x+c)+3/d*cos(d*x+c)*sin(d*x+c)^2*a*b^2+6*a*b^2
*cos(d*x+c)/d+1/2/d*b^3*sin(d*x+c)^5/cos(d*x+c)^2+1/2*b^3*sin(d*x+c)^3/d+3/
2*b^3*sin(d*x+c)/d-3/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.32, size = 128, normalized size = 0.96

$$-\frac{b^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+3\log(\sin(dx+c)+1)-3\log(\sin(dx+c)-1)-4\sin(dx+c)\right)-12ab^2\left(\frac{1}{\cos(dx+c)}+\cos(\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) -
3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 12*a*b^2*(1/cos(d*x + c) + cos(
d*x + c)) - 6*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(
d*x + c)) + 4*a^3*cos(d*x + c))/d
```

mupad [B] time = 5.80, size = 193, normalized size = 1.45

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a^2\,b-3\,b^3\right)+2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-12\,a\,b^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(12\,a\,b^2-4\,a^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(6\,a^2\right.}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(6*a^2*b - 3*b^3) + 2*a^3*tan(c/2 + (d*x)/2)^4 - 12*a*b
^2 + tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 4*a^3) + tan(c/2 + (d*x)/2)^5*(6*a^2*
b - 3*b^3) - tan(c/2 + (d*x)/2)^3*(12*a^2*b - 2*b^3) + 2*a^3)/(d*(tan(c/2 +
 (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 - 1)) + (atanh(ta
n(c/2 + (d*x)/2))*(6*a^2*b - 3*b^3))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^3 \sin(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sin(c + d*x), x)
```

3.35 $\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=86

$$-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}-\frac{b^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^3\tan(c+dx)}{2}$$

```
[Out] -a^3*arctanh(cos(d*x+c))/d+3*a^2*b*arctanh(sin(d*x+c))/d-1/2*b^3*arctanh(si
n(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3517, 3770, 2606, 8, 2611}

$$\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}-\frac{b^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^3\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a^3*ArcTanh[Cos[c + d*x]])/d) + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (b^3
*ArcTanh[Sin[c + d*x]])/(2*d) + (3*a*b^2*Sec[c + d*x])/d + (b^3*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a^3\csc(c+dx)+3a^2b\sec(c+dx)+3ab^2\sec(c+dx)\tan(c+dx)+b^3\right.$$

$$= a^3\int \csc(c+dx)\,dx + \left(3a^2b\right)\int \sec(c+dx)\,dx + \left(3ab^2\right)\int \sec(c+dx)\,\text{ta}$$

$$= -\frac{a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d} + \frac{b^3\sec(c+dx)\tan}{2d}$$

$$= -\frac{a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d} - \frac{b^3\tanh^{-1}(\sin(c+}{2d}$$

Mathematica [B] time = 2.29, size = 241, normalized size = 2.80

$$4a^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 4a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 12a^2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + 12a^2b\log$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (12*a*b^2 - 4*a^3*Log[Cos[(c + d*x)/2]] - 12*a^2*b*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 2*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*a^3*L
og[Sin[(c + d*x)/2]] + 12*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] -
2*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^3/(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2])^2 + 24*a*b^2*Sec[c + d*x]*Sin[(c + d*x)/2]^2 - b^3/(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^2)/(4*d)
```

fricas [A] time = 0.53, size = 148, normalized size = 1.72

$$-\frac{2\,a^3\cos(dx+c)^2\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - 2\,a^3\cos(dx+c)^2\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - 12\,ab^2\cos(dx+c) -}{4\,d\,\text{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^3*cos(d*x + c)^2*log(1/2*cos(d*x + c) + 1/2) - 2*a^3*cos(d*x + c)
^2*log(-1/2*cos(d*x + c) + 1/2) - 12*a*b^2*cos(d*x + c) - (6*a^2*b - b^3)*c
os(d*x + c)^2*log(sin(d*x + c) + 1) + (6*a^2*b - b^3)*cos(d*x + c)^2*log(-s
in(d*x + c) + 1) - 2*b^3*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [A] time = 7.58, size = 144, normalized size = 1.67

$$\frac{2\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) + \left(6\,a^2b-b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - \left(6\,a^2b-b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + (6*a^2*b - b^3)*log(abs(tan(1/2
*d*x + 1/2*c) + 1)) - (6*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) +
2*(b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^2 + b^3*tan(1/
2*d*x + 1/2*c) + 6*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.35, size = 125, normalized size = 1.45

$$\frac{a^3\ln(\csc(dx+c)-\cot(dx+c))}{d}+\frac{3a^2b\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{3b^2a}{d\cos(dx+c)}+\frac{b^3\left(\sin^3(dx+c)\right)}{2d\cos(dx+c)^2}+\frac{b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+3/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3/d*b
^2*a/cos(d*x+c)+1/2/d*b^3*sin(d*x+c)^3/cos(d*x+c)^2+1/2*b^3*sin(d*x+c)/d-1/
2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.33, size = 111, normalized size = 1.29

$$-\frac{b^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)-6a^2b\left(\log(\sin(dx+c)+1)-\log(\sin(dx+\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) - 6*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) -
1)) + 4*a^3*log(cot(d*x + c) + csc(d*x + c)) - 12*a*b^2/cos(d*x + c))/d
```

mupad [B] time = 4.21, size = 278, normalized size = 3.23

$$\frac{2\left(\frac{a^3\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{2}-\frac{b^3\,\mathrm{atan}\left(\frac{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^3-6\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)b^3}{2i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^3-6i\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b+1i\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^3}\right)1i}{2}+a^2b\,\mathrm{atan}\left(\frac{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^3-6\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b+c}{2i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^3-6i\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b+1}\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/sin(c + d*x),x)
```

```
[Out] (2*((a^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 - (b^3*atan((b^3*cos
(c/2 + (d*x)/2) + 2*a^3*sin(c/2 + (d*x)/2) - 6*a^2*b*cos(c/2 + (d*x)/2))/(a
^3*cos(c/2 + (d*x)/2)*2i + b^3*sin(c/2 + (d*x)/2)*1i - a^2*b*sin(c/2 + (d*x
)/2)*6i))*1i)/2 + a^2*b*atan((b^3*cos(c/2 + (d*x)/2) + 2*a^3*sin(c/2 + (d*x
)/2) - 6*a^2*b*cos(c/2 + (d*x)/2))/(a^3*cos(c/2 + (d*x)/2)*2i + b^3*sin(c/2
 + (d*x)/2)*1i - a^2*b*sin(c/2 + (d*x)/2)*6i))*3i))/d + ((b^3*sin(c + d*x))
/2 + 3*a*b^2*cos(c + d*x))/(d*(cos(2*c + 2*d*x)/2 + 1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\tan(c+dx))^3\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*csc(c + d*x), x)
```

3.36 $\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=64

$$-\frac{a^3\cot(c+dx)}{d}+\frac{3a^2b\log(\tan(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\tan^2(c+dx)}{2d}$$

```
[Out] -a^3*cot(d*x+c)/d+3*a^2*b*ln(tan(d*x+c))/d+3*a*b^2*tan(d*x+c)/d+1/2*b^3*tan
(d*x+c)^2/d
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 43}

$$\frac{3a^2b\log(\tan(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\tan^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a^3*Cot[c + d*x])/d) + (3*a^2*b*Log[Tan[c + d*x]])/d + (3*a*b^2*Tan[c +
d*x])/d + (b^3*Tan[c + d*x]^2)/(2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx &= \frac{b\,\text{Subst}\left(\int \frac{(a+x)^3}{x^2}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\,\text{Subst}\left(\int \left(3a+\frac{a^3}{x^2}+\frac{3a^2}{x}+x\right)dx,x,b\tan(c+dx)\right)}{d}\\ &= -\frac{a^3\cot(c+dx)}{d}+\frac{3a^2b\log(\tan(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\tan^2(c}{2d}\end{aligned}$$

Mathematica [A] time = 1.01, size = 126, normalized size = 1.97

$$-\frac{\csc(c+dx)\sec^2(c+dx)\left(\left(a^3+3ab^2\right)\cos(3(c+dx))+3a\left(a^2-b^2\right)\cos(c+dx)-2b\sin(c+dx)\left(3a^2\log(\sin(c+\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/4*(Csc[c + d*x]*Sec[c + d*x]^2*(3*a*(a^2 - b^2)*Cos[c + d*x] + (a^3 + 3*
a*b^2)*Cos[3*(c + d*x)] - 2*b*(b^2 - 3*a^2*Log[Cos[c + d*x]] - 3*a^2*Cos[2*
```

```
(c + d*x)]*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]]) + 3*a^2*Log[Sin[c + d*x]
])*Sin[c + d*x]))/d
```

fricas [B] time = 0.48, size = 127, normalized size = 1.98

$$-\frac{3\,a^2b\cos\left(dx+c\right)^2\log\left(\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)-3\,a^2b\cos\left(dx+c\right)^2\log\left(-\frac{1}{4}\,\cos\left(dx+c\right)^2+\frac{1}{4}\right)\sin\left(dx+c\right.}{2\,d\cos\left(dx+c\right)^2\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a^2*b*cos(d*x + c)^2*log(cos(d*x + c)^2)*sin(d*x + c) - 3*a^2*b*cos
(d*x + c)^2*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 6*a*b^2*cos(d*x +
 c) + 2*(a^3 + 3*a*b^2)*cos(d*x + c)^3 - b^3*sin(d*x + c))/(d*cos(d*x + c)^
2*sin(d*x + c))
```

giac [A] time = 1.80, size = 70, normalized size = 1.09

$$\frac{b^3\tan\left(dx+c\right)^2+6\,a^2b\log\left(\left|\tan\left(dx+c\right)\right|\right)+6\,ab^2\tan\left(dx+c\right)-\frac{2\left(3\,a^2b\tan\left(dx+c\right)+a^3\right)}{\tan\left(dx+c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*tan(d*x + c)^2 + 6*a^2*b*log(abs(tan(d*x + c))) + 6*a*b^2*tan(d*x
+ c) - 2*(3*a^2*b*tan(d*x + c) + a^3)/tan(d*x + c))/d
```

maple [A] time = 0.52, size = 63, normalized size = 0.98

$$-\frac{a^3\cot\left(dx+c\right)}{d}+\frac{3a^2b\ln\left(\tan\left(dx+c\right)\right)}{d}+\frac{3a\,b^2\tan\left(dx+c\right)}{d}+\frac{b^3}{2d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x)
```

```
[Out] -a^3*cot(d*x+c)/d+3*a^2*b*ln(tan(d*x+c))/d+3*a*b^2*tan(d*x+c)/d+1/2/d*b^3/c
os(d*x+c)^2
```

maxima [A] time = 0.32, size = 56, normalized size = 0.88

$$\frac{b^3\tan\left(dx+c\right)^2+6\,a^2b\log\left(\tan\left(dx+c\right)\right)+6\,ab^2\tan\left(dx+c\right)-\frac{2\,a^3}{\tan\left(dx+c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*tan(d*x + c)^2 + 6*a^2*b*log(tan(d*x + c)) + 6*a*b^2*tan(d*x + c)
- 2*a^3/tan(d*x + c))/d
```

mupad [B] time = 3.66, size = 62, normalized size = 0.97

$$\frac{b^3\tan\left(c+d\,x\right)^2}{2\,d}-\frac{a^3\cot\left(c+d\,x\right)}{d}+\frac{3\,a^2\,b\,\ln\left(\tan\left(c+d\,x\right)\right)}{d}+\frac{3\,a\,b^2\tan\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/sin(c + d*x)^2,x)
```

```
[Out] (b^3*tan(c + d*x)^2)/(2*d) - (a^3*cot(c + d*x))/d + (3*a^2*b*log(tan(c + d*
x)))/d + (3*a*b^2*tan(c + d*x))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^3 \csc^2(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*csc(c + d*x)**2, x)
```

3.37 $\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=141

$$-\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{3ab^2\sec(c}{d}$$

```
[Out] -1/2*a^3*arctanh(cos(d*x+c))/d-3*a*b^2*arctanh(cos(d*x+c))/d+3*a^2*b*arctan
h(sin(d*x+c))/d+1/2*b^3*arctanh(sin(d*x+c))/d-3*a^2*b*csc(d*x+c)/d-1/2*a^3*
cot(d*x+c)*csc(d*x+c)/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)*tan(d*x+c)/
d
```

Rubi [A] time = 0.13, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3517, 3768, 3770, 2621, 321, 207, 2622}

$$-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}+\frac{3ab^2\sec(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*b^2*ArcTanh[Cos[c + d*x]])/d + (3
*a^2*b*ArcTanh[Sin[c + d*x]])/d + (b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^
2*b*Csc[c + d*x])/d - (a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (3*a*b^2*Sec[
c + d*x])/d + (b^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
```

```
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a^3\csc^3(c+dx)+3a^2b\csc^2(c+dx)\sec(c+dx)+3ab^2\csc(c+dx)\,\mathrm{se}\right.$$

$$= a^3\int \csc^3(c+dx)\,dx+\left(3a^2b\right)\int \csc^2(c+dx)\sec(c+dx)\,dx+\left(3ab^2\right)\int$$

$$= -\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}+\frac{b^3\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}a^3\int \csc(c$$

$$= -\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}+\frac{b^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{3a^2b\csc(c+dx)}{d}-$$

$$= -\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3ab^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^2b\tanh^{-1}(\sin}{d}$$

Mathematica [B] time = 6.19, size = 897, normalized size = 6.36

$$-\frac{3a^2b\cos^3(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{3ab^2\cos^3(c+dx)(a+b\tan(c+dx))^3}{d(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{a^3\cos^3(c+dx)\,\mathrm{c}}{8d(a\,\mathrm{co}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3) - (3*a^2*b*Cos[c + d*x]^3*Cot[(c + d*x)/2]*(a + b*Tan[c + d*x]
)^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a^3*Cos[c + d*x]^3*Csc[(c
 + d*x)/2]^2*(a + b*Tan[c + d*x])^3)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])
^3) + ((-a^3 - 6*a*b^2)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2]]*(a + b*Tan[c +
 d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + ((-6*a^2*b - b^3)*Cos
[c + d*x]^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3
)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + ((a^3 + 6*a*b^2)*Cos[c + d*x]
^3*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*S
in[c + d*x])^3) + ((6*a^2*b + b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*
x])^3) + (a^3*Cos[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^3)/(8*
d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (b^3*Cos[c + d*x]^3*(a + b*Tan[c +
 d*x])^3)/(4*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*
Sin[c + d*x])^3) + (3*a*b^2*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[c +
d*x])^3)/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^3) - (b^3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*a*b^2
```

```
*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*a^2*b*Cos[
c + d*x]^3*Tan[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] +
b*Sin[c + d*x])^3)
```

fricas [B] time = 0.56, size = 299, normalized size = 2.12

$$-\frac{12\,ab^2\cos(dx+c) - 2\left(a^3+6\,ab^2\right)\cos(dx+c)^3 + \left(\left(a^3+6\,ab^2\right)\cos(dx+c)^4 - \left(a^3+6\,ab^2\right)\cos(dx+c)^2\right)\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(12*a*b^2*cos(d*x + c) - 2*(a^3 + 6*a*b^2)*cos(d*x + c)^3 + ((a^3 + 6*
a*b^2)*cos(d*x + c)^4 - (a^3 + 6*a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d*x + c
) + 1/2) - ((a^3 + 6*a*b^2)*cos(d*x + c)^4 - (a^3 + 6*a*b^2)*cos(d*x + c)^2
)*log(-1/2*cos(d*x + c) + 1/2) - ((6*a^2*b + b^3)*cos(d*x + c)^4 - (6*a^2*b
 + b^3)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((6*a^2*b + b^3)*cos(d*x +
c)^4 - (6*a^2*b + b^3)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(b^3 - (6
*a^2*b + b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4 - d*cos(d*x +
 c)^2)
```

giac [B] time = 2.11, size = 304, normalized size = 2.16

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 12\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 4\left(6\,a^2b+b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 4\left(6\,a^2b+b^3\right)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan(1/2*d*x + 1/2*c) + 4*(6*a^2*
b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 4*(6*a^2*b + b^3)*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 4*(a^3 + 6*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))
- (2*a^3*tan(1/2*d*x + 1/2*c)^6 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 12*a^2*
b*tan(1/2*d*x + 1/2*c)^5 - 8*b^3*tan(1/2*d*x + 1/2*c)^5 - 3*a^3*tan(1/2*d*x
 + 1/2*c)^4 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 24*a^2*b*tan(1/2*d*x + 1/2*
c)^3 - 8*b^3*tan(1/2*d*x + 1/2*c)^3 - 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 12*
a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1
/2*c))^2)/d
```

maple [A] time = 0.52, size = 170, normalized size = 1.21

$$-\frac{a^3\cot(dx+c)\csc(dx+c)}{2d} + \frac{a^3\ln(\csc(dx+c)-\cot(dx+c))}{2d} - \frac{3a^2b}{d\sin(dx+c)} + \frac{3a^2b\ln(\sec(dx+c)+\tan(d}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x)
```

```
[Out] -1/2*a^3*cot(d*x+c)*csc(d*x+c)/d+1/2/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-3/d*a^
2*b/sin(d*x+c)+3/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3/d*b^2*a/cos(d*x+c)+3/d
*b^2*a*ln(csc(d*x+c)-cot(d*x+c))+1/2*b^3*sec(d*x+c)*tan(d*x+c)/d+1/2/d*b^3*
ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.33, size = 171, normalized size = 1.21

$$\frac{a^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1)\right) - b^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + log
(cos(d*x + c) - 1)) - b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a*b^2*(2/cos(d*x + c) - log(cos(d*
x + c) + 1) + log(cos(d*x + c) - 1)) - 6*a^2*b*(2/sin(d*x + c) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 3.97, size = 581, normalized size = 4.12

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{a^3}{2}+24ab^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(a^3+24ab^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(6a^2b-4b^3\right)-t}{d\left(4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/sin(c + d*x)^3,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) - (tan(c/2 + (d*x)/2)^4*(24*a*b^2 + a^3/2)
 - tan(c/2 + (d*x)/2)^2*(24*a*b^2 + a^3) + tan(c/2 + (d*x)/2)^5*(6*a^2*b -
4*b^3) - tan(c/2 + (d*x)/2)^3*(12*a^2*b + 4*b^3) + a^3/2 + 6*a^2*b*tan(c/2
+ (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 - 8*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2
 + (d*x)/2)^6)) + (log(tan(c/2 + (d*x)/2))*(3*a*b^2 + a^3/2))/d - (atan(-((
3*a^2*b + b^3/2)*(6*tan(c/2 + (d*x)/2)*(3*a^2*b + b^3/2) + 6*a^2*b + b^3 -
tan(c/2 + (d*x)/2)*(6*a*b^2 + a^3))*1i - (3*a^2*b + b^3/2)*(6*tan(c/2 + (d*
x)/2)*(3*a^2*b + b^3/2) - 6*a^2*b - b^3 + tan(c/2 + (d*x)/2)*(6*a*b^2 + a^3
))*1i)/(2*tan(c/2 + (d*x)/2)*(b^6 + 12*a^2*b^4 + 36*a^4*b^2) - (3*a^2*b + b
^3/2)*(6*tan(c/2 + (d*x)/2)*(3*a^2*b + b^3/2) + 6*a^2*b + b^3 - tan(c/2 + (
d*x)/2)*(6*a*b^2 + a^3)) - (3*a^2*b + b^3/2)*(6*tan(c/2 + (d*x)/2)*(3*a^2*b
 + b^3/2) - 6*a^2*b - b^3 + tan(c/2 + (d*x)/2)*(6*a*b^2 + a^3)) + 6*a*b^5 +
 6*a^5*b + 37*a^3*b^3))*(a^2*b*6i + b^3*1i))/d - (3*a^2*b*tan(c/2 + (d*x)/2
))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^3\csc^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*csc(c + d*x)**3, x)
```

3.38 $\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=113

$$-\frac{a^3\cot^3(c+dx)}{3d}-\frac{a\left(a^2+3b^2\right)\cot(c+dx)}{d}+\frac{b\left(3a^2+b^2\right)\log(\tan(c+dx))}{d}-\frac{3a^2b\cot^2(c+dx)}{2d}+\frac{3ab^2\tan(c+dx)}{d}$$

```
[Out] -a*(a^2+3*b^2)*cot(d*x+c)/d-3/2*a^2*b*cot(d*x+c)^2/d-1/3*a^3*cot(d*x+c)^3/d
+b*(3*a^2+b^2)*ln(tan(d*x+c))/d+3*a*b^2*tan(d*x+c)/d+1/2*b^3*tan(d*x+c)^2/d
```

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 894}

$$-\frac{a\left(a^2+3b^2\right)\cot(c+dx)}{d}+\frac{b\left(3a^2+b^2\right)\log(\tan(c+dx))}{d}-\frac{3a^2b\cot^2(c+dx)}{2d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{3ab^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a*(a^2 + 3*b^2)*Cot[c + d*x])/d) - (3*a^2*b*Cot[c + d*x]^2)/(2*d) - (a^3
*Cot[c + d*x]^3)/(3*d) + (b*(3*a^2 + b^2)*Log[Tan[c + d*x]])/d + (3*a*b^2*T
an[c + d*x])/d + (b^3*Tan[c + d*x]^2)/(2*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx &= \frac{b\,\text{Subst}\left(\int \frac{(a+x)^3\left(b^2+x^2\right)}{x^4}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\,\text{Subst}\left(\int \left(3a+\frac{a^3b^2}{x^4}+\frac{3a^2b^2}{x^3}+\frac{a^3+3ab^2}{x^2}+\frac{3a^2+b^2}{x}+x\right)dx,x,b\tan(c+\right.}{d}\\ &= -\frac{a\left(a^2+3b^2\right)\cot(c+dx)}{d}-\frac{3a^2b\cot^2(c+dx)}{2d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{b\left(\right.}{}\end{aligned}$$

Mathematica [A] time = 2.25, size = 212, normalized size = 1.88

$$\frac{\sec^2(c+dx)(a\cot(c+dx)+b)^3\left(-16a^3\cos(c+dx)-2\sin(c+dx)\left(2a^3\sin(4(c+dx))+6\left(3a^2b+b^3\right)\cos(2(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((b + a*Cot[c + d*x])^3*Sec[c + d*x]^2*(-16*a^3*Cos[c + d*x] - 2*Sin[c + d*
x]*(18*a^2*b - 6*b^3 + 6*(3*a^2*b + b^3)*Cos[2*(c + d*x)] + 9*a^2*b*Log[Cos
[c + d*x]] + 3*b^3*Log[Cos[c + d*x]] - 3*b*(3*a^2 + b^2)*Cos[4*(c + d*x)]*(
Log[Cos[c + d*x]] - Log[Sin[c + d*x]]) - 9*a^2*b*Log[Sin[c + d*x]] - 3*b^3*
Log[Sin[c + d*x]] + 2*a^3*Sin[4*(c + d*x)] + 18*a*b^2*Sin[4*(c + d*x)])))/(
48*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
```

fricas [B] time = 0.46, size = 237, normalized size = 2.10

$$-\frac{4\left(a^3+9\,ab^2\right)\cos\left(dx+c\right)^5+18\,ab^2\cos\left(dx+c\right)-6\left(a^3+9\,ab^2\right)\cos\left(dx+c\right)^3+3\left(\left(3\,a^2b+b^3\right)\cos\left(dx+c\right)^4-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(4*(a^3 + 9*a*b^2)*cos(d*x + c)^5 + 18*a*b^2*cos(d*x + c) - 6*(a^3 + 9
*a*b^2)*cos(d*x + c)^3 + 3*((3*a^2*b + b^3)*cos(d*x + c)^4 - (3*a^2*b + b^3
)*cos(d*x + c)^2)*log(cos(d*x + c)^2)*sin(d*x + c) - 3*((3*a^2*b + b^3)*cos
(d*x + c)^4 - (3*a^2*b + b^3)*cos(d*x + c)^2)*log(-1/4*cos(d*x + c)^2 + 1/4
)*sin(d*x + c) + 3*(b^3 - (3*a^2*b + b^3)*cos(d*x + c)^2)*sin(d*x + c))/((d
*cos(d*x + c)^4 - d*cos(d*x + c)^2)*sin(d*x + c))
```

giac [A] time = 2.51, size = 133, normalized size = 1.18

$$\frac{3\,b^3\tan\left(dx+c\right)^2+18\,ab^2\tan\left(dx+c\right)+6\left(3\,a^2b+b^3\right)\log\left(\left|\tan\left(dx+c\right)\right|\right)-\frac{33\,a^2b\tan(dx+c)^3+11\,b^3\tan(dx+c)^3+6\,a^3\tan(dx\cdots}{\tan(dx\cdots}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*b^3*tan(d*x + c)^2 + 18*a*b^2*tan(d*x + c) + 6*(3*a^2*b + b^3)*log(a
bs(tan(d*x + c))) - (33*a^2*b*tan(d*x + c)^3 + 11*b^3*tan(d*x + c)^3 + 6*a^
3*tan(d*x + c)^2 + 18*a*b^2*tan(d*x + c)^2 + 9*a^2*b*tan(d*x + c) + 2*a^3)/
tan(d*x + c)^3)/d
```

maple [A] time = 0.54, size = 141, normalized size = 1.25

$$-\frac{2a^3\cot\left(dx+c\right)}{3d}-\frac{a^3\cot\left(dx+c\right)\left(\csc^2\left(dx+c\right)\right)}{3d}-\frac{3a^2b}{2d\sin\left(dx+c\right)^2}+\frac{3a^2b\ln\left(\tan\left(dx+c\right)\right)}{d}+\frac{3b^2a}{d\sin\left(dx+c\right)\cos\left(dx\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x)
```

```
[Out] -2/3*a^3*cot(d*x+c)/d-1/3/d*a^3*cot(d*x+c)*csc(d*x+c)^2-3/2/d*a^2*b/sin(d*x
+c)^2+3*a^2*b*ln(tan(d*x+c))/d+3/d*b^2*a/sin(d*x+c)/cos(d*x+c)-6/d*b^2*a*co
t(d*x+c)+1/2/d*b^3/cos(d*x+c)^2+1/d*b^3*ln(tan(d*x+c))
```

maxima [A] time = 0.46, size = 98, normalized size = 0.87

$$\frac{3\,b^3\tan\left(dx+c\right)^2+18\,ab^2\tan\left(dx+c\right)+6\left(3\,a^2b+b^3\right)\log\left(\tan\left(dx+c\right)\right)-\frac{9\,a^2b\tan(dx+c)+2\,a^3+6\left(a^3+3\,ab^2\right)\tan(dx+c)^2}{\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*b^3*tan(d*x + c)^2 + 18*a*b^2*tan(d*x + c) + 6*(3*a^2*b + b^3)*log(t
an(d*x + c)) - (9*a^2*b*tan(d*x + c) + 2*a^3 + 6*(a^3 + 3*a*b^2)*tan(d*x +
c)^2)/tan(d*x + c)^3)/d
```

mupad [B] time = 3.72, size = 103, normalized size = 0.91

$$\frac{\ln\left(\tan\left(c+d\,x\right)\right)\,\left(3\,a^2\,b+b^3\right)}{d}-\frac{\cot\left(c+d\,x\right)^3\,\left(\frac{a^3}{3}+\tan\left(c+d\,x\right)^2\,\left(a^3+3\,a\,b^2\right)+\frac{3\,a^2\,b\,\tan\left(c+d\,x\right)}{2}\right)}{d}+\frac{b^3\,\tan\left(c+d\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/sin(c + d*x)^4,x)
```

```
[Out] (log(tan(c + d*x))*(3*a^2*b + b^3))/d - (cot(c + d*x)^3*(a^3/3 + tan(c + d*
x)^2*(3*a*b^2 + a^3) + (3*a^2*b*tan(c + d*x))/2))/d + (b^3*tan(c + d*x)^2)/
(2*d) + (3*a*b^2*tan(c + d*x))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)^3\csc^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*csc(c + d*x)**4, x)
```

3.39 $\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=229

$$-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{8d}-\frac{a^2b\csc^3(c+dx)}{d}-\frac{3a^2b\csc(c}{d}$$

```
[Out] -3/8*a^3*arctanh(cos(d*x+c))/d-9/2*a*b^2*arctanh(cos(d*x+c))/d+3*a^2*b*arct
anh(sin(d*x+c))/d+3/2*b^3*arctanh(sin(d*x+c))/d-3*a^2*b*csc(d*x+c)/d-3/2*b^
3*csc(d*x+c)/d-3/8*a^3*cot(d*x+c)*csc(d*x+c)/d-a^2*b*csc(d*x+c)^3/d-1/4*a^3
*cot(d*x+c)*csc(d*x+c)^3/d+9/2*a*b^2*sec(d*x+c)/d-3/2*a*b^2*csc(d*x+c)^2*se
c(d*x+c)/d+1/2*b^3*csc(d*x+c)*sec(d*x+c)^2/d
```

Rubi [A] time = 0.21, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3517, 3768, 3770, 2621, 302, 207, 2622, 288, 321}

$$-\frac{a^2b\csc^3(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3\cot(c+dx)\csc^3(c}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-3*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (9*a*b^2*ArcTanh[Cos[c + d*x]])/(2*d
) + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (3*b^3*ArcTanh[Sin[c + d*x]])/(2*d)
 - (3*a^2*b*Csc[c + d*x])/d - (3*b^3*Csc[c + d*x])/(2*d) - (3*a^3*Cot[c + d
*x]*Csc[c + d*x])/(8*d) - (a^2*b*Csc[c + d*x]^3)/d - (a^3*Cot[c + d*x]*Csc[
c + d*x]^3)/(4*d) + (9*a*b^2*Sec[c + d*x])/(2*d) - (3*a*b^2*Csc[c + d*x]^2*
Sec[c + d*x])/(2*d) + (b^3*Csc[c + d*x]*Sec[c + d*x]^2)/(2*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a^3\csc^5(c+dx)+3a^2b\csc^4(c+dx)\sec(c+dx)+3ab^2\csc^3(c+d$$

$$= a^3\int \csc^5(c+dx)\,dx+\left(3a^2b\right)\int \csc^4(c+dx)\sec(c+dx)\,dx+\left(3ab^2\right.$$

$$= -\frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{1}{4}\left(3a^3\right)\int \csc^3(c+dx)\,dx-\frac{\left(3a^2b\right)\text{Su}}{}$$

$$= -\frac{3a^3\cot(c+dx)\csc(c+dx)}{8d}-\frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3ab^2\csc}{}$$

$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{3a^2b\csc(c+dx)}{d}-\frac{3b^3\csc(c+dx)}{2d}-\frac{3a}{}$$

$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{9ab^2\tanh^{-1}(\cos(c+dx))}{2d}+\frac{3a^2b\tanh^{-1}}{}$$

Mathematica [B] time = 6.22, size = 1229, normalized size = 5.37

$$-\frac{a^3\cos^3(c+dx)(a+b\tan(c+dx))^3\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{3\left(a^3+4b^2a\right)\cos^3(c+dx)(a+b\tan(c+dx))^3\csc^2\left(\frac{1}{2}(c\right.}{32d(a\cos(c+dx)+b\sin(c+dx))^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3) + ((-7*a^2*b*Cos[(c + d*x)/2] - 2*b^3*Cos[(c + d*x)/2])*Cos[c
+ d*x]^3*Csc[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(4*d*(a*Cos[c + d*x] + b*
Sin[c + d*x])^3) - (3*(a^3 + 4*a*b^2)*Cos[c + d*x]^3*Csc[(c + d*x)/2]^2*(a
+ b*Tan[c + d*x])^3)/(32*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a^2*b*Co
s[c + d*x]^3*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^3)/(8
*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a^3*Cos[c + d*x]^3*Csc[(c + d*x)
/2]^4*(a + b*Tan[c + d*x])^3)/(64*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) -
(3*(a^3 + 12*a*b^2)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2]]*(a + b*Tan[c + d*x
])^3)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*(2*a^2*b + b^3)*Cos[c
+ d*x]^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(
2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (3*(a^3 + 12*a*b^2)*Cos[c + d*x]
^3*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(8*d*(a*Cos[c + d*x] + b*S
in[c + d*x])^3) + (3*(2*a^2*b + b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c +
d*x])^3) + (3*(a^3 + 4*a*b^2)*Cos[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + b*Tan[
c + d*x])^3)/(32*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (a^3*Cos[c + d*x]
^3*Sec[(c + d*x)/2]^4*(a + b*Tan[c + d*x])^3)/(64*d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^3) + (b^3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c +
d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (3*a*b
^2*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (b^3*Cos[c
+ d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*a*b^2*Cos[c + d*x]^3*Sin[(c +
d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a
*Cos[c + d*x] + b*Sin[c + d*x])^3) + (Cos[c + d*x]^3*Sec[(c + d*x)/2]*(-7*a
^2*b*Sin[(c + d*x)/2] - 2*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^3)/(4*
d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a^2*b*Cos[c + d*x]^3*Sec[(c + d*x
)/2]^2*Tan[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(8*d*(a*Cos[c + d*x] + b*Si
n[c + d*x])^3)
```

fricas [B] time = 0.57, size = 427, normalized size = 1.86

$$6\left(a^3+12\,ab^2\right)\cos\left(dx+c\right)^5+48\,ab^2\cos\left(dx+c\right)-10\left(a^3+12\,ab^2\right)\cos\left(dx+c\right)^3-3\left(\left(a^3+12\,ab^2\right)\cos\left(dx+c\right)^6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(a^3 + 12*a*b^2)*cos(d*x + c)^5 + 48*a*b^2*cos(d*x + c) - 10*(a^3 +
 12*a*b^2)*cos(d*x + c)^3 - 3*((a^3 + 12*a*b^2)*cos(d*x + c)^6 - 2*(a^3 + 1
2*a*b^2)*cos(d*x + c)^4 + (a^3 + 12*a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d*x
+ c) + 1/2) + 3*((a^3 + 12*a*b^2)*cos(d*x + c)^6 - 2*(a^3 + 12*a*b^2)*cos(d
*x + c)^4 + (a^3 + 12*a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) +
 12*((2*a^2*b + b^3)*cos(d*x + c)^6 - 2*(2*a^2*b + b^3)*cos(d*x + c)^4 + (2
*a^2*b + b^3)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 12*((2*a^2*b + b^3)*c
os(d*x + c)^6 - 2*(2*a^2*b + b^3)*cos(d*x + c)^4 + (2*a^2*b + b^3)*cos(d*x
+ c)^2)*log(-sin(d*x + c) + 1) + 8*(3*(2*a^2*b + b^3)*cos(d*x + c)^4 + b^3
- 4*(2*a^2*b + b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^6 - 2*d*c
os(d*x + c)^4 + d*cos(d*x + c)^2)
```

giac [A] time = 5.33, size = 373, normalized size = 1.63

$$a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-8\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-120\,a^2b\tan\left(\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(a^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 8*a^3*t
an(1/2*d*x + 1/2*c)^2 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 120*a^2*b*tan(1/2
*d*x + 1/2*c) - 32*b^3*tan(1/2*d*x + 1/2*c) + 96*(2*a^2*b + b^3)*log(abs(ta
n(1/2*d*x + 1/2*c) + 1)) - 96*(2*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + 24*(a^3 + 12*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + 64*(b^3*tan(1/
2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^2 + b^3*tan(1/2*d*x + 1/2*c
) + 6*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - (50*a^3*tan(1/2*d*x + 1/2*c)^
4 + 600*a*b^2*tan(1/2*d*x + 1/2*c)^4 + 120*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 3
2*b^3*tan(1/2*d*x + 1/2*c)^3 + 8*a^3*tan(1/2*d*x + 1/2*c)^2 + 24*a*b^2*tan(
1/2*d*x + 1/2*c)^2 + 8*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*
c)^4)/d
```

maple [A] time = 0.48, size = 254, normalized size = 1.11

$$-\frac{a^3\cot(dx+c)\left(\csc^3(dx+c)\right)}{4d}-\frac{3a^3\cot(dx+c)\csc(dx+c)}{8d}+\frac{3a^3\ln(\csc(dx+c)-\cot(dx+c))}{8d}-\frac{a^2b}{d\sin(dx+ \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x)
```

```
[Out] -1/4*a^3*cot(d*x+c)*csc(d*x+c)^3/d-3/8*a^3*cot(d*x+c)*csc(d*x+c)/d+3/8/d*a^
3*ln(csc(d*x+c)-cot(d*x+c))-1/d*a^2*b/sin(d*x+c)^3-3/d*a^2*b/sin(d*x+c)+3/d
*a^2*b*ln(sec(d*x+c)+tan(d*x+c))-3/2/d*b^2*a/sin(d*x+c)^2/cos(d*x+c)+9/2/d*
b^2*a/cos(d*x+c)+9/2/d*b^2*a*ln(csc(d*x+c)-cot(d*x+c))+1/2/d*b^3/sin(d*x+c)
/cos(d*x+c)^2-3/2/d*b^3/sin(d*x+c)+3/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.51, size = 250, normalized size = 1.09

$$a^3\left(\frac{2\left(3\cos(dx+c)^3-5\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)+12ab^2\left(\frac{2\left(3\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(a^3*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*
x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 12*a*b
^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(cos(d*
x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 4*b^3*(2*(3*sin(d*x + c)^2 - 2)/(s
in(d*x + c)^3 - sin(d*x + c)) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 8*a^2*b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 4.04, size = 698, normalized size = 3.05

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{a^3}{8}+\frac{3\,a\,b^2}{8}\right)}{d}-\frac{\mathrm{atan}\left(\frac{\left(3\,a^2\,b+\frac{3\,b^3}{2}\right)\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,a^3}{4}+9\,a\,b^2\right)+6\,\cdots\right.}{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(36\,a^4\,b^2+36\,a^2\,b^4+9\,b^6\right)+27\,a\,b^5+\frac{9\,a^5\,b}{2}-\left(3\,a^2\,b+\frac{3\,b^3}{2}\right)\left(\tan\left(\cdots\right.\right.}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/sin(c + d*x)^5,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^4)/(64*d) + (tan(c/2 + (d*x)/2)^2*((3*a*b^2)/8 + a^
3/8))/d - (atan(((3*a^2*b + (3*b^3)/2)*(tan(c/2 + (d*x)/2)*(9*a*b^2 + (3*a^
3)/4) + 6*tan(c/2 + (d*x)/2)*(3*a^2*b + (3*b^3)/2) - 6*a^2*b - 3*b^3)*1i -
(3*a^2*b + (3*b^3)/2)*(6*tan(c/2 + (d*x)/2)*(3*a^2*b + (3*b^3)/2) - tan(c/2
 + (d*x)/2)*(9*a*b^2 + (3*a^3)/4) + 6*a^2*b + 3*b^3)*1i)/(2*tan(c/2 + (d*x)
/2)*(9*b^6 + 36*a^2*b^4 + 36*a^4*b^2) + 27*a*b^5 + (9*a^5*b)/2 - (3*a^2*b +
 (3*b^3)/2)*(tan(c/2 + (d*x)/2)*(9*a*b^2 + (3*a^3)/4) + 6*tan(c/2 + (d*x)/2
)*(3*a^2*b + (3*b^3)/2) - 6*a^2*b - 3*b^3) - (3*a^2*b + (3*b^3)/2)*(6*tan(c
/2 + (d*x)/2)*(3*a^2*b + (3*b^3)/2) - tan(c/2 + (d*x)/2)*(9*a*b^2 + (3*a^3)
/4) + 6*a^2*b + 3*b^3) + (225*a^3*b^3)/4))*(a^2*b*6i + b^3*3i))/d - (tan(c/
2 + (d*x)/2)^2*(6*a*b^2 + (3*a^3)/2) + tan(c/2 + (d*x)/2)^6*(102*a*b^2 + 2*
a^3) - tan(c/2 + (d*x)/2)^4*(108*a*b^2 + (15*a^3)/4) + tan(c/2 + (d*x)/2)^3
*(26*a^2*b + 8*b^3) + tan(c/2 + (d*x)/2)^7*(30*a^2*b - 8*b^3) - tan(c/2 + (
d*x)/2)^5*(58*a^2*b + 32*b^3) + a^3/4 + 2*a^2*b*tan(c/2 + (d*x)/2))/(d*(16*
tan(c/2 + (d*x)/2)^4 - 32*tan(c/2 + (d*x)/2)^6 + 16*tan(c/2 + (d*x)/2)^8))
- (tan(c/2 + (d*x)/2)*((15*a^2*b)/8 + b^3/2))/d + (3*a*log(tan(c/2 + (d*x)/
2))*(a^2 + 12*b^2))/(8*d) - (a^2*b*tan(c/2 + (d*x)/2)^3)/(8*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan (c + dx))^3 \csc^5 (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*csc(c + d*x)**5, x)
```

3.40 $\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=167

$$-\frac{a^3\cot^5(c+dx)}{5d}-\frac{a\left(2a^2+3b^2\right)\cot^3(c+dx)}{3d}-\frac{b\left(6a^2+b^2\right)\cot^2(c+dx)}{2d}-\frac{a\left(a^2+6b^2\right)\cot(c+dx)}{d}+\frac{b\left(3a^2+2b\right.}{}$$

```
[Out] -a*(a^2+6*b^2)*cot(d*x+c)/d-1/2*b*(6*a^2+b^2)*cot(d*x+c)^2/d-1/3*a*(2*a^2+3
*b^2)*cot(d*x+c)^3/d-3/4*a^2*b*cot(d*x+c)^4/d-1/5*a^3*cot(d*x+c)^5/d+b*(3*a
^2+2*b^2)*ln(tan(d*x+c))/d+3*a*b^2*tan(d*x+c)/d+1/2*b^3*tan(d*x+c)^2/d
```

Rubi [A] time = 0.13, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 948}

$$-\frac{a\left(2a^2+3b^2\right)\cot^3(c+dx)}{3d}-\frac{b\left(6a^2+b^2\right)\cot^2(c+dx)}{2d}-\frac{a\left(a^2+6b^2\right)\cot(c+dx)}{d}+\frac{b\left(3a^2+2b^2\right)\log(\tan(c+d}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a*(a^2 + 6*b^2)*Cot[c + d*x])/d) - (b*(6*a^2 + b^2)*Cot[c + d*x]^2)/(2*d
) - (a*(2*a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*d) - (3*a^2*b*Cot[c + d*x]^4)/(4*
d) - (a^3*Cot[c + d*x]^5)/(5*d) + (b*(3*a^2 + 2*b^2)*Log[Tan[c + d*x]])/d +
 (3*a*b^2*Tan[c + d*x])/d + (b^3*Tan[c + d*x]^2)/(2*d)
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx &= \frac{b\,\text{Subst}\left(\int \frac{(a+x)^3\left(b^2+x^2\right)^2}{x^6}\,dx, x, b\tan(c+dx)\right)}{d}\\
&= \frac{b\,\text{Subst}\left(\int \left(3a+\frac{a^3b^4}{x^6}+\frac{3a^2b^4}{x^5}+\frac{2a^3b^2+3ab^4}{x^4}+\frac{6a^2b^2+b^4}{x^3}+\frac{a^3+6ab^2}{x^2}+\frac{3a^2+2}{x}\right.\right.}{d}\\
&= -\frac{a\left(a^2+6b^2\right)\cot(c+dx)}{d}-\frac{b\left(6a^2+b^2\right)\cot^2(c+dx)}{2d}-\frac{a\left(2a^2+3b^2\right)}{3}
\end{aligned}$$

Mathematica [B] time = 1.84, size = 515, normalized size = 3.08

$$-\frac{\csc^5(c+dx)\sec^2(c+dx)\left(8\left(a^3+15ab^2\right)\cos(3(c+dx))-24a^3\cos(5(c+dx))+8a^3\cos(7(c+dx))+40a\left(5\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/960*(Csc[c + d*x]^5*Sec[c + d*x]^2*(40*a*(5*a^2 + 3*b^2)*Cos[c + d*x] +
8*(a^3 + 15*a*b^2)*Cos[3*(c + d*x)] - 24*a^3*Cos[5*(c + d*x)] - 360*a*b^2*C
os[5*(c + d*x)] + 8*a^3*Cos[7*(c + d*x)] + 120*a*b^2*Cos[7*(c + d*x)] + 360
*a^2*b*Sin[c + d*x] - 240*b^3*Sin[c + d*x] + 225*a^2*b*Log[Cos[c + d*x]]*Si
n[c + d*x] + 150*b^3*Log[Cos[c + d*x]]*Sin[c + d*x] - 225*a^2*b*Log[Sin[c +
 d*x]]*Sin[c + d*x] - 150*b^3*Log[Sin[c + d*x]]*Sin[c + d*x] + 270*a^2*b*Si
n[3*(c + d*x)] + 180*b^3*Sin[3*(c + d*x)] + 45*a^2*b*Log[Cos[c + d*x]]*Sin[
3*(c + d*x)] + 30*b^3*Log[Cos[c + d*x]]*Sin[3*(c + d*x)] - 45*a^2*b*Log[Sin
[c + d*x]]*Sin[3*(c + d*x)] - 30*b^3*Log[Sin[c + d*x]]*Sin[3*(c + d*x)] - 9
0*a^2*b*Sin[5*(c + d*x)] - 60*b^3*Sin[5*(c + d*x)] - 135*a^2*b*Log[Cos[c +
d*x]]*Sin[5*(c + d*x)] - 90*b^3*Log[Cos[c + d*x]]*Sin[5*(c + d*x)] + 135*a^
2*b*Log[Sin[c + d*x]]*Sin[5*(c + d*x)] + 90*b^3*Log[Sin[c + d*x]]*Sin[5*(c
+ d*x)] + 45*a^2*b*Log[Cos[c + d*x]]*Sin[7*(c + d*x)] + 30*b^3*Log[Cos[c +
d*x]]*Sin[7*(c + d*x)] - 45*a^2*b*Log[Sin[c + d*x]]*Sin[7*(c + d*x)] - 30*b
^3*Log[Sin[c + d*x]]*Sin[7*(c + d*x)]))/d
```

fricas [B] time = 0.46, size = 343, normalized size = 2.05

$$-\frac{32\left(a^{3}+15\,ab^{2}\right)\cos\left(dx+c\right)^{7}-80\left(a^{3}+15\,ab^{2}\right)\cos\left(dx+c\right)^{5}-180\,ab^{2}\cos\left(dx+c\right)+60\left(a^{3}+15\,ab^{2}\right)\cos\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(32*(a^3 + 15*a*b^2)*cos(d*x + c)^7 - 80*(a^3 + 15*a*b^2)*cos(d*x + c
)^5 - 180*a*b^2*cos(d*x + c) + 60*(a^3 + 15*a*b^2)*cos(d*x + c)^3 + 30*((3*
a^2*b + 2*b^3)*cos(d*x + c)^6 - 2*(3*a^2*b + 2*b^3)*cos(d*x + c)^4 + (3*a^2
*b + 2*b^3)*cos(d*x + c)^2)*log(cos(d*x + c)^2)*sin(d*x + c) - 30*((3*a^2*b
 + 2*b^3)*cos(d*x + c)^6 - 2*(3*a^2*b + 2*b^3)*cos(d*x + c)^4 + (3*a^2*b +
2*b^3)*cos(d*x + c)^2)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 15*(2*
(3*a^2*b + 2*b^3)*cos(d*x + c)^4 + 2*b^3 - 3*(3*a^2*b + 2*b^3)*cos(d*x + c)
^2)*sin(d*x + c))/((d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^4 + d*cos(d*x + c)^
2)*sin(d*x + c))
```

giac [A] time = 1.85, size = 189, normalized size = 1.13

$$\frac{30\,b^{3}\tan\left(dx+c\right)^{2}+180\,ab^{2}\tan\left(dx+c\right)+60\left(3\,a^{2}b+2\,b^{3}\right)\log\left(\left|\tan\left(dx+c\right)\right|\right)-\frac{411\,a^{2}b\tan(dx+c)^{5}+274\,b^{3}\tan(dx+c)^{5}+6}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(30*b^3*tan(d*x + c)^2 + 180*a*b^2*tan(d*x + c) + 60*(3*a^2*b + 2*b^3)
*log(abs(tan(d*x + c))) - (411*a^2*b*tan(d*x + c)^5 + 274*b^3*tan(d*x + c)^
5 + 60*a^3*tan(d*x + c)^4 + 360*a*b^2*tan(d*x + c)^4 + 180*a^2*b*tan(d*x +
c)^3 + 30*b^3*tan(d*x + c)^3 + 40*a^3*tan(d*x + c)^2 + 60*a*b^2*tan(d*x + c
)^2 + 45*a^2*b*tan(d*x + c) + 12*a^3)/tan(d*x + c)^5)/d
```

maple [A] time = 0.57, size = 230, normalized size = 1.38

$$-\frac{8a^{3}\cot\left(dx+c\right)}{15d}-\frac{a^{3}\cot\left(dx+c\right)\left(\csc^{4}\left(dx+c\right)\right)}{5d}-\frac{4a^{3}\cot\left(dx+c\right)\left(\csc^{2}\left(dx+c\right)\right)}{15d}-\frac{3a^{2}b}{4d\sin\left(dx+c\right)^{4}}-\frac{3a^{2}b}{2d\sin\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x)
```

```
[Out] -8/15*a^3*cot(d*x+c)/d-1/5/d*a^3*cot(d*x+c)*csc(d*x+c)^4-4/15/d*a^3*cot(d*x
+c)*csc(d*x+c)^2-3/4/d*a^2*b/sin(d*x+c)^4-3/2/d*a^2*b/sin(d*x+c)^2+3*a^2*b*
ln(tan(d*x+c))/d-1/d*b^2*a/sin(d*x+c)^3/cos(d*x+c)+4/d*b^2*a/sin(d*x+c)/cos
(d*x+c)-8/d*b^2*a*cot(d*x+c)+1/2/d*b^3/sin(d*x+c)^2/cos(d*x+c)^2-1/d*b^3/si
n(d*x+c)^2+2/d*b^3*ln(tan(d*x+c))
```

maxima [A] time = 0.32, size = 142, normalized size = 0.85

$$\frac{30\,b^3\tan\left(dx+c\right)^2+180\,ab^2\tan\left(dx+c\right)+60\left(3\,a^2b+2\,b^3\right)\log\left(\tan\left(dx+c\right)\right)-\frac{60\left(a^3+6\,ab^2\right)\tan(dx+c)^4+45\,a^2b\tan(\cdots}{\cdots}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(30*b^3*tan(d*x + c)^2 + 180*a*b^2*tan(d*x + c) + 60*(3*a^2*b + 2*b^3)
*log(tan(d*x + c)) - (60*(a^3 + 6*a*b^2)*tan(d*x + c)^4 + 45*a^2*b*tan(d*x
+ c) + 30*(6*a^2*b + b^3)*tan(d*x + c)^3 + 12*a^3 + 20*(2*a^3 + 3*a*b^2)*ta
n(d*x + c)^2)/tan(d*x + c)^5)/d
```

mupad [B] time = 3.88, size = 146, normalized size = 0.87

$$\frac{\ln\left(\tan\left(c+d\,x\right)\right)\left(3\,a^2\,b+2\,b^3\right)}{d}-\frac{\cot\left(c+d\,x\right)^5\left(\tan\left(c+d\,x\right)^2\left(\frac{2\,a^3}{3}+a\,b^2\right)+\tan\left(c+d\,x\right)^3\left(3\,a^2\,b+\frac{b^3}{2}\right)+\frac{a^3}{5}\cdots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/sin(c + d*x)^6,x)
```

```
[Out] (log(tan(c + d*x))*(3*a^2*b + 2*b^3))/d - (cot(c + d*x)^5*(tan(c + d*x)^2*(
a*b^2 + (2*a^3)/3) + tan(c + d*x)^3*(3*a^2*b + b^3/2) + a^3/5 + tan(c + d*x
)^4*(6*a*b^2 + a^3) + (3*a^2*b*tan(c + d*x))/4))/d + (b^3*tan(c + d*x)^2)/(
2*d) + (3*a*b^2*tan(c + d*x))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.41 $\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=275

$$\frac{a^4\cos^3(c+dx)}{3d}-\frac{a^4\cos(c+dx)}{d}-\frac{4a^3b\sin^3(c+dx)}{3d}-\frac{4a^3b\sin(c+dx)}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2b^2\cos^3(c+}{d}$$

```
[Out] 4*a^3*b*arctanh(sin(d*x+c))/d-10*a*b^3*arctanh(sin(d*x+c))/d-a^4*cos(d*x+c)
/d+12*a^2*b^2*cos(d*x+c)/d-3*b^4*cos(d*x+c)/d+1/3*a^4*cos(d*x+c)^3/d-2*a^2*
b^2*cos(d*x+c)^3/d+1/3*b^4*cos(d*x+c)^3/d+6*a^2*b^2*sec(d*x+c)/d-3*b^4*sec(
d*x+c)/d+1/3*b^4*sec(d*x+c)^3/d-4*a^3*b*sin(d*x+c)/d+10*a*b^3*sin(d*x+c)/d-
4/3*a^3*b*sin(d*x+c)^3/d+10/3*a*b^3*sin(d*x+c)^3/d+2*a*b^3*sin(d*x+c)^3*tan
(d*x+c)^2/d
```

Rubi [A] time = 0.25, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3517, 2633, 2592, 302, 206, 2590, 270, 288}

$$-\frac{2a^2b^2\cos^3(c+dx)}{d}+\frac{12a^2b^2\cos(c+dx)}{d}+\frac{6a^2b^2\sec(c+dx)}{d}-\frac{4a^3b\sin^3(c+dx)}{3d}-\frac{4a^3b\sin(c+dx)}{d}+\frac{4a^3b\tanh^{-1}(}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (10*a*b^3*ArcTanh[Sin[c + d*x]])/d - (a
^4*Cos[c + d*x])/d + (12*a^2*b^2*Cos[c + d*x])/d - (3*b^4*Cos[c + d*x])/d +
 (a^4*Cos[c + d*x]^3)/(3*d) - (2*a^2*b^2*Cos[c + d*x]^3)/d + (b^4*Cos[c + d
*x]^3)/(3*d) + (6*a^2*b^2*Sec[c + d*x])/d - (3*b^4*Sec[c + d*x])/d + (b^4*S
ec[c + d*x]^3)/(3*d) - (4*a^3*b*Sin[c + d*x])/d + (10*a*b^3*Sin[c + d*x])/d
 - (4*a^3*b*Sin[c + d*x]^3)/(3*d) + (10*a*b^3*Sin[c + d*x]^3)/(3*d) + (2*a*
b^3*Sin[c + d*x]^3*Tan[c + d*x]^2)/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx = \int \left(a^4\sin^3(c+dx)+4a^3b\sin^3(c+dx)\tan(c+dx)+6a^2b^2\sin^3(c+a$$

$$= a^4\int \sin^3(c+dx)\,dx+\left(4a^3b\right)\int \sin^3(c+dx)\tan(c+dx)\,dx+\left(6a^2b^2$$

$$= -\frac{a^4\,\mathrm{Subst}\left(\int\left(1-x^2\right)\,dx,x,\cos(c+dx)\right)}{d}+\frac{\left(4a^3b\right)\mathrm{Subst}\left(\int\frac{x^4}{1-x^2}\,dx,\right.}{d}$$

$$= -\frac{a^4\cos(c+dx)}{d}+\frac{a^4\cos^3(c+dx)}{3d}+\frac{2ab^3\sin^3(c+dx)\tan^2(c+dx)}{d}$$

$$= -\frac{a^4\cos(c+dx)}{d}+\frac{12a^2b^2\cos(c+dx)}{d}-\frac{3b^4\cos(c+dx)}{d}+\frac{a^4\cos^3(c}{3d}$$

$$= \frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\cos(c+dx)}{d}+\frac{12a^2b^2\cos(c+dx)}{d}-\frac{3l}{}$$

$$= \frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{10ab^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\cos(c+a}{d}$$

Mathematica [B] time = 6.27, size = 1017, normalized size = 3.70

$$-\frac{\left(3a^4-42b^2a^2+11b^4\right)(a+b\tan(c+dx))^4\cos^5(c+dx)}{4d(a\cos(c+dx)+b\sin(c+dx))^4}+\frac{ab\left(a^2-b^2\right)\sin(3(c+dx))(a+b\tan(c+dx))^4\cos^4}{3d(a\cos(c+dx)+b\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/6*(b^2*(-36*a^2 + 17*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(d*(a*C
os[c + d*x] + b*Sin[c + d*x])^4) - ((3*a^4 - 42*a^2*b^2 + 11*b^4)*Cos[c + d
```

```
*x]^5*(a + b*Tan[c + d*x])^4)/(4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (
(a^4 - 6*a^2*b^2 + b^4)*Cos[c + d*x]^4*Cos[3*(c + d*x)]*(a + b*Tan[c + d*x]
)^4)/(12*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (2*(2*a^3*b - 5*a*b^3)*Co
s[c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^
4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (2*(2*a^3*b - 5*a*b^3)*Cos[c +
 d*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(d
*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((12*a*b^3 + b^4)*Cos[c + d*x]^4*(a
 + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[
c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b
*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(a*Cos[c + d
*x] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b*Tan[
c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^4) + ((-12*a*b^3 + b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x]
)^4)/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[
c + d*x])^4) + (Cos[c + d*x]^4*(36*a^2*b^2*Sin[(c + d*x)/2] - 17*b^4*Sin[(c
 + d*x)/2])*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/
2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(-36*a^2*b^2*Sin
[(c + d*x)/2] + 17*b^4*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (a*
b*(5*a^2 - 9*b^2)*Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Tan[c + d*x])^4)/(d*(a
*Cos[c + d*x] + b*Sin[c + d*x])^4) + (a*b*(a^2 - b^2)*Cos[c + d*x]^4*Sin[3*
(c + d*x)]*(a + b*Tan[c + d*x])^4)/(3*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4
)
```

fricas [A] time = 0.49, size = 224, normalized size = 0.81

$$\frac{\left(a^4 - 6\,a^2b^2 + b^4\right)\cos\left(dx+c\right)^6 - 3\left(a^4 - 12\,a^2b^2 + 3\,b^4\right)\cos\left(dx+c\right)^4 + 3\left(2\,a^3b - 5\,ab^3\right)\cos\left(dx+c\right)^3\log\left(\sin\left(d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*((a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^6 - 3*(a^4 - 12*a^2*b^2 + 3*b^4)*
cos(d*x + c)^4 + 3*(2*a^3*b - 5*a*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1)
 - 3*(2*a^3*b - 5*a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + b^4 + 9*(2
*a^2*b^2 - b^4)*cos(d*x + c)^2 + 2*(2*(a^3*b - a*b^3)*cos(d*x + c)^5 + 3*a*
b^3*cos(d*x + c) - 2*(4*a^3*b - 7*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*c
os(d*x + c)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.53, size = 412, normalized size = 1.50

$$-\frac{\cos\left(dx+c\right)\left(\sin^2\left(dx+c\right)\right)a^4}{3d}-\frac{2a^4\cos\left(dx+c\right)}{3d}-\frac{4a^3b\left(\sin^3\left(dx+c\right)\right)}{3d}-\frac{4a^3b\sin\left(dx+c\right)}{d}+\frac{4a^3b\ln\left(\sec\left(dx+c\right)-\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x)
```

```
[Out] -1/3/d*cos(d*x+c)*sin(d*x+c)^2*a^4-2/3*a^4*cos(d*x+c)/d-4/3*a^3*b*sin(d*x+c
)^3/d-4*a^3*b*sin(d*x+c)/d+4/d*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+6/d*a^2*b^2*
```

```
sin(d*x+c)^6/cos(d*x+c)+16*a^2*b^2*cos(d*x+c)/d+6/d*a^2*b^2*cos(d*x+c)*sin(
d*x+c)^4+8/d*a^2*b^2*cos(d*x+c)*sin(d*x+c)^2+2/d*a*b^3*sin(d*x+c)^7/cos(d*x
+c)^2+2/d*a*b^3*sin(d*x+c)^5+10/3*a*b^3*sin(d*x+c)^3/d+10*a*b^3*sin(d*x+c)/
d-10/d*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+1/3/d*b^4*sin(d*x+c)^8/cos(d*x+c)^3-
5/3/d*b^4*sin(d*x+c)^8/cos(d*x+c)-16/3*b^4*cos(d*x+c)/d-5/3/d*b^4*cos(d*x+c
)*sin(d*x+c)^6-2/d*b^4*cos(d*x+c)*sin(d*x+c)^4-8/3/d*b^4*cos(d*x+c)*sin(d*x
+c)^2
```

maxima [A] time = 0.54, size = 218, normalized size = 0.79

$$\frac{\left(\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)a^{4}-2\left(2\,\sin\left(dx+c\right)^{3}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)+6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*((cos(d*x + c)^3 - 3*cos(d*x + c))*a^4 - 2*(2*sin(d*x + c)^3 - 3*log(si
n(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*a^3*b - 6*(cos(
d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^2*b^2 + (4*sin(d*x + c)^3 -
 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(si
n(d*x + c) - 1) + 24*sin(d*x + c))*a*b^3 + (cos(d*x + c)^3 - (9*cos(d*x + c
)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x + c))*b^4)/d
```

mupad [B] time = 7.21, size = 319, normalized size = 1.16

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\,\left(8\,a^{4}-96\,a^{2}\,b^{2}+32\,b^{4}\right)+4\,a^{4}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(20\,a\,b^{3}-8\,a^{3}\,b\right)-\frac{4\,a^{4}}{3}-\frac{32\,b^{4}}{3}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b*tan(c + d*x))^4,x)
```

```
[Out] - (tan(c/2 + (d*x)/2)^4*(8*a^4 + 32*b^4 - 96*a^2*b^2) + 4*a^4*tan(c/2 + (d*
x)/2)^8 + tan(c/2 + (d*x)/2)*(20*a*b^3 - 8*a^3*b) - (4*a^4)/3 - (32*b^4)/3
+ 32*a^2*b^2 - tan(c/2 + (d*x)/2)^6*((32*a^4)/3 - 64*a^2*b^2) + tan(c/2 + (
d*x)/2)^3*((20*a*b^3)/3 - (8*a^3*b)/3) - tan(c/2 + (d*x)/2)^11*(20*a*b^3 -
8*a^3*b) - tan(c/2 + (d*x)/2)^9*((20*a*b^3)/3 - (8*a^3*b)/3) - tan(c/2 + (d
*x)/2)^5*(56*a*b^3 - 48*a^3*b) + tan(c/2 + (d*x)/2)^7*(56*a*b^3 - 48*a^3*b)
)/(d*(3*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^
12 - 1)) - (atanh(tan(c/2 + (d*x)/2))*(20*a*b^3 - 8*a^3*b))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)^{4}\sin^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**4*sin(c + d*x)**3, x)
```

3.42 $\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=139

$$\frac{b^2\left(18a^2-5b^2\right)\tan(c+dx)}{2d}-\frac{4ab\left(a^2-2b^2\right)\log(\cos(c+dx))}{d}+\frac{1}{2}x\left(a^4-18a^2b^2+5b^4\right)+\frac{4ab^3\tan^2(c+dx)}{d}-\frac{\sin(c$$

```
[Out] 1/2*(a^4-18*a^2*b^2+5*b^4)*x-4*a*b*(a^2-2*b^2)*ln(cos(d*x+c))/d+1/2*b^2*(18
*a^2-5*b^2)*tan(d*x+c)/d+4*a*b^3*tan(d*x+c)^2/d+5/6*b^4*tan(d*x+c)^3/d-1/2*
cos(d*x+c)*sin(d*x+c)*(a+b*tan(d*x+c))^4/d
```

Rubi [A] time = 0.17, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3516, 1645, 801, 635, 203, 260}

$$\frac{b^2\left(18a^2-5b^2\right)\tan(c+dx)}{2d}-\frac{4ab\left(a^2-2b^2\right)\log(\cos(c+dx))}{d}+\frac{1}{2}x\left(-18a^2b^2+a^4+5b^4\right)+\frac{4ab^3\tan^2(c+dx)}{d}-\frac{\sin($$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] ((a^4 - 18*a^2*b^2 + 5*b^4)*x)/2 - (4*a*b*(a^2 - 2*b^2)*Log[Cos[c + d*x]])/
d + (b^2*(18*a^2 - 5*b^2)*Tan[c + d*x])/(2*d) + (4*a*b^3*Tan[c + d*x]^2)/d
+ (5*b^4*Tan[c + d*x]^3)/(6*d) - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c +
d*x])^4)/(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
```

```
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^2(a+x)^4}{\left(b^2+x^2\right)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^4}{2d} - \frac{\text{Subst}\left(\int \frac{(a+x)^3\left(-ab^2-5\right.}{b^2+x^2}\right.}{2}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^4}{2d} - \frac{\text{Subst}\left(\int \left(-18a^2b^2+\right.\right.}{}$$

$$= \frac{b^2\left(18a^2-5b^2\right)\tan(c+dx)}{2d} + \frac{4ab^3\tan^2(c+dx)}{d} + \frac{5b^4\tan^3(c+dx)}{6d} -$$

$$= \frac{b^2\left(18a^2-5b^2\right)\tan(c+dx)}{2d} + \frac{4ab^3\tan^2(c+dx)}{d} + \frac{5b^4\tan^3(c+dx)}{6d} -$$

$$= \frac{1}{2}\left(a^4-18a^2b^2+5b^4\right)x - \frac{4ab\left(a^2-2b^2\right)\log(\cos(c+dx))}{d} + \frac{b^2\left(18a^2\right.}{}$$

Mathematica [A] time = 6.29, size = 263, normalized size = 1.89

$$\frac{b\left(2b\left(3a^2-b^2\right)\tan(c+dx) - \frac{\left(a^4-6a^2b^2+b^4\right)\tan^{-1}(\tan(c+dx))}{2b} - \frac{\left(a^4-6a^2b^2+b^4\right)\sin(c+dx)\cos(c+dx)}{2b} + \frac{1}{2}\left(4a^3+\frac{a^4-12a^2b^2+3b^4}{\sqrt{-b^2}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (b*(-1/2*((a^4 - 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]])/b + 2*a*(a - b)*(a
+ b)*Cos[c + d*x]^2 + ((4*a^3 - 8*a*b^2 + (a^4 - 12*a^2*b^2 + 3*b^4)/Sqrt[-
b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((4*a^3 - 8*a*b^2 - (a^4 - 12*a
^2*b^2 + 3*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/2 - ((a^4 - 6
*a^2*b^2 + b^4)*Cos[c + d*x]*Sin[c + d*x])/(2*b) + 2*b*(3*a^2 - b^2)*Tan[c
+ d*x] + 2*a*b^2*Tan[c + d*x]^2 + (b^3*Tan[c + d*x]^3)/3))/d
```

fricas [A] time = 0.47, size = 186, normalized size = 1.34

$$\frac{12\left(a^3b-ab^3\right)\cos(dx+c)^5 + 12\,ab^3\cos(dx+c) - 24\left(a^3b-2\,ab^3\right)\cos(dx+c)^3\log(-\cos(dx+c)) - 3\left(2\,a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(12*(a^3*b - a*b^3)*cos(d*x + c)^5 + 12*a*b^3*cos(d*x + c) - 24*(a^3*b
- 2*a*b^3)*cos(d*x + c)^3*log(-cos(d*x + c)) - 3*(2*a^3*b - 2*a*b^3 - (a^4
```

```
- 18*a^2*b^2 + 5*b^4)*d*x)*cos(d*x + c)^3 - (3*(a^4 - 6*a^2*b^2 + b^4)*cos(
d*x + c)^4 - 2*b^4 - 2*(18*a^2*b^2 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(
d*cos(d*x + c)^3)
```

giac [B] time = 22.34, size = 3931, normalized size = 28.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*a^4*d*x*tan(d*x)^5*tan(c)^5 - 54*a^2*b^2*d*x*tan(d*x)^5*tan(c)^5 + 1
5*b^4*d*x*tan(d*x)^5*tan(c)^5 - 12*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan
(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/
(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 + 24*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 -
 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
 + 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 + 3*a^4*d*x*tan(d*x)^5*tan(c)^3 -
 54*a^2*b^2*d*x*tan(d*x)^5*tan(c)^3 + 15*b^4*d*x*tan(d*x)^5*tan(c)^3 - 9*a^
4*d*x*tan(d*x)^4*tan(c)^4 + 162*a^2*b^2*d*x*tan(d*x)^4*tan(c)^4 - 45*b^4*d*
x*tan(d*x)^4*tan(c)^4 + 3*a^4*d*x*tan(d*x)^3*tan(c)^5 - 54*a^2*b^2*d*x*tan(
d*x)^3*tan(c)^5 + 15*b^4*d*x*tan(d*x)^3*tan(c)^5 + 6*a^3*b*tan(d*x)^5*tan(c
)^5 + 6*a*b^3*tan(d*x)^5*tan(c)^5 - 12*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2
*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^3 + 24*a*b^3*log(4*(tan(d*x)^4*tan(c)
^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*ta
n(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^3 + 36*a^3*b*log(4*(tan(d*x)^4*
tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d
*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 - 72*a*b^3*log(4*(tan(d
*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2
*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 + 3*a^4*tan(d*x)^
5*tan(c)^4 - 54*a^2*b^2*tan(d*x)^5*tan(c)^4 + 15*b^4*tan(d*x)^5*tan(c)^4 -
12*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(
c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c
)^5 + 24*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^
2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3
*tan(c)^5 + 3*a^4*tan(d*x)^4*tan(c)^5 - 54*a^2*b^2*tan(d*x)^4*tan(c)^5 + 15
*b^4*tan(d*x)^4*tan(c)^5 - 9*a^4*d*x*tan(d*x)^4*tan(c)^2 + 162*a^2*b^2*d*x*
tan(d*x)^4*tan(c)^2 - 45*b^4*d*x*tan(d*x)^4*tan(c)^2 + 12*a^4*d*x*tan(d*x)^
3*tan(c)^3 - 216*a^2*b^2*d*x*tan(d*x)^3*tan(c)^3 + 60*b^4*d*x*tan(d*x)^3*ta
n(c)^3 - 6*a^3*b*tan(d*x)^5*tan(c)^3 + 30*a*b^3*tan(d*x)^5*tan(c)^3 - 9*a^4
*d*x*tan(d*x)^2*tan(c)^4 + 162*a^2*b^2*d*x*tan(d*x)^2*tan(c)^4 - 45*b^4*d*x
*tan(d*x)^2*tan(c)^4 - 42*a^3*b*tan(d*x)^4*tan(c)^4 + 30*a*b^3*tan(d*x)^4*t
an(c)^4 - 6*a^3*b*tan(d*x)^3*tan(c)^5 + 30*a*b^3*tan(d*x)^3*tan(c)^5 + 36*a
^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^2
- 72*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*ta
n(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan
(c)^2 - 36*a^2*b^2*tan(d*x)^5*tan(c)^2 + 10*b^4*tan(d*x)^5*tan(c)^2 - 48*a^
3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
+ tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 +
 96*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan
(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(
c)^3 - 12*a^4*tan(d*x)^4*tan(c)^3 + 108*a^2*b^2*tan(d*x)^4*tan(c)^3 - 30*b^
4*tan(d*x)^4*tan(c)^3 + 36*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*
tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^
2 + 1))*tan(d*x)^2*tan(c)^4 - 72*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d
*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(t
an(c)^2 + 1))*tan(d*x)^2*tan(c)^4 - 12*a^4*tan(d*x)^3*tan(c)^4 + 108*a^2*b^
2*tan(d*x)^3*tan(c)^4 - 30*b^4*tan(d*x)^3*tan(c)^4 - 36*a^2*b^2*tan(d*x)^2*
tan(c)^5 + 10*b^4*tan(d*x)^2*tan(c)^5 + 9*a^4*d*x*tan(d*x)^3*tan(c) - 162*a
```

```
^2*b^2*d*x*tan(d*x)^3*tan(c) + 45*b^4*d*x*tan(d*x)^3*tan(c) + 12*a*b^3*tan(
d*x)^5*tan(c) - 12*a^4*d*x*tan(d*x)^2*tan(c)^2 + 216*a^2*b^2*d*x*tan(d*x)^2
*tan(c)^2 - 60*b^4*d*x*tan(d*x)^2*tan(c)^2 + 18*a^3*b*tan(d*x)^4*tan(c)^2 -
 42*a*b^3*tan(d*x)^4*tan(c)^2 + 9*a^4*d*x*tan(d*x)*tan(c)^3 - 162*a^2*b^2*d
*x*tan(d*x)*tan(c)^3 + 45*b^4*d*x*tan(d*x)*tan(c)^3 + 96*a^3*b*tan(d*x)^3*t
an(c)^3 - 48*a*b^3*tan(d*x)^3*tan(c)^3 + 18*a^3*b*tan(d*x)^2*tan(c)^4 - 42*
a*b^3*tan(d*x)^2*tan(c)^4 + 12*a*b^3*tan(d*x)*tan(c)^5 - 2*b^4*tan(d*x)^5 -
 36*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan
(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(
c) + 72*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2
*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*
tan(c) + 72*a^2*b^2*tan(d*x)^4*tan(c) - 30*b^4*tan(d*x)^4*tan(c) + 48*a^3*b
*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + t
an(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 96
*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)
^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^
2 + 18*a^4*tan(d*x)^3*tan(c)^2 - 108*a^2*b^2*tan(d*x)^3*tan(c)^2 + 10*b^4*t
an(d*x)^3*tan(c)^2 - 36*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan
(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 +
 1))*tan(d*x)*tan(c)^3 + 72*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3
*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)
^2 + 1))*tan(d*x)*tan(c)^3 + 18*a^4*tan(d*x)^2*tan(c)^3 - 108*a^2*b^2*tan(d
*x)^2*tan(c)^3 + 10*b^4*tan(d*x)^2*tan(c)^3 + 72*a^2*b^2*tan(d*x)*tan(c)^4
- 30*b^4*tan(d*x)*tan(c)^4 - 2*b^4*tan(c)^5 - 3*a^4*d*x*tan(d*x)^2 + 54*a^2
*b^2*d*x*tan(d*x)^2 - 15*b^4*d*x*tan(d*x)^2 - 12*a*b^3*tan(d*x)^4 + 9*a^4*d
*x*tan(d*x)*tan(c) - 162*a^2*b^2*d*x*tan(d*x)*tan(c) + 45*b^4*d*x*tan(d*x)*
tan(c) - 18*a^3*b*tan(d*x)^3*tan(c) + 42*a*b^3*tan(d*x)^3*tan(c) - 3*a^4*d*
x*tan(c)^2 + 54*a^2*b^2*d*x*tan(c)^2 - 15*b^4*d*x*tan(c)^2 - 96*a^3*b*tan(d
*x)^2*tan(c)^2 + 48*a*b^3*tan(d*x)^2*tan(c)^2 - 18*a^3*b*tan(d*x)*tan(c)^3
+ 42*a*b^3*tan(d*x)*tan(c)^3 - 12*a*b^3*tan(c)^4 + 12*a^3*b*log(4*(tan(d*x)
^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*ta
n(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2 - 24*a*b^3*log(4*(tan(d*x)^4*
tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d
*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2 - 36*a^2*b^2*tan(d*x)^3 + 10*b^4
*tan(d*x)^3 - 36*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + t
an(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*ta
n(d*x)*tan(c) + 72*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) +
 tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*
tan(d*x)*tan(c) - 12*a^4*tan(d*x)^2*tan(c) + 108*a^2*b^2*tan(d*x)^2*tan(c)
- 30*b^4*tan(d*x)^2*tan(c) + 12*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*
x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(ta
n(c)^2 + 1))*tan(c)^2 - 24*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*
tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^
2 + 1))*tan(c)^2 - 12*a^4*tan(d*x)*tan(c)^2 + 108*a^2*b^2*tan(d*x)*tan(c)^2
 - 30*b^4*tan(d*x)*tan(c)^2 - 36*a^2*b^2*tan(c)^3 + 10*b^4*tan(c)^3 - 3*a^4
*d*x + 54*a^2*b^2*d*x - 15*b^4*d*x + 6*a^3*b*tan(d*x)^2 - 30*a*b^3*tan(d*x)
^2 + 42*a^3*b*tan(d*x)*tan(c) - 30*a*b^3*tan(d*x)*tan(c) + 6*a^3*b*tan(c)^2
 - 30*a*b^3*tan(c)^2 + 12*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*t
an(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2
 + 1)) - 24*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*
x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 3*a^4
*tan(d*x) - 54*a^2*b^2*tan(d*x) + 15*b^4*tan(d*x) + 3*a^4*tan(c) - 54*a^2*b
^2*tan(c) + 15*b^4*tan(c) - 6*a^3*b - 6*a*b^3)/(d*tan(d*x)^5*tan(c)^5 + d*t
an(d*x)^5*tan(c)^3 - 3*d*tan(d*x)^4*tan(c)^4 + d*tan(d*x)^3*tan(c)^5 - 3*d*
tan(d*x)^4*tan(c)^2 + 4*d*tan(d*x)^3*tan(c)^3 - 3*d*tan(d*x)^2*tan(c)^4 + 3
*d*tan(d*x)^3*tan(c) - 4*d*tan(d*x)^2*tan(c)^2 + 3*d*tan(d*x)*tan(c)^3 - d*
tan(d*x)^2 + 3*d*tan(d*x)*tan(c) - d*tan(c)^2 - d)
```

maple [B] time = 0.47, size = 368, normalized size = 2.65

$$-\frac{a^4 \sin(dx+c)\cos(dx+c)}{2d}+\frac{a^4 x}{2}+\frac{a^4 c}{2d}-\frac{2a^3 b\left(\sin^2(dx+c)\right)}{d}-\frac{4a^3 b\ln(\cos(dx+c))}{d}+\frac{6a^2 b^2\left(\sin^5(dx+c)\right)}{d\cos(dx+c)}+\frac{6a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x)
```

```
[Out] -1/2/d*a^4*sin(d*x+c)*cos(d*x+c)+1/2*a^4*x+1/2/d*a^4*c-2/d*a^3*b*sin(d*x+c)
^2-4/d*a^3*b*ln(cos(d*x+c))+6/d*a^2*b^2*sin(d*x+c)^5/cos(d*x+c)+6/d*a^2*b^2
*cos(d*x+c)*sin(d*x+c)^3+9/d*a^2*b^2*sin(d*x+c)*cos(d*x+c)-9*a^2*b^2*x-9/d*
a^2*b^2*c+2/d*a*b^3*sin(d*x+c)^6/cos(d*x+c)^2+2/d*a*b^3*sin(d*x+c)^4+4/d*a*
b^3*sin(d*x+c)^2+8/d*a*b^3*ln(cos(d*x+c))+1/3/d*b^4*sin(d*x+c)^7/cos(d*x+c)
^3-4/3/d*b^4*sin(d*x+c)^7/cos(d*x+c)-4/3/d*b^4*cos(d*x+c)*sin(d*x+c)^5-5/3/
d*b^4*cos(d*x+c)*sin(d*x+c)^3-5/2/d*b^4*sin(d*x+c)*cos(d*x+c)+5/2*b^4*x+5/2
/d*b^4*c
```

maxima [A] time = 0.45, size = 154, normalized size = 1.11

$$\frac{2\,b^4\tan(dx+c)^3+12\,ab^3\tan(dx+c)^2+3\left(a^4-18\,a^2b^2+5\,b^4\right)(dx+c)+12\left(a^3b-2\,ab^3\right)\log\left(\tan(dx+c)^2+1\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^4*tan(d*x + c)^3 + 12*a*b^3*tan(d*x + c)^2 + 3*(a^4 - 18*a^2*b^2 +
 5*b^4)*(d*x + c) + 12*(a^3*b - 2*a*b^3)*log(tan(d*x + c)^2 + 1) + 12*(3*a^
2*b^2 - b^4)*tan(d*x + c) + 3*(4*a^3*b - 4*a*b^3 - (a^4 - 6*a^2*b^2 + b^4)*
tan(d*x + c))/(tan(d*x + c)^2 + 1))/d
```

mupad [B] time = 3.79, size = 161, normalized size = 1.16

$$x\left(\frac{a^4}{2}-9\,a^2\,b^2+\frac{5\,b^4}{2}\right)-\frac{\ln\left(\tan(c+d\,x)^2+1\right)\left(4\,a\,b^3-2\,a^3\,b\right)}{d}-\frac{\cos(c+d\,x)^2\left(\tan(c+d\,x)\left(\frac{a^4}{2}-3\,a^2\,b^2+\frac{b^4}{2}\right)\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b*tan(c + d*x))^4,x)
```

```
[Out] x*(a^4/2 + (5*b^4)/2 - 9*a^2*b^2) - (log(tan(c + d*x)^2 + 1)*(4*a*b^3 - 2*a
^3*b))/d - (cos(c + d*x)^2*(tan(c + d*x)*(a^4/2 + b^4/2 - 3*a^2*b^2) + 2*a*
b^3 - 2*a^3*b))/d + (b^4*tan(c + d*x)^3)/(3*d) - (tan(c + d*x)*(2*b^4 - 6*a
^2*b^2))/d + (2*a*b^3*tan(c + d*x)^2)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^4\sin^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**4*sin(c + d*x)**2, x)
```

3.43 $\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=180

$$-\frac{a^4\cos(c+dx)}{d}-\frac{4a^3b\sin(c+dx)}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}+\frac{6a^2b^2\cos(c+dx)}{d}+\frac{6a^2b^2\sec(c+dx)}{d}+\frac{6ab^3\sin}{}$$

```
[Out] 4*a^3*b*arctanh(sin(d*x+c))/d-6*a*b^3*arctanh(sin(d*x+c))/d-a^4*cos(d*x+c)/
d+6*a^2*b^2*cos(d*x+c)/d-b^4*cos(d*x+c)/d+6*a^2*b^2*sec(d*x+c)/d-2*b^4*sec(
d*x+c)/d+1/3*b^4*sec(d*x+c)^3/d-4*a^3*b*sin(d*x+c)/d+6*a*b^3*sin(d*x+c)/d+2
*a*b^3*sin(d*x+c)*tan(d*x+c)^2/d
```

Rubi [A] time = 0.16, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {3517, 2638, 2592, 321, 206, 2590, 14, 288, 270}

$$\frac{6a^2b^2\cos(c+dx)}{d}+\frac{6a^2b^2\sec(c+dx)}{d}-\frac{4a^3b\sin(c+dx)}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\cos(c+dx)}{d}+\frac{6ab^3\sin(}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (6*a*b^3*ArcTanh[Sin[c + d*x]])/d - (a^
4*Cos[c + d*x])/d + (6*a^2*b^2*Cos[c + d*x])/d - (b^4*Cos[c + d*x])/d + (6*
a^2*b^2*Sec[c + d*x])/d - (2*b^4*Sec[c + d*x])/d + (b^4*Sec[c + d*x]^3)/(3*
d) - (4*a^3*b*Sin[c + d*x])/d + (6*a*b^3*Sin[c + d*x])/d + (2*a*b^3*Sin[c +
 d*x]*Tan[c + d*x]^2)/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx = \int \left(a^4\sin(c+dx)+4a^3b\sin(c+dx)\tan(c+dx)+6a^2b^2\sin(c+dx)\tan^2\right.$$

$$= a^4\int \sin(c+dx)\,dx+\left(4a^3b\right)\int \sin(c+dx)\tan(c+dx)\,dx+\left(6a^2b^2\right)\int \text{s}$$

$$= -\frac{a^4\cos(c+dx)}{d}+\frac{\left(4a^3b\right)\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d}-\frac{\left(6a^2b^2\right)\text{Sub}}{}$$

$$= -\frac{a^4\cos(c+dx)}{d}-\frac{4a^3b\sin(c+dx)}{d}+\frac{2ab^3\sin(c+dx)\tan^2(c+dx)}{d}+\frac{(4}{}$$

$$= \frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\cos(c+dx)}{d}+\frac{6a^2b^2\cos(c+dx)}{d}-\frac{b^4\cos(}{}$$

$$= \frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{6ab^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\cos(c+dx)}{d}+$$

Mathematica [B] time = 5.26, size = 383, normalized size = 2.13

$$-48ab\left(a^2-b^2\right)\sin(c+dx)+\frac{2b^2\left(36a^2-11b^2\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{2b^2\left(11b^2-36a^2\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)}-24ab\left(2a^2-3b^2\right)\log\left(\cos\left(\frac{1}{2}(c\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (72*a^2*b^2 - 22*b^4 - 12*(a^4 - 6*a^2*b^2 + b^4)*Cos[c + d*x] - 24*a*b*(2*
a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 24*a*b*(2*a^2 - 3*b
```

```
^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (b^3*(12*a + b))/(Cos[(c + d
*x)/2] - Sin[(c + d*x)/2])^2 + (2*b^4*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^3 + (2*b^2*(36*a^2 - 11*b^2)*Sin[(c + d*x)/2])/(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]) - (2*b^4*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^3 + (b^3*(-12*a + b))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2 + (2*b^2*(-36*a^2 + 11*b^2)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]) - 48*a*b*(a^2 - b^2)*Sin[c + d*x])/(12*d)
```

fricas [A] time = 0.50, size = 176, normalized size = 0.98

$$-\frac{3\left(a^{4}-6\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{4}-3\left(2\,a^{3}b-3\,ab^{3}\right)\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)+3\left(2\,a^{3}b-3\,ab^{3}\right)\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^4 - 3*(2*a^3*b - 3*a*b^3)*cos(
d*x + c)^3*log(sin(d*x + c) + 1) + 3*(2*a^3*b - 3*a*b^3)*cos(d*x + c)^3*log
(-sin(d*x + c) + 1) - b^4 - 6*(3*a^2*b^2 - b^4)*cos(d*x + c)^2 - 6*(a*b^3*c
os(d*x + c) - 2*(a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x +
c)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.44, size = 309, normalized size = 1.72

$$-\frac{a^{4}\cos\left(dx+c\right)}{d}-\frac{4a^{3}b\sin\left(dx+c\right)}{d}+\frac{4a^{3}b\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{6a^{2}b^{2}\left(\sin^{4}\left(dx+c\right)\right)}{d\cos\left(dx+c\right)}+\frac{6a^{2}b^{2}\cos\left(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*tan(d*x+c))^4,x)
```

```
[Out] -a^4*cos(d*x+c)/d-4*a^3*b*sin(d*x+c)/d+4/d*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+
6/d*a^2*b^2*sin(d*x+c)^4/cos(d*x+c)+6/d*a^2*b^2*cos(d*x+c)*sin(d*x+c)^2+12*
a^2*b^2*cos(d*x+c)/d+2/d*a*b^3*sin(d*x+c)^5/cos(d*x+c)^2+2*a*b^3*sin(d*x+c)
^3/d+6*a*b^3*sin(d*x+c)/d-6/d*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+1/3/d*b^4*sin
(d*x+c)^6/cos(d*x+c)^3-1/d*b^4*sin(d*x+c)^6/cos(d*x+c)-8/3*b^4*cos(d*x+c)/d
-1/d*b^4*cos(d*x+c)*sin(d*x+c)^4-4/3/d*b^4*cos(d*x+c)*sin(d*x+c)^2
```

maxima [A] time = 0.70, size = 166, normalized size = 0.92

$$-\frac{3\,ab^{3}\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)-4\,\sin\left(dx+c\right)\right)-18\,a^{2}b^{2}\left(\frac{1}{\cos\left(dx+c\right)}+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1
) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 18*a^2*b^2*(1/cos(d*x + c)
+ cos(d*x + c)) + b^4*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x +
```

```
c)) - 6*a^3*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x +
c)) + 3*a^4*cos(d*x + c))/d
```

mupad [B] time = 7.29, size = 268, normalized size = 1.49

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(6\,a^4-48\,a^2\,b^2+\frac{32\,b^4}{3}\right)+2\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(12\,a\,b^3-8\,a^3\,b\right)-2\,a^4-\frac{16\,b^4}{3}+24}{d\,\left(\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*tan(c + d*x))^4,x)
```

```
[Out] - (tan(c/2 + (d*x)/2)^2*(6*a^4 + (32*b^4)/3 - 48*a^2*b^2) + 2*a^4*tan(c/2 +
 (d*x)/2)^6 + tan(c/2 + (d*x)/2)*(12*a*b^3 - 8*a^3*b) - 2*a^4 - (16*b^4)/3
+ 24*a^2*b^2 - tan(c/2 + (d*x)/2)^4*(6*a^4 - 24*a^2*b^2) - tan(c/2 + (d*x)/
2)^7*(12*a*b^3 - 8*a^3*b) - tan(c/2 + (d*x)/2)^3*(20*a*b^3 - 24*a^3*b) + ta
n(c/2 + (d*x)/2)^5*(20*a*b^3 - 24*a^3*b))/(d*(2*tan(c/2 + (d*x)/2)^2 - 2*ta
n(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 - 1)) - (atanh(tan(c/2 + (d*x)/2)
)*(12*a*b^3 - 8*a^3*b))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan{(c + dx)})^4 \sin{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**4*sin(c + d*x), x)
```

3.44 $\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=118

$$-\frac{a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}+\frac{6a^2b^2\sec(c+dx)}{d}-\frac{2ab^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ab^3\tan(}{}$$

```
[Out] -a^4*arctanh(cos(d*x+c))/d+4*a^3*b*arctanh(sin(d*x+c))/d-2*a*b^3*arctanh(si
n(d*x+c))/d+6*a^2*b^2*sec(d*x+c)/d-b^4*sec(d*x+c)/d+1/3*b^4*sec(d*x+c)^3/d+
2*a*b^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3517, 3770, 2606, 8, 2611}

$$\frac{6a^2b^2\sec(c+dx)}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\tanh^{-1}(\cos(c+dx))}{d}-\frac{2ab^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ab^3\tan(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -((a^4*ArcTanh[Cos[c + d*x]])/d) + (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (2*a
*b^3*ArcTanh[Sin[c + d*x]])/d + (6*a^2*b^2*Sec[c + d*x])/d - (b^4*Sec[c + d
*x])/d + (b^4*Sec[c + d*x]^3)/(3*d) + (2*a*b^3*Sec[c + d*x]*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx = \int \left(a^4\csc(c+dx)+4a^3b\sec(c+dx)+6a^2b^2\sec(c+dx)\tan(c+dx)+4\right.$$
$$= a^4\int\csc(c+dx)\,dx+\left(4a^3b\right)\int\sec(c+dx)\,dx+\left(6a^2b^2\right)\int\sec(c+dx)\,t$$
$$= -\frac{a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ab^3\sec(c+dx)}{d}$$
$$= -\frac{a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab^3\tanh^{-1}(\sin(}{d}$$

Mathematica [B] time = 5.00, size = 352, normalized size = 2.98

$$12a^4\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)-12a^4\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)-48a^3b\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+48a^3b$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (72*a^2*b^2 - 10*b^4 - 12*a^4*Log[Cos[(c + d*x)/2]] - 48*a^3*b*Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]] + 24*a*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] + 12*a^4*Log[Sin[(c + d*x)/2]] + 48*a^3*b*Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] - 24*a*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (12*a*b
^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + b^4/(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])^2 + 2*b^2*(36*a^2 - b^2 + 2*b^2*Cos[c + d*x] + (36*a^2 - 5*b^2
)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*Sin[(c + d*x)/2]^2 - (12*a*b^3)/(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^2 + b^4/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^2)/(12*d)
```

fricas [A] time = 0.51, size = 175, normalized size = 1.48

$$-\frac{3\,a^4\cos(dx+c)^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\,a^4\cos(dx+c)^3\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-12\,ab^3\cos(dx+c)\,\mathrm{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*a^4*cos(d*x + c)^3*log(1/2*cos(d*x + c) + 1/2) - 3*a^4*cos(d*x + c)
^3*log(-1/2*cos(d*x + c) + 1/2) - 12*a*b^3*cos(d*x + c)*sin(d*x + c) - 6*(2
*a^3*b - a*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 6*(2*a^3*b - a*b^3)*
cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*b^4 - 6*(6*a^2*b^2 - b^4)*cos(d*x
 + c)^2)/(d*cos(d*x + c)^3)
```

giac [A] time = 6.00, size = 193, normalized size = 1.64

$$\frac{3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+6\left(2\,a^3b-ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\left(2\,a^3b-ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 6*(2*a^3*b - a*b^3)*log(abs(tan
(1/2*d*x + 1/2*c) + 1)) - 6*(2*a^3*b - a*b^3)*log(abs(tan(1/2*d*x + 1/2*c)
```

```
- 1)) + 4*(3*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^2*b^2*tan(1/2*d*x + 1/2*c)^
4 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*b^4*tan(1/2*d*x + 1/2*c)^2 - 3*a*
b^3*tan(1/2*d*x + 1/2*c) - 9*a^2*b^2 + b^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)
/d
```

maple [A] time = 0.46, size = 214, normalized size = 1.81

$$\frac{a^4 \ln(\csc(dx+c) - \cot(dx+c))}{d} + \frac{4a^3 b \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{6a^2b^2}{d\cos(dx+c)} + \frac{2a\,b^3 \left(\sin^3(dx+c)\right)}{d\cos(dx+c)^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*tan(d*x+c))^4,x)
```

```
[Out] 1/d*a^4*ln(csc(d*x+c)-cot(d*x+c))+4/d*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+6/d*a
^2*b^2/cos(d*x+c)+2/d*a*b^3*sin(d*x+c)^3/cos(d*x+c)^2+2*a*b^3*sin(d*x+c)/d-
2/d*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+1/3/d*b^4*sin(d*x+c)^4/cos(d*x+c)^3-1/3
/d*b^4*sin(d*x+c)^4/cos(d*x+c)-1/3/d*b^4*cos(d*x+c)*sin(d*x+c)^2-2/3*b^4*co
s(d*x+c)/d
```

maxima [A] time = 0.68, size = 139, normalized size = 1.18

$$-\frac{3\,ab^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} + \log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)\right) - 6\,a^3b\left(\log(\sin(dx+c)+1) - \log(\sin(d\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1)
- log(sin(d*x + c) - 1)) - 6*a^3*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 3*a^4*log(cot(d*x + c) + csc(d*x + c)) - 18*a^2*b^2/cos(d*x + c)
+ (3*cos(d*x + c)^2 - 1)*b^4/cos(d*x + c)^3)/d
```

mupad [B] time = 4.14, size = 496, normalized size = 4.20

$$\frac{a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{12\,a^2\,b^2 - \frac{4\,b^4}{3} + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(4\,b^4 - 24\,a^2\,b^2\right) + 12\,a^2\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 4\,a\,b^3\,\tan\left(\frac{c}{2}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - 3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^4/sin(c + d*x),x)
```

```
[Out] (a^4*log(tan(c/2 + (d*x)/2)))/d - (12*a^2*b^2 - (4*b^4)/3 + tan(c/2 + (d*x)
/2)^2*(4*b^4 - 24*a^2*b^2) + 12*a^2*b^2*tan(c/2 + (d*x)/2)^4 + 4*a*b^3*tan(
c/2 + (d*x)/2) - 4*a*b^3*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d*x)/2)^2 -
 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) - (a*b*atan((a*b*(2*a^
2 - b^2)*(4*a*b^3 - 8*a^3*b + 2*a^4*tan(c/2 + (d*x)/2) - 12*a*b*tan(c/2 + (
d*x)/2)*(2*a^2 - b^2))*2i + a*b*(2*a^2 - b^2)*(4*a*b^3 - 8*a^3*b + 2*a^4*ta
n(c/2 + (d*x)/2) + 12*a*b*tan(c/2 + (d*x)/2)*(2*a^2 - b^2))*2i)/(2*tan(c/2
+ (d*x)/2)*(16*a^2*b^6 - 64*a^4*b^4 + 64*a^6*b^2) + 16*a^7*b - 8*a^5*b^3 +
2*a*b*(2*a^2 - b^2)*(4*a*b^3 - 8*a^3*b + 2*a^4*tan(c/2 + (d*x)/2) - 12*a*b*
tan(c/2 + (d*x)/2)*(2*a^2 - b^2)) - 2*a*b*(2*a^2 - b^2)*(4*a*b^3 - 8*a^3*b
+ 2*a^4*tan(c/2 + (d*x)/2) + 12*a*b*tan(c/2 + (d*x)/2)*(2*a^2 - b^2))))*(2*
a^2 - b^2)*4i)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^4 \csc(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**4*csc(c + d*x), x)
```

3.45 $\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=83

$$-\frac{a^4\cot(c+dx)}{d}+\frac{4a^3b\log(\tan(c+dx))}{d}+\frac{6a^2b^2\tan(c+dx)}{d}+\frac{2ab^3\tan^2(c+dx)}{d}+\frac{b^4\tan^3(c+dx)}{3d}$$

```
[Out] -a^4*cot(d*x+c)/d+4*a^3*b*ln(tan(d*x+c))/d+6*a^2*b^2*tan(d*x+c)/d+2*a*b^3*t
an(d*x+c)^2/d+1/3*b^4*tan(d*x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 43}

$$\frac{6a^2b^2\tan(c+dx)}{d}+\frac{4a^3b\log(\tan(c+dx))}{d}-\frac{a^4\cot(c+dx)}{d}+\frac{2ab^3\tan^2(c+dx)}{d}+\frac{b^4\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -((a^4*Cot[c + d*x])/d) + (4*a^3*b*Log[Tan[c + d*x]])/d + (6*a^2*b^2*Tan[c
+ d*x])/d + (2*a*b^3*Tan[c + d*x]^2)/d + (b^4*Tan[c + d*x]^3)/(3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx &= \frac{b\,\text{Subst}\left(\int \frac{(a+x)^4}{x^2}\,dx,x,b\tan(c+dx)\right)}{d}\\
&= \frac{b\,\text{Subst}\left(\int \left(6a^2+\frac{a^4}{x^2}+\frac{4a^3}{x}+4ax+x^2\right)dx,x,b\tan(c+dx)\right)}{d}\\
&= -\frac{a^4\cot(c+dx)}{d}+\frac{4a^3b\log(\tan(c+dx))}{d}+\frac{6a^2b^2\tan(c+dx)}{d}+\frac{2ab^3}{}
\end{aligned}$$

Mathematica [A] time = 1.21, size = 162, normalized size = 1.95

$$-\frac{\csc(c+dx)\sec^3(c+dx)\left(4\left(3a^4+b^4\right)\cos(2(c+dx))+\left(3a^4+18a^2b^2-b^4\right)\cos(4(c+dx))+3\left(3a^4+4a^3b\,\text{si}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/24*(Csc[c + d*x]*Sec[c + d*x]^3*(4*(3*a^4 + b^4)*Cos[2*(c + d*x)] + (3*a
^4 + 18*a^2*b^2 - b^4)*Cos[4*(c + d*x)] + 3*(3*a^4 - 6*a^2*b^2 - b^4 + 8*a*
```

```
b*(-b^2 + a^2*Log[Cos[c + d*x]] - a^2*Log[Sin[c + d*x]])*Sin[2*(c + d*x)] +
 4*a^3*b*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]])*Sin[4*(c + d*x)])))/d
```

fricas [A] time = 0.45, size = 159, normalized size = 1.92

$$-\frac{6\,a^3b\cos\left(dx+c\right)^3\log\left(\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)-6\,a^3b\cos\left(dx+c\right)^3\log\left(-\frac{1}{4}\,\cos\left(dx+c\right)^2+\frac{1}{4}\right)\sin\left(dx+c\right)-}{3\,d\cos\left(dx+c\right)^3\,\mathrm{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a^3*b*cos(d*x + c)^3*log(cos(d*x + c)^2)*sin(d*x + c) - 6*a^3*b*cos
(d*x + c)^3*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 6*a*b^3*cos(d*x +
 c)*sin(d*x + c) + (3*a^4 + 18*a^2*b^2 - b^4)*cos(d*x + c)^4 - b^4 - 2*(9*a
^2*b^2 - b^4)*cos(d*x + c)^2)/(d*cos(d*x + c)^3*sin(d*x + c))
```

giac [A] time = 4.04, size = 86, normalized size = 1.04

$$\frac{b^4\tan\left(dx+c\right)^3+6\,ab^3\tan\left(dx+c\right)^2+12\,a^3b\log\left(\left|\tan\left(dx+c\right)\right|\right)+18\,a^2b^2\tan\left(dx+c\right)-\frac{3\left(4\,a^3b\tan\left(dx+c\right)+a^4\right)}{\tan\left(dx+c\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 12*a^3*b*log(abs(tan(d*x
 + c))) + 18*a^2*b^2*tan(d*x + c) - 3*(4*a^3*b*tan(d*x + c) + a^4)/tan(d*x
+ c))/d
```

maple [A] time = 0.54, size = 90, normalized size = 1.08

$$-\frac{a^4\cot\left(dx+c\right)}{d}+\frac{4a^3b\ln\left(\tan\left(dx+c\right)\right)}{d}+\frac{6a^2b^2\tan\left(dx+c\right)}{d}+\frac{2a\,b^3}{d\cos\left(dx+c\right)^2}+\frac{b^4\left(\sin^3\left(dx+c\right)\right)}{3d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x)
```

```
[Out] -a^4*cot(d*x+c)/d+4*a^3*b*ln(tan(d*x+c))/d+6*a^2*b^2*tan(d*x+c)/d+2/d*a*b^3
/cos(d*x+c)^2+1/3/d*b^4*sin(d*x+c)^3/cos(d*x+c)^3
```

maxima [A] time = 0.42, size = 72, normalized size = 0.87

$$\frac{b^4\tan\left(dx+c\right)^3+6\,ab^3\tan\left(dx+c\right)^2+12\,a^3b\log\left(\tan\left(dx+c\right)\right)+18\,a^2b^2\tan\left(dx+c\right)-\frac{3\,a^4}{\tan\left(dx+c\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 12*a^3*b*log(tan(d*x + c
)) + 18*a^2*b^2*tan(d*x + c) - 3*a^4/tan(d*x + c))/d
```

mupad [B] time = 3.66, size = 81, normalized size = 0.98

$$\frac{b^4\tan\left(c+d\,x\right)^3}{3\,d}-\frac{a^4\cot\left(c+d\,x\right)}{d}+\frac{6\,a^2\,b^2\tan\left(c+d\,x\right)}{d}+\frac{2\,a\,b^3\tan\left(c+d\,x\right)^2}{d}+\frac{4\,a^3\,b\,\ln\left(\tan\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^4/sin(c + d*x)^2,x)
```

```
[Out] (b^4*tan(c + d*x)^3)/(3*d) - (a^4*cot(c + d*x))/d + (6*a^2*b^2*tan(c + d*x)
)/d + (2*a*b^3*tan(c + d*x)^2)/d + (4*a^3*b*log(tan(c + d*x)))/d
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan (c+dx))^4 \csc^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**4*csc(c + d*x)**2, x)
```

3.46 $\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=161

$$-\frac{a^4\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^4\cot(c+dx)\csc(c+dx)}{2d}-\frac{4a^3b\csc(c+dx)}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}+\frac{6a^2b^2\sec(c+}{d}$$

```
[Out] -1/2*a^4*arctanh(cos(d*x+c))/d-6*a^2*b^2*arctanh(cos(d*x+c))/d+4*a^3*b*arct
anh(sin(d*x+c))/d+2*a*b^3*arctanh(sin(d*x+c))/d-4*a^3*b*csc(d*x+c)/d-1/2*a^
4*cot(d*x+c)*csc(d*x+c)/d+6*a^2*b^2*sec(d*x+c)/d+1/3*b^4*sec(d*x+c)^3/d+2*a
*b^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.16, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3517, 3768, 3770, 2621, 321, 207, 2622, 2606, 30}

$$\frac{6a^2b^2\sec(c+dx)}{d}-\frac{6a^2b^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{4a^3b\csc(c+dx)}{d}+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\tanh^{-1}(\cos(c+a}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(a^4*ArcTanh[Cos[c + d*x]])/(2*d) - (6*a^2*b^2*ArcTanh[Cos[c + d*x]])/d +
(4*a^3*b*ArcTanh[Sin[c + d*x]])/d + (2*a*b^3*ArcTanh[Sin[c + d*x]])/d - (4*
a^3*b*Csc[c + d*x])/d - (a^4*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (6*a^2*b^2*
Sec[c + d*x])/d + (b^4*Sec[c + d*x]^3)/(3*d) + (2*a*b^3*Sec[c + d*x]*Tan[c
+ d*x])/d
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
```

```
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx = \int \left(a^4\csc^3(c+dx)+4a^3b\csc^2(c+dx)\sec(c+dx)+6a^2b^2\csc(c+d\right.$$

$$= a^4\int \csc^3(c+dx)\,dx+\left(4a^3b\right)\int \csc^2(c+dx)\sec(c+dx)\,dx+\left(6a^2b^{\right.$$

$$= -\frac{a^4\cot(c+dx)\csc(c+dx)}{2d}+\frac{2ab^3\sec(c+dx)\tan(c+dx)}{d}+\frac{1}{2}a^4\int$$

$$= -\frac{a^4\tanh^{-1}(\cos(c+dx))}{2d}+\frac{2ab^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{4a^3b\csc(c+}{d}$$

$$= -\frac{a^4\tanh^{-1}(\cos(c+dx))}{2d}-\frac{6a^2b^2\tanh^{-1}(\cos(c+dx))}{d}+4a^3b\tanh^{-1}$$

Mathematica [B] time = 6.21, size = 1128, normalized size = 7.01

$$-\frac{2a^3b\cos^4(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4}+\frac{b^2\left(36a^2+b^2\right)\cos^4(c+dx)(a+b\tan(c+dx))^4}{6d(a\cos(c+dx)+b\sin(c+dx))^4}-\frac{a^4 c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (b^2*(36*a^2 + b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(6*d*(a*Cos[c +
d*x] + b*Sin[c + d*x])^4) - (2*a^3*b*Cos[c + d*x]^4*Cot[(c + d*x)/2]*(a + b
*Tan[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (a^4*Cos[c + d*
x]^4*Csc[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^4)/(8*d*(a*Cos[c + d*x] + b*Si
n[c + d*x])^4) + ((-a^4 - 12*a^2*b^2)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2]]*
```

```
(a + b*Tan[c + d*x])^4)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (2*(2*a
^3*b + a*b^3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a +
b*Tan[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((a^4 + 12*a^2
*b^2)*Cos[c + d*x]^4*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(2*d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) + (2*(2*a^3*b + a*b^3)*Cos[c + d*x]^4*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(d*(a*Cos[c +
 d*x] + b*Sin[c + d*x])^4) + (a^4*Cos[c + d*x]^4*Sec[(c + d*x)/2]^2*(a + b*
Tan[c + d*x])^4)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((12*a*b^3 + b
^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*Si
n[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*Sin[(c
+ d*x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((-12*a*b^3 + b^4)*Cos[c + d*x]^
4*(a + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(-36*a^2*b^2*Sin[(c + d
*x)/2] - b^4*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]
^4*(36*a^2*b^2*Sin[(c + d*x)/2] + b^4*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x]
)^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c +
 d*x])^4) - (2*a^3*b*Cos[c + d*x]^4*Tan[(c + d*x)/2]*(a + b*Tan[c + d*x])^4
)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

fricas [B] time = 0.55, size = 346, normalized size = 2.15

$$6\left(a^4+12\,a^2b^2\right)\cos\left(dx+c\right)^4-4\,b^4-4\left(18\,a^2b^2-b^4\right)\cos\left(dx+c\right)^2-3\left(\left(a^4+12\,a^2b^2\right)\cos\left(dx+c\right)^5-\left(a^4+12\,a^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^4 + 12*a^2*b^2)*cos(d*x + c)^4 - 4*b^4 - 4*(18*a^2*b^2 - b^4)*co
s(d*x + c)^2 - 3*((a^4 + 12*a^2*b^2)*cos(d*x + c)^5 - (a^4 + 12*a^2*b^2)*co
s(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2) + 3*((a^4 + 12*a^2*b^2)*cos(d*x +
 c)^5 - (a^4 + 12*a^2*b^2)*cos(d*x + c)^3)*log(-1/2*cos(d*x + c) + 1/2) + 1
2*((2*a^3*b + a*b^3)*cos(d*x + c)^5 - (2*a^3*b + a*b^3)*cos(d*x + c)^3)*log
(sin(d*x + c) + 1) - 12*((2*a^3*b + a*b^3)*cos(d*x + c)^5 - (2*a^3*b + a*b^
3)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 24*(a*b^3*cos(d*x + c) - (2*a^3
*b + a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^5 - d*cos(d*x + c
)^3)
```

giac [A] time = 12.82, size = 300, normalized size = 1.86

$$3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-48\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+48\left(2\,a^3b+ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-48\left(2\,a^3b+ab^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^4*tan(1/2*d*x + 1/2*c)^2 - 48*a^3*b*tan(1/2*d*x + 1/2*c) + 48*(2*
a^3*b + a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 48*(2*a^3*b + a*b^3)*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 12*(a^4 + 12*a^2*b^2)*log(abs(tan(1/2*d*
x + 1/2*c))) - 3*(6*a^4*tan(1/2*d*x + 1/2*c)^2 + 72*a^2*b^2*tan(1/2*d*x + 1
/2*c)^2 + 16*a^3*b*tan(1/2*d*x + 1/2*c) + a^4)/tan(1/2*d*x + 1/2*c)^2 + 16*
(6*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 3*b^4
```

```
*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a*b^3*tan(1
/2*d*x + 1/2*c) - 18*a^2*b^2 - b^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.52, size = 192, normalized size = 1.19

$$-\frac{a^4 \cot(dx+c)\csc(dx+c)}{2d}+\frac{a^4 \ln(\csc(dx+c)-\cot(dx+c))}{2d}-\frac{4a^3 b}{d\sin(dx+c)}+\frac{4a^3 b\ln(\sec(dx+c)+\tan(d}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x)
```

```
[Out] -1/2*a^4*cot(d*x+c)*csc(d*x+c)/d+1/2/d*a^4*ln(csc(d*x+c)-cot(d*x+c))-4/d*a^
3*b/sin(d*x+c)+4/d*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+6/d*a^2*b^2/cos(d*x+c)+6
/d*a^2*b^2*ln(csc(d*x+c)-cot(d*x+c))+2*a*b^3*sec(d*x+c)*tan(d*x+c)/d+2/d*a*
b^3*ln(sec(d*x+c)+tan(d*x+c))+1/3/d*b^4/cos(d*x+c)^3
```

maxima [A] time = 0.48, size = 188, normalized size = 1.17

$$\frac{3\,a^4\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-12\,ab^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^4*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) +
log(cos(d*x + c) - 1)) - 12*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - lo
g(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 36*a^2*b^2*(2/cos(d*x + c) -
 log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 24*a^3*b*(2/sin(d*x + c)
- log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 4*b^4/cos(d*x + c)^3)/d
```

mupad [B] time = 4.07, size = 670, normalized size = 4.16

$$\frac{a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{\frac{a^4}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{3\,a^4}{2}+48\,a^2\,b^2+\frac{8\,b^4}{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{a^4}{2}+48\,a^2\,b^2+8\,b^4\right)+\tan\left(\frac{c}{2}+\right.}{d\left(-4\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^4/sin(c + d*x)^3,x)
```

```
[Out] (a^4*tan(c/2 + (d*x)/2)^2)/(8*d) - (a^4/2 - tan(c/2 + (d*x)/2)^2*((3*a^4)/2
 + (8*b^4)/3 + 48*a^2*b^2) - tan(c/2 + (d*x)/2)^6*(a^4/2 + 8*b^4 + 48*a^2*b
^2) + tan(c/2 + (d*x)/2)^4*((3*a^4)/2 + 96*a^2*b^2) + tan(c/2 + (d*x)/2)^7*
(16*a*b^3 - 8*a^3*b) - tan(c/2 + (d*x)/2)^3*(16*a*b^3 + 24*a^3*b) + 8*a^3*b
*tan(c/2 + (d*x)/2) + 24*a^3*b*tan(c/2 + (d*x)/2)^5)/(d*(4*tan(c/2 + (d*x)/
2)^2 - 12*tan(c/2 + (d*x)/2)^4 + 12*tan(c/2 + (d*x)/2)^6 - 4*tan(c/2 + (d*x
)/2)^8)) + (log(tan(c/2 + (d*x)/2))*(a^4/2 + 6*a^2*b^2))/d - (2*a^3*b*tan(c
/2 + (d*x)/2))/d - (a*b*atan((a*b*(2*a^2 + b^2)*(tan(c/2 + (d*x)/2)*(a^4 +
12*a^2*b^2) - 4*a*b^3 - 8*a^3*b + 12*a*b*tan(c/2 + (d*x)/2)*(2*a^2 + b^2))*
2i - a*b*(2*a^2 + b^2)*(4*a*b^3 - tan(c/2 + (d*x)/2)*(a^4 + 12*a^2*b^2) + 8
*a^3*b + 12*a*b*tan(c/2 + (d*x)/2)*(2*a^2 + b^2))*2i)/(2*tan(c/2 + (d*x)/2)
*(16*a^2*b^6 + 64*a^4*b^4 + 64*a^6*b^2) + 8*a^7*b + 48*a^3*b^5 + 100*a^5*b^
3 - 2*a*b*(2*a^2 + b^2)*(tan(c/2 + (d*x)/2)*(a^4 + 12*a^2*b^2) - 4*a*b^3 -
8*a^3*b + 12*a*b*tan(c/2 + (d*x)/2)*(2*a^2 + b^2)) - 2*a*b*(2*a^2 + b^2)*(4
*a*b^3 - tan(c/2 + (d*x)/2)*(a^4 + 12*a^2*b^2) + 8*a^3*b + 12*a*b*tan(c/2 +
 (d*x)/2)*(2*a^2 + b^2))))*(2*a^2 + b^2)*4i)/d
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^4 \csc^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**4*csc(c + d*x)**3, x)
```

3.47 $\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=137

$$-\frac{a^4\cot^3(c+dx)}{3d}-\frac{2a^3b\cot^2(c+dx)}{d}+\frac{b^2\left(6a^2+b^2\right)\tan(c+dx)}{d}-\frac{a^2\left(a^2+6b^2\right)\cot(c+dx)}{d}+\frac{4ab\left(a^2+b^2\right)\log(\mathrm{t}}{d}$$

```
[Out] -a^2*(a^2+6*b^2)*cot(d*x+c)/d-2*a^3*b*cot(d*x+c)^2/d-1/3*a^4*cot(d*x+c)^3/d
+4*a*b*(a^2+b^2)*ln(tan(d*x+c))/d+b^2*(6*a^2+b^2)*tan(d*x+c)/d+2*a*b^3*tan(
d*x+c)^2/d+1/3*b^4*tan(d*x+c)^3/d
```

Rubi [A] time = 0.10, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 894}

$$\frac{b^2\left(6a^2+b^2\right)\tan(c+dx)}{d}-\frac{a^2\left(a^2+6b^2\right)\cot(c+dx)}{d}+\frac{4ab\left(a^2+b^2\right)\log(\tan(c+dx))}{d}-\frac{2a^3b\cot^2(c+dx)}{d}-\frac{a^4\,\mathrm{cc}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -((a^2*(a^2 + 6*b^2)*Cot[c + d*x])/d) - (2*a^3*b*Cot[c + d*x]^2)/d - (a^4*C
ot[c + d*x]^3)/(3*d) + (4*a*b*(a^2 + b^2)*Log[Tan[c + d*x]])/d + (b^2*(6*a^
2 + b^2)*Tan[c + d*x])/d + (2*a*b^3*Tan[c + d*x]^2)/d + (b^4*Tan[c + d*x]^3
)/(3*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx &= \frac{b\,\mathrm{Subst}\left(\int \frac{(a+x)^4\left(b^2+x^2\right)}{x^4}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\,\mathrm{Subst}\left(\int \left(6a^2\left(1+\frac{b^2}{6a^2}\right)+\frac{a^4b^2}{x^4}+\frac{4a^3b^2}{x^3}+\frac{a^4+6a^2b^2}{x^2}+\frac{4a\left(a^2+b^2\right)}{x}+4ax\right.\right.}{d}\\ &= -\frac{a^2\left(a^2+6b^2\right)\cot(c+dx)}{d}-\frac{2a^3b\cot^2(c+dx)}{d}-\frac{a^4\cot^3(c+dx)}{3d}+\frac{4}{}\end{aligned}$$

Mathematica [A] time = 3.84, size = 188, normalized size = 1.37

$$-\frac{\sin(c+dx)\tan^3(c+dx)(a\cot(c+dx)+b)^4\left(-2b^2\left(9a^2+b^2\right)\sin(c+dx)\cos^2(c+dx)+2a\cos^3(c+dx)\left(a\left(a\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/3*((b + a*Cot[c + d*x])^4*Sin[c + d*x]*(Cos[c + d*x]*(-6*a*b^3 + 6*a^3*b
*Cot[c + d*x]^2 + a^4*Cot[c + d*x]^3) + 2*a*Cos[c + d*x]^3*(a*(a^2 + 9*b^2)
*Cot[c + d*x] + 6*b*(a^2 + b^2)*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]])) -
b^4*Sin[c + d*x] - 2*b^2*(9*a^2 + b^2)*Cos[c + d*x]^2*Sin[c + d*x])*Tan[c +
 d*x]^3)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

fricas [B] time = 0.46, size = 267, normalized size = 1.95

$$-\frac{2\left(a^4+18\,a^2b^2+b^4\right)\cos\left(dx+c\right)^6+18\,a^2b^2\cos\left(dx+c\right)^2-3\left(a^4+18\,a^2b^2+b^4\right)\cos\left(dx+c\right)^4+b^4+6\left(\left(a^3b+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^4 + 18*a^2*b^2 + b^4)*cos(d*x + c)^6 + 18*a^2*b^2*cos(d*x + c)^2
 - 3*(a^4 + 18*a^2*b^2 + b^4)*cos(d*x + c)^4 + b^4 + 6*((a^3*b + a*b^3)*cos
(d*x + c)^5 - (a^3*b + a*b^3)*cos(d*x + c)^3)*log(cos(d*x + c)^2)*sin(d*x +
 c) - 6*((a^3*b + a*b^3)*cos(d*x + c)^5 - (a^3*b + a*b^3)*cos(d*x + c)^3)*l
og(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) + 6*(a*b^3*cos(d*x + c) - (a^3*b
 + a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/((d*cos(d*x + c)^5 - d*cos(d*x + c)
^3)*sin(d*x + c))
```

giac [A] time = 7.80, size = 161, normalized size = 1.18

$$\frac{b^4\tan\left(dx+c\right)^3+6\,ab^3\tan\left(dx+c\right)^2+18\,a^2b^2\tan\left(dx+c\right)+3\,b^4\tan\left(dx+c\right)+12\left(a^3b+ab^3\right)\log\left(\left|\tan\left(dx+c\right)\right|\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 18*a^2*b^2*tan(d*x + c)
+ 3*b^4*tan(d*x + c) + 12*(a^3*b + a*b^3)*log(abs(tan(d*x + c))) - (22*a^3*
b*tan(d*x + c)^3 + 22*a*b^3*tan(d*x + c)^3 + 3*a^4*tan(d*x + c)^2 + 18*a^2*
b^2*tan(d*x + c)^2 + 6*a^3*b*tan(d*x + c) + a^4)/tan(d*x + c)^3)/d
```

maple [A] time = 0.66, size = 184, normalized size = 1.34

$$-\frac{2a^4\cot\left(dx+c\right)}{3d}-\frac{a^4\cot\left(dx+c\right)\left(\csc^2\left(dx+c\right)\right)}{3d}-\frac{2a^3b}{d\sin\left(dx+c\right)^2}+\frac{4a^3b\ln\left(\tan\left(dx+c\right)\right)}{d}+\frac{6a^2b^2}{d\sin\left(dx+c\right)\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x)
```

```
[Out] -2/3*a^4*cot(d*x+c)/d-1/3/d*a^4*cot(d*x+c)*csc(d*x+c)^2-2/d*a^3*b/sin(d*x+c
)^2+4*a^3*b*ln(tan(d*x+c))/d+6/d*a^2*b^2/sin(d*x+c)/cos(d*x+c)-12/d*a^2*b^2
*cot(d*x+c)+2/d*a*b^3/cos(d*x+c)^2+4/d*a*b^3*ln(tan(d*x+c))+2/3/d*b^4*tan(d
*x+c)+1/3/d*b^4*tan(d*x+c)*sec(d*x+c)^2
```

maxima [A] time = 0.40, size = 120, normalized size = 0.88

$$\frac{b^4\tan\left(dx+c\right)^3+6\,ab^3\tan\left(dx+c\right)^2+12\left(a^3b+ab^3\right)\log\left(\tan\left(dx+c\right)\right)+3\left(6\,a^2b^2+b^4\right)\tan\left(dx+c\right)-\frac{6\,a^3b\tan(d}{}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 12*(a^3*b + a*b^3)*log(t
an(d*x + c)) + 3*(6*a^2*b^2 + b^4)*tan(d*x + c) - (6*a^3*b*tan(d*x + c) + a
^4 + 3*(a^4 + 6*a^2*b^2)*tan(d*x + c)^2)/tan(d*x + c)^3)/d
```

mupad [B] time = 3.76, size = 132, normalized size = 0.96

$$\frac{\ln\left(\tan\left(c+d\,x\right)\right)\,\left(4\,a^3\,b+4\,a\,b^3\right)}{d}-\frac{\cot\left(c+d\,x\right)^3\,\left(\tan\left(c+d\,x\right)^2\,\left(a^4+6\,a^2\,b^2\right)+\frac{a^4}{3}+2\,a^3\,b\,\tan\left(c+d\,x\right)\right)}{d}+\frac{\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^4/sin(c + d*x)^4,x)
```

```
[Out] (log(tan(c + d*x))*(4*a*b^3 + 4*a^3*b))/d - (cot(c + d*x)^3*(tan(c + d*x)^2
*(a^4 + 6*a^2*b^2) + a^4/3 + 2*a^3*b*tan(c + d*x)))/d + (tan(c + d*x)*(b^4
+ 6*a^2*b^2))/d + (b^4*tan(c + d*x)^3)/(3*d) + (2*a*b^3*tan(c + d*x)^2)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^4 \csc^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**4*csc(c + d*x)**4, x)
```

3.48 $\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=274

$$-\frac{3a^4\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^4\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^4\cot(c+dx)\csc(c+dx)}{8d}-\frac{4a^3b\csc^3(c+dx)}{3d}-\frac{4a^3b\csc}{d}$$

```
[Out] -3/8*a^4*arctanh(cos(d*x+c))/d-9*a^2*b^2*arctanh(cos(d*x+c))/d-b^4*arctanh(
cos(d*x+c))/d+4*a^3*b*arctanh(sin(d*x+c))/d+6*a*b^3*arctanh(sin(d*x+c))/d-4
*a^3*b*csc(d*x+c)/d-6*a*b^3*csc(d*x+c)/d-3/8*a^4*cot(d*x+c)*csc(d*x+c)/d-4/
3*a^3*b*csc(d*x+c)^3/d-1/4*a^4*cot(d*x+c)*csc(d*x+c)^3/d+9*a^2*b^2*sec(d*x+
c)/d+b^4*sec(d*x+c)/d-3*a^2*b^2*csc(d*x+c)^2*sec(d*x+c)/d+2*a*b^3*csc(d*x+c
)*sec(d*x+c)^2/d+1/3*b^4*sec(d*x+c)^3/d
```

Rubi [A] time = 0.24, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3517, 3768, 3770, 2621, 302, 207, 2622, 288, 321}

$$\frac{9a^2b^2\sec(c+dx)}{d}-\frac{9a^2b^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{3a^2b^2\csc^2(c+dx)\sec(c+dx)}{d}-\frac{4a^3b\csc^3(c+dx)}{3d}-\frac{4a^3b\csc(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (-3*a^4*ArcTanh[Cos[c + d*x]])/(8*d) - (9*a^2*b^2*ArcTanh[Cos[c + d*x]])/d
- (b^4*ArcTanh[Cos[c + d*x]])/d + (4*a^3*b*ArcTanh[Sin[c + d*x]])/d + (6*a*
b^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*b*Csc[c + d*x])/d - (6*a*b^3*Csc[c +
d*x])/d - (3*a^4*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (4*a^3*b*Csc[c + d*x]^3
)/(3*d) - (a^4*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d) + (9*a^2*b^2*Sec[c + d*x]
)/d + (b^4*Sec[c + d*x])/d - (3*a^2*b^2*Csc[c + d*x]^2*Sec[c + d*x])/d + (2
*a*b^3*Csc[c + d*x]*Sec[c + d*x]^2)/d + (b^4*Sec[c + d*x]^3)/(3*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx = \int \left(a^4\csc^5(c+dx)+4a^3b\csc^4(c+dx)\sec(c+dx)+6a^2b^2\csc^3(c+
$$

$$= a^4\int \csc^5(c+dx)\,dx+\left(4a^3b\right)\int \csc^4(c+dx)\sec(c+dx)\,dx+\left(6a^2b^
$$

$$= -\frac{a^4\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{1}{4}\left(3a^4\right)\int \csc^3(c+dx)\,dx-\frac{\left(4a^3b\right)\mathrm{Su}}{}
$$

$$= -\frac{3a^4\cot(c+dx)\csc(c+dx)}{8d}-\frac{a^4\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^2b^2\,\mathrm{cs}}{}
$$

$$= -\frac{3a^4\tanh^{-1}(\cos(c+dx))}{8d}-\frac{4a^3b\csc(c+dx)}{d}-\frac{6ab^3\csc(c+dx)}{d}-3
$$

$$= -\frac{3a^4\tanh^{-1}(\cos(c+dx))}{8d}-\frac{9a^2b^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{b^4\tanh^{-1}(}{}
$$

Mathematica [B] time = 6.30, size = 1491, normalized size = 5.44

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (b^2*(36*a^2 + 7*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(6*d*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^4) + ((-7*a^3*b*Cos[(c + d*x)/2] - 6*a*b^3*Cos[(c
+ d*x)/2])*Cos[c + d*x]^4*Csc[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(3*d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) - (3*(a^4 + 8*a^2*b^2)*Cos[c + d*x]^4*Csc
[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^4)/(32*d*(a*Cos[c + d*x] + b*Sin[c + d
*x])^4) - (a^3*b*Cos[c + d*x]^4*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(a + b*
Tan[c + d*x])^4)/(6*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (a^4*Cos[c + d
*x]^4*Csc[(c + d*x)/2]^4*(a + b*Tan[c + d*x])^4)/(64*d*(a*Cos[c + d*x] + b*
Sin[c + d*x])^4) + ((-3*a^4 - 72*a^2*b^2 - 8*b^4)*Cos[c + d*x]^4*Log[Cos[(c
 + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^
4) - (2*(2*a^3*b + 3*a*b^3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]]*(a + b*Tan[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) +
((3*a^4 + 72*a^2*b^2 + 8*b^4)*Cos[c + d*x]^4*Log[Sin[(c + d*x)/2]]*(a + b*T
an[c + d*x])^4)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (2*(2*a^3*b + 3
*a*b^3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[
c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (3*(a^4 + 8*a^2*b^2)
*Cos[c + d*x]^4*Sec[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^4)/(32*d*(a*Cos[c +
 d*x] + b*Sin[c + d*x])^4) + (a^4*Cos[c + d*x]^4*Sec[(c + d*x)/2]^4*(a + b*
Tan[c + d*x])^4)/(64*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((12*a*b^3 +
b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*S
in[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*Sin[(c
 + d*x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((-12*a*b^3 + b^4)*Cos[c + d*x]
^4*(a + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a
*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*Sec[(c + d*x)/2]*(-7*a
^3*b*Sin[(c + d*x)/2] - 6*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(
3*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(-36*a^2*b^2*Sin
[(c + d*x)/2] - 7*b^4*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos
[c + d*x]^4*(36*a^2*b^2*Sin[(c + d*x)/2] + 7*b^4*Sin[(c + d*x)/2])*(a + b*T
an[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x]
+ b*Sin[c + d*x])^4) - (a^3*b*Cos[c + d*x]^4*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

fricas [B] time = 0.69, size = 547, normalized size = 2.00

$$\frac{6\left(3\,a^4+72\,a^2b^2+8\,b^4\right)\cos\left(dx+c\right)^6-10\left(3\,a^4+72\,a^2b^2+8\,b^4\right)\cos\left(dx+c\right)^4+16\,b^4+16\left(18\,a^2b^2+b^4\right)\cos\left(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/48*(6*(3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^6 - 10*(3*a^4 + 72*a^2*b^
2 + 8*b^4)*cos(d*x + c)^4 + 16*b^4 + 16*(18*a^2*b^2 + b^4)*cos(d*x + c)^2 -
 3*((3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^7 - 2*(3*a^4 + 72*a^2*b^2 + 8
*b^4)*cos(d*x + c)^5 + (3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^3)*log(1/2
*cos(d*x + c) + 1/2) + 3*((3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^7 - 2*(
3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^5 + (3*a^4 + 72*a^2*b^2 + 8*b^4)*c
os(d*x + c)^3)*log(-1/2*cos(d*x + c) + 1/2) + 48*((2*a^3*b + 3*a*b^3)*cos(d
*x + c)^7 - 2*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^5 + (2*a^3*b + 3*a*b^3)*cos(
d*x + c)^3)*log(sin(d*x + c) + 1) - 48*((2*a^3*b + 3*a*b^3)*cos(d*x + c)^7
- 2*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^5 + (2*a^3*b + 3*a*b^3)*cos(d*x + c)^3
)*log(-sin(d*x + c) + 1) + 32*(3*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^5 + 3*a*b
^3*cos(d*x + c) - 4*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*co
s(d*x + c)^7 - 2*d*cos(d*x + c)^5 + d*cos(d*x + c)^3)
```

giac [A] time = 8.97, size = 479, normalized size = 1.75

$$3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-32\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+144\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-480\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^4*tan(1/2*d*x + 1/2*c)^4 - 32*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 24*
a^4*tan(1/2*d*x + 1/2*c)^2 + 144*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 480*a^3*b
*tan(1/2*d*x + 1/2*c) - 384*a*b^3*tan(1/2*d*x + 1/2*c) + 384*(2*a^3*b + 3*a
*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 384*(2*a^3*b + 3*a*b^3)*log(abs(
tan(1/2*d*x + 1/2*c) - 1)) + 24*(3*a^4 + 72*a^2*b^2 + 8*b^4)*log(abs(tan(1/
2*d*x + 1/2*c))) + 256*(3*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^2*b^2*tan(1/2*
d*x + 1/2*c)^4 - 3*b^4*tan(1/2*d*x + 1/2*c)^4 + 18*a^2*b^2*tan(1/2*d*x + 1/
2*c)^2 + 3*b^4*tan(1/2*d*x + 1/2*c)^2 - 3*a*b^3*tan(1/2*d*x + 1/2*c) - 9*a^
2*b^2 - 2*b^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3 - (150*a^4*tan(1/2*d*x + 1/2*
c)^4 + 3600*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 400*b^4*tan(1/2*d*x + 1/2*c)^4
 + 480*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 384*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 24
*a^4*tan(1/2*d*x + 1/2*c)^2 + 144*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 32*a^3*b
*tan(1/2*d*x + 1/2*c) + 3*a^4)/tan(1/2*d*x + 1/2*c)^4)/d
```

maple [A] time = 0.50, size = 317, normalized size = 1.16

$$-\frac{a^4\cot(dx+c)\left(\csc^3(dx+c)\right)}{4d}-\frac{3a^4\cot(dx+c)\csc(dx+c)}{8d}+\frac{3a^4\ln(\csc(dx+c)-\cot(dx+c))}{8d}-\frac{4a^3b}{3d\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x)
```

```
[Out] -1/4*a^4*cot(d*x+c)*csc(d*x+c)^3/d-3/8*a^4*cot(d*x+c)*csc(d*x+c)/d+3/8/d*a^
4*ln(csc(d*x+c)-cot(d*x+c))-4/3/d*a^3*b/sin(d*x+c)^3-4/d*a^3*b/sin(d*x+c)+4
/d*a^3*b*ln(sec(d*x+c)+tan(d*x+c))-3/d*a^2*b^2/sin(d*x+c)^2/cos(d*x+c)+9/d*
a^2*b^2/cos(d*x+c)+9/d*a^2*b^2*ln(csc(d*x+c)-cot(d*x+c))+2/d*a*b^3/sin(d*x+
c)/cos(d*x+c)^2-6/d*a*b^3/sin(d*x+c)+6/d*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+1/
3/d*b^4/cos(d*x+c)^3+1/d*b^4/cos(d*x+c)+1/d*b^4*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.63, size = 304, normalized size = 1.11

$$3\,a^4\left(\frac{2\left(3\,\cos(dx+c)^3-5\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-3\,\log\left(\cos(dx+c)+1\right)+3\,\log\left(\cos(dx+c)-1\right)\right)+72\,a^2b^2\left(\frac{2\left(3\,\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*a^4*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(
d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 72*a
^2*b^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(co
s(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 48*a*b^3*(2*(3*sin(d*x + c)^2
- 2)/(sin(d*x + c)^3 - sin(d*x + c)) - 3*log(sin(d*x + c) + 1) + 3*log(sin(
d*x + c) - 1)) + 8*b^4*(2*(3*cos(d*x + c)^2 + 1)/cos(d*x + c)^3 - 3*log(cos
(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 32*a^3*b*(2*(3*sin(d*x + c)^2 +
 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 4.17, size = 857, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^4/sin(c + d*x)^5,x)
```

```
[Out] (a^4*tan(c/2 + (d*x)/2)^4)/(64*d) - (atan(-((6*a*b^3 + 4*a^3*b)*(12*a*b^3 +
 8*a^3*b - 6*tan(c/2 + (d*x)/2)*(6*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((
3*a^4)/4 + 2*b^4 + 18*a^2*b^2))*1i + (6*a*b^3 + 4*a^3*b)*(12*a*b^3 + 8*a^3*
b + 6*tan(c/2 + (d*x)/2)*(6*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((3*a^4)/
4 + 2*b^4 + 18*a^2*b^2))*1i)/(2*tan(c/2 + (d*x)/2)*(144*a^2*b^6 + 192*a^4*b
^4 + 64*a^6*b^2) + (6*a*b^3 + 4*a^3*b)*(12*a*b^3 + 8*a^3*b - 6*tan(c/2 + (d
*x)/2)*(6*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((3*a^4)/4 + 2*b^4 + 18*a^2
*b^2)) - (6*a*b^3 + 4*a^3*b)*(12*a*b^3 + 8*a^3*b + 6*tan(c/2 + (d*x)/2)*(6*
a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((3*a^4)/4 + 2*b^4 + 18*a^2*b^2)) + 2
4*a*b^7 + 6*a^7*b + 232*a^3*b^5 + 153*a^5*b^3))*(a*b^3*12i + a^3*b*8i))/d -
 (tan(c/2 + (d*x)/2)*(2*a^3*b + (a*b*(a^2 + 4*b^2))/2))/d + (log(tan(c/2 +
(d*x)/2))*((3*a^4)/8 + b^4 + 9*a^2*b^2))/d + (tan(c/2 + (d*x)/2)^2*(a^4/8 +
 (3*a^2*b^2)/4))/d - (tan(c/2 + (d*x)/2)^6*((23*a^4)/4 + 64*b^4 + 420*a^2*b
^2) - tan(c/2 + (d*x)/2)^4*((21*a^4)/4 + (128*b^4)/3 + 228*a^2*b^2) - tan(c
/2 + (d*x)/2)^8*(2*a^4 + 64*b^4 + 204*a^2*b^2) + a^4/4 + tan(c/2 + (d*x)/2)
^2*((5*a^4)/4 + 12*a^2*b^2) + tan(c/2 + (d*x)/2)^3*(32*a*b^3 + 32*a^3*b) +
tan(c/2 + (d*x)/2)^9*(32*a*b^3 - 40*a^3*b) - tan(c/2 + (d*x)/2)^5*(160*a*b^
3 + 112*a^3*b) + tan(c/2 + (d*x)/2)^7*(96*a*b^3 + (352*a^3*b)/3) + (8*a^3*b
*tan(c/2 + (d*x)/2))/3)/(d*(16*tan(c/2 + (d*x)/2)^4 - 48*tan(c/2 + (d*x)/2)
^6 + 48*tan(c/2 + (d*x)/2)^8 - 16*tan(c/2 + (d*x)/2)^10)) - (a^3*b*tan(c/2
+ (d*x)/2)^3)/(6*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.49 $\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=194

$$-\frac{a^4\cot^5(c+dx)}{5d}-\frac{a^3b\cot^4(c+dx)}{d}+\frac{2b^2\left(3a^2+b^2\right)\tan(c+dx)}{d}-\frac{2a^2\left(a^2+3b^2\right)\cot^3(c+dx)}{3d}-\frac{2ab\left(2a^2+b^2\right)c}{d}$$

```
[Out] -(a^4+12*a^2*b^2+b^4)*cot(d*x+c)/d-2*a*b*(2*a^2+b^2)*cot(d*x+c)^2/d-2/3*a^2
*(a^2+3*b^2)*cot(d*x+c)^3/d-a^3*b*cot(d*x+c)^4/d-1/5*a^4*cot(d*x+c)^5/d+4*a
*b*(a^2+2*b^2)*ln(tan(d*x+c))/d+2*b^2*(3*a^2+b^2)*tan(d*x+c)/d+2*a*b^3*tan(
d*x+c)^2/d+1/3*b^4*tan(d*x+c)^3/d
```

Rubi [A] time = 0.16, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 948}

$$\frac{2b^2\left(3a^2+b^2\right)\tan(c+dx)}{d}-\frac{2a^2\left(a^2+3b^2\right)\cot^3(c+dx)}{3d}-\frac{2ab\left(2a^2+b^2\right)\cot^2(c+dx)}{d}-\frac{\left(12a^2b^2+a^4+b^4\right)\cot(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(((a^4 + 12*a^2*b^2 + b^4)*Cot[c + d*x])/d) - (2*a*b*(2*a^2 + b^2)*Cot[c +
 d*x]^2)/d - (2*a^2*(a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*d) - (a^3*b*Cot[c + d*
x]^4)/d - (a^4*Cot[c + d*x]^5)/(5*d) + (4*a*b*(a^2 + 2*b^2)*Log[Tan[c + d*x
]])/d + (2*b^2*(3*a^2 + b^2)*Tan[c + d*x])/d + (2*a*b^3*Tan[c + d*x]^2)/d +
 (b^4*Tan[c + d*x]^3)/(3*d)
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx &= \frac{b\,\text{Subst}\left(\int \frac{(a+x)^4\left(b^2+x^2\right)^2}{x^6}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\,\text{Subst}\left(\int \left(2\left(3a^2+b^2\right)+\frac{a^4b^4}{x^6}+\frac{4a^3b^4}{x^5}+\frac{2a^2b^2\left(a^2+3b^2\right)}{x^4}+\frac{4ab^2\left(2a^2+b^2\right)}{x^3}+\right.\right.}{d}\\ &= -\frac{\left(a^4+12a^2b^2+b^4\right)\cot(c+dx)}{d}-\frac{2ab\left(2a^2+b^2\right)\cot^2(c+dx)}{d}-\frac{2a^2}{}\end{aligned}$$

Mathematica [A] time = 4.01, size = 233, normalized size = 1.20

$$-\frac{(a+b\tan(c+dx))^4\left(3a^4\cot^5(c+dx)+15a^3b\cot^4(c+dx)-5b^2\left(18a^2+5b^2\right)\sin(c+dx)\cos^3(c+dx)+2a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/15*((15*a^3*b*Cot[c + d*x]^4 + 3*a^4*Cot[c + d*x]^5 + 2*a*Cos[c + d*x]^2
*(-15*b^3 + 15*b*(a^2 + b^2)*Cot[c + d*x]^2 + a*(2*a^2 + 15*b^2)*Cot[c + d*
x]^3) + Cos[c + d*x]^4*((8*a^4 + 150*a^2*b^2 + 15*b^4)*Cot[c + d*x] + 60*a*
b*(a^2 + 2*b^2)*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]])) - 5*b^2*(18*a^2 +
5*b^2)*Cos[c + d*x]^3*Sin[c + d*x] - (5*b^4*Sin[2*(c + d*x)])/2)*(a + b*Tan
[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

fricas [B] time = 0.48, size = 386, normalized size = 1.99

$$-\frac{8\left(a^4+30\,a^2b^2+5\,b^4\right)\cos\left(dx+c\right)^8-20\left(a^4+30\,a^2b^2+5\,b^4\right)\cos\left(dx+c\right)^6+15\left(a^4+30\,a^2b^2+5\,b^4\right)\cos\left(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^4 + 30*a^2*b^2 + 5*b^4)*cos(d*x + c)^8 - 20*(a^4 + 30*a^2*b^2 +
 5*b^4)*cos(d*x + c)^6 + 15*(a^4 + 30*a^2*b^2 + 5*b^4)*cos(d*x + c)^4 - 5*b
^4 - 10*(9*a^2*b^2 + b^4)*cos(d*x + c)^2 + 30*((a^3*b + 2*a*b^3)*cos(d*x +
c)^7 - 2*(a^3*b + 2*a*b^3)*cos(d*x + c)^5 + (a^3*b + 2*a*b^3)*cos(d*x + c)^
3)*log(cos(d*x + c)^2)*sin(d*x + c) - 30*((a^3*b + 2*a*b^3)*cos(d*x + c)^7
- 2*(a^3*b + 2*a*b^3)*cos(d*x + c)^5 + (a^3*b + 2*a*b^3)*cos(d*x + c)^3)*lo
g(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 15*(2*(a^3*b + 2*a*b^3)*cos(d*x
 + c)^5 + 2*a*b^3*cos(d*x + c) - 3*(a^3*b + 2*a*b^3)*cos(d*x + c)^3)*sin(d*
x + c))/((d*cos(d*x + c)^7 - 2*d*cos(d*x + c)^5 + d*cos(d*x + c)^3)*sin(d*x
 + c))
```

giac [A] time = 3.64, size = 235, normalized size = 1.21

$$\frac{5\,b^4\tan\left(dx+c\right)^3+30\,ab^3\tan\left(dx+c\right)^2+90\,a^2b^2\tan\left(dx+c\right)+30\,b^4\tan\left(dx+c\right)+60\left(a^3b+2\,ab^3\right)\log\left(\left|\tan\left(d\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/15*(5*b^4*tan(d*x + c)^3 + 30*a*b^3*tan(d*x + c)^2 + 90*a^2*b^2*tan(d*x +
 c) + 30*b^4*tan(d*x + c) + 60*(a^3*b + 2*a*b^3)*log(abs(tan(d*x + c))) - (
137*a^3*b*tan(d*x + c)^5 + 274*a*b^3*tan(d*x + c)^5 + 15*a^4*tan(d*x + c)^4
 + 180*a^2*b^2*tan(d*x + c)^4 + 15*b^4*tan(d*x + c)^4 + 60*a^3*b*tan(d*x +
c)^3 + 30*a*b^3*tan(d*x + c)^3 + 10*a^4*tan(d*x + c)^2 + 30*a^2*b^2*tan(d*x
 + c)^2 + 15*a^3*b*tan(d*x + c) + 3*a^4)/tan(d*x + c)^5)/d
```

maple [A] time = 0.59, size = 301, normalized size = 1.55

$$-\frac{8a^4\cot\left(dx+c\right)}{15d}-\frac{a^4\cot\left(dx+c\right)\left(\csc^4\left(dx+c\right)\right)}{5d}-\frac{4a^4\cot\left(dx+c\right)\left(\csc^2\left(dx+c\right)\right)}{15d}-\frac{a^3b}{d\sin\left(dx+c\right)^4}-\frac{2a^3b}{d\sin\left(dx+c\right)^{\cdot}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x)
```

```
[Out] -8/15*a^4*cot(d*x+c)/d-1/5/d*a^4*cot(d*x+c)*csc(d*x+c)^4-4/15/d*a^4*cot(d*x
+c)*csc(d*x+c)^2-1/d*a^3*b/sin(d*x+c)^4-2/d*a^3*b/sin(d*x+c)^2+4*a^3*b*ln(t
an(d*x+c))/d-2/d*a^2*b^2/sin(d*x+c)^3/cos(d*x+c)+8/d*a^2*b^2/sin(d*x+c)/cos
(d*x+c)-16/d*a^2*b^2*cot(d*x+c)+2/d*a*b^3/sin(d*x+c)^2/cos(d*x+c)^2-4/d*a*b
```

```
^3/sin(d*x+c)^2+8/d*a*b^3*ln(tan(d*x+c))+1/3/d*b^4/sin(d*x+c)/cos(d*x+c)^3+
4/3/d*b^4/sin(d*x+c)/cos(d*x+c)-8/3/d*b^4*cot(d*x+c)
```

maxima [A] time = 0.63, size = 171, normalized size = 0.88

$$\frac{5\,b^4\tan\left(dx+c\right)^3+30\,ab^3\tan\left(dx+c\right)^2+60\left(a^3b+2\,ab^3\right)\log\left(\tan\left(dx+c\right)\right)+30\left(3\,a^2b^2+b^4\right)\tan\left(dx+c\right)-}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/15*(5*b^4*tan(d*x + c)^3 + 30*a*b^3*tan(d*x + c)^2 + 60*(a^3*b + 2*a*b^3)
*log(tan(d*x + c)) + 30*(3*a^2*b^2 + b^4)*tan(d*x + c) - (15*a^3*b*tan(d*x
+ c) + 15*(a^4 + 12*a^2*b^2 + b^4)*tan(d*x + c)^4 + 3*a^4 + 30*(2*a^3*b + a
*b^3)*tan(d*x + c)^3 + 10*(a^4 + 3*a^2*b^2)*tan(d*x + c)^2)/tan(d*x + c)^5)
/d
```

mupad [B] time = 3.83, size = 181, normalized size = 0.93

$$\frac{\ln\left(\tan\left(c+d\,x\right)\right)\,\left(4\,a^3\,b+8\,a\,b^3\right)}{d}-\frac{\cot\left(c+d\,x\right)^5\,\left(\tan\left(c+d\,x\right)^2\,\left(\frac{2\,a^4}{3}+2\,a^2\,b^2\right)+\tan\left(c+d\,x\right)^3\,\left(4\,a^3\,b+2\,a\,b\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^4/sin(c + d*x)^6,x)
```

```
[Out] (log(tan(c + d*x))*(8*a*b^3 + 4*a^3*b))/d - (cot(c + d*x)^5*(tan(c + d*x)^2
*((2*a^4)/3 + 2*a^2*b^2) + tan(c + d*x)^3*(2*a*b^3 + 4*a^3*b) + a^4/5 + tan
(c + d*x)^4*(a^4 + b^4 + 12*a^2*b^2) + a^3*b*tan(c + d*x)))/d + (b^4*tan(c
+ d*x)^3)/(3*d) + (tan(c + d*x)*(2*b^4 + 6*a^2*b^2))/d + (2*a*b^3*tan(c + d
*x)^2)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.50 $\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal. Leaf size=402

$$-\frac{5a^4\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^4\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a^4\cot(c+dx)\csc^3(c+dx)}{24d}-\frac{5a^4\cot(c+dx)\csc(c+dx)}{16d}$$

```
[Out] -5/16*a^4*arctanh(cos(d*x+c))/d-45/4*a^2*b^2*arctanh(cos(d*x+c))/d-5/2*b^4*
arctanh(cos(d*x+c))/d+4*a^3*b*arctanh(sin(d*x+c))/d+10*a*b^3*arctanh(sin(d*
x+c))/d-4*a^3*b*csc(d*x+c)/d-10*a*b^3*csc(d*x+c)/d-5/16*a^4*cot(d*x+c)*csc(
d*x+c)/d-4/3*a^3*b*csc(d*x+c)^3/d-10/3*a*b^3*csc(d*x+c)^3/d-5/24*a^4*cot(d*
x+c)*csc(d*x+c)^3/d-4/5*a^3*b*csc(d*x+c)^5/d-1/6*a^4*cot(d*x+c)*csc(d*x+c)^
5/d+45/4*a^2*b^2*sec(d*x+c)/d+5/2*b^4*sec(d*x+c)/d-15/4*a^2*b^2*csc(d*x+c)^
2*sec(d*x+c)/d-3/2*a^2*b^2*csc(d*x+c)^4*sec(d*x+c)/d+2*a*b^3*csc(d*x+c)^3*s
ec(d*x+c)^2/d+5/6*b^4*sec(d*x+c)^3/d-1/2*b^4*csc(d*x+c)^2*sec(d*x+c)^3/d
```

Rubi [A] time = 0.31, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3517, 3768, 3770, 2621, 302, 207, 2622, 288, 321}

$$\frac{45a^2b^2\sec(c+dx)}{4d}-\frac{45a^2b^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{3a^2b^2\csc^4(c+dx)\sec(c+dx)}{2d}-\frac{15a^2b^2\csc^2(c+dx)\sec(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^7*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (-5*a^4*ArcTanh[Cos[c + d*x]])/(16*d) - (45*a^2*b^2*ArcTanh[Cos[c + d*x]])/
(4*d) - (5*b^4*ArcTanh[Cos[c + d*x]])/(2*d) + (4*a^3*b*ArcTanh[Sin[c + d*x]
])/d + (10*a*b^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*b*Csc[c + d*x])/d - (10*
a*b^3*Csc[c + d*x])/d - (5*a^4*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (4*a^3*b
*Csc[c + d*x]^3)/(3*d) - (10*a*b^3*Csc[c + d*x]^3)/(3*d) - (5*a^4*Cot[c + d
*x]*Csc[c + d*x]^3)/(24*d) - (4*a^3*b*Csc[c + d*x]^5)/(5*d) - (a^4*Cot[c +
d*x]*Csc[c + d*x]^5)/(6*d) + (45*a^2*b^2*Sec[c + d*x])/(4*d) + (5*b^4*Sec[c
 + d*x])/(2*d) - (15*a^2*b^2*Csc[c + d*x]^2*Sec[c + d*x])/(4*d) - (3*a^2*b^
2*Csc[c + d*x]^4*Sec[c + d*x])/(2*d) + (2*a*b^3*Csc[c + d*x]^3*Sec[c + d*x]
^2)/d + (5*b^4*Sec[c + d*x]^3)/(6*d) - (b^4*Csc[c + d*x]^2*Sec[c + d*x]^3)/
(2*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx = \int \left(a^4\csc^7(c+dx)+4a^3b\csc^6(c+dx)\sec(c+dx)+6a^2b^2\csc^5(c+dx)\right.$$

$$= a^4\int \csc^7(c+dx)\,dx + \left(4a^3b\right)\int \csc^6(c+dx)\sec(c+dx)\,dx + \left(6a^2b^2\right)$$

$$= -\frac{a^4\cot(c+dx)\csc^5(c+dx)}{6d} + \frac{1}{6}\left(5a^4\right)\int \csc^5(c+dx)\,dx - \frac{\left(4a^3b\right)\text{Subs}}{}$$

$$= -\frac{5a^4\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{a^4\cot(c+dx)\csc^5(c+dx)}{6d} - \frac{3a^2b^2\csc}{}$$

$$= -\frac{4a^3b\csc(c+dx)}{d} - \frac{5a^4\cot(c+dx)\csc(c+dx)}{16d} - \frac{4a^3b\csc^3(c+dx)}{3d} - $$

$$= -\frac{5a^4\tanh^{-1}(\cos(c+dx))}{16d} + \frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d} - \frac{4a^3b\csc(c+dx}{d}$$

$$= -\frac{5a^4\tanh^{-1}(\cos(c+dx))}{16d} - \frac{45a^2b^2\tanh^{-1}(\cos(c+dx))}{4d} - \frac{5b^4\tanh^{-1}(c}{2}$$

Mathematica [A] time = 6.27, size = 660, normalized size = 1.64

$$-\frac{2\left(2a^3b+5ab^3\right)\cos^4(c+dx)(a+b\tan(c+dx))^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d(a\cos(c+dx)+b\sin(c+dx))^4}+\frac{2\left(2a^3b+5ab^3\right)\cos^4(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^7*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (-5*(a^4 + 36*a^2*b^2 + 8*b^4)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2]]*(a + b*
Tan[c + d*x])^4)/(16*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (2*(2*a^3*b +
 5*a*b^3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Ta
n[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (5*(a^4 + 36*a^2*b
^2 + 8*b^4)*Cos[c + d*x]^4*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(1
6*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (2*(2*a^3*b + 5*a*b^3)*Cos[c + d
*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(d*(
a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cot[c + d*x]*Csc[c + d*x]^5*(-2545*a
^4 + 540*a^2*b^2 + 5240*b^4 - 2760*a^4*Cos[2*(c + d*x)] - 7200*a^2*b^2*Cos[
2*(c + d*x)] - 6720*b^4*Cos[2*(c + d*x)] + 60*a^4*Cos[4*(c + d*x)] + 2160*a
^2*b^2*Cos[4*(c + d*x)] + 480*b^4*Cos[4*(c + d*x)] + 200*a^4*Cos[6*(c + d*x
)] + 7200*a^2*b^2*Cos[6*(c + d*x)] + 1600*b^4*Cos[6*(c + d*x)] - 75*a^4*Cos
[8*(c + d*x)] - 2700*a^2*b^2*Cos[8*(c + d*x)] - 600*b^4*Cos[8*(c + d*x)] -
15744*a^3*b*Sin[2*(c + d*x)] - 8640*a*b^3*Sin[2*(c + d*x)] - 1152*a^3*b*Sin
[4*(c + d*x)] - 2880*a*b^3*Sin[4*(c + d*x)] + 3200*a^3*b*Sin[6*(c + d*x)] +
 8000*a*b^3*Sin[6*(c + d*x)] - 960*a^3*b*Sin[8*(c + d*x)] - 2400*a*b^3*Sin[
8*(c + d*x)])*(a + b*Tan[c + d*x])^4)/(30720*d*(a*Cos[c + d*x] + b*Sin[c +
d*x])^4)
```

fricas [A] time = 0.71, size = 697, normalized size = 1.73

$$\frac{150\left(a^4+36\,a^2b^2+8\,b^4\right)\cos\left(dx+c\right)^8-400\left(a^4+36\,a^2b^2+8\,b^4\right)\cos\left(dx+c\right)^6+330\left(a^4+36\,a^2b^2+8\,b^4\right)\cos\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/480*(150*(a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^8 - 400*(a^4 + 36*a^2*b^
2 + 8*b^4)*cos(d*x + c)^6 + 330*(a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 -
 160*b^4 - 480*(6*a^2*b^2 + b^4)*cos(d*x + c)^2 - 75*((a^4 + 36*a^2*b^2 + 8
*b^4)*cos(d*x + c)^9 - 3*(a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^7 + 3*(a^4
 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^5 - (a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x
+ c)^3)*log(1/2*cos(d*x + c) + 1/2) + 75*((a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*
x + c)^9 - 3*(a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^7 + 3*(a^4 + 36*a^2*b^
2 + 8*b^4)*cos(d*x + c)^5 - (a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^3)*log(
-1/2*cos(d*x + c) + 1/2) + 480*((2*a^3*b + 5*a*b^3)*cos(d*x + c)^9 - 3*(2*a
^3*b + 5*a*b^3)*cos(d*x + c)^7 + 3*(2*a^3*b + 5*a*b^3)*cos(d*x + c)^5 - (2*
a^3*b + 5*a*b^3)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 480*((2*a^3*b + 5*
a*b^3)*cos(d*x + c)^9 - 3*(2*a^3*b + 5*a*b^3)*cos(d*x + c)^7 + 3*(2*a^3*b +
 5*a*b^3)*cos(d*x + c)^5 - (2*a^3*b + 5*a*b^3)*cos(d*x + c)^3)*log(-sin(d*x
 + c) + 1) + 64*(15*(2*a^3*b + 5*a*b^3)*cos(d*x + c)^7 - 35*(2*a^3*b + 5*a*
b^3)*cos(d*x + c)^5 - 15*a*b^3*cos(d*x + c) + 23*(2*a^3*b + 5*a*b^3)*cos(d*
x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^9 - 3*d*cos(d*x + c)^7 + 3*d*cos(d*
x + c)^5 - d*cos(d*x + c)^3)
```

giac [A] time = 3.19, size = 647, normalized size = 1.61

$$\frac{5\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-48\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+180\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-560\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^4*tan(1/2*d*x + 1/2*c)^6 - 48*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 45
*a^4*tan(1/2*d*x + 1/2*c)^4 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 560*a^3*
b*tan(1/2*d*x + 1/2*c)^3 - 320*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 225*a^4*tan(1
/2*d*x + 1/2*c)^2 + 2880*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 240*b^4*tan(1/2*d
*x + 1/2*c)^2 - 5280*a^3*b*tan(1/2*d*x + 1/2*c) - 8640*a*b^3*tan(1/2*d*x +
1/2*c) + 3840*(2*a^3*b + 5*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3840
*(2*a^3*b + 5*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 600*(a^4 + 36*a^2
*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c))) + 1280*(6*a*b^3*tan(1/2*d*x +
1/2*c)^5 - 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 9*b^4*tan(1/2*d*x + 1/2*c)^4
 + 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^4*tan(1/2*d*x + 1/2*c)^2 - 6*a*
b^3*tan(1/2*d*x + 1/2*c) - 18*a^2*b^2 - 7*b^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)
^3 - (1470*a^4*tan(1/2*d*x + 1/2*c)^6 + 52920*a^2*b^2*tan(1/2*d*x + 1/2*c)^
6 + 11760*b^4*tan(1/2*d*x + 1/2*c)^6 + 5280*a^3*b*tan(1/2*d*x + 1/2*c)^5 +
8640*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 225*a^4*tan(1/2*d*x + 1/2*c)^4 + 2880*a
^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 240*b^4*tan(1/2*d*x + 1/2*c)^4 + 560*a^3*b*
tan(1/2*d*x + 1/2*c)^3 + 320*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 45*a^4*tan(1/2*
d*x + 1/2*c)^2 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 48*a^3*b*tan(1/2*d*x
+ 1/2*c) + 5*a^4)/tan(1/2*d*x + 1/2*c)^6)/d
```

maple [A] time = 0.54, size = 442, normalized size = 1.10

$$-\frac{a^4\cot(dx+c)\left(\csc^5(dx+c)\right)}{6d}-\frac{5a^4\cot(dx+c)\left(\csc^3(dx+c)\right)}{24d}-\frac{5a^4\cot(dx+c)\csc(dx+c)}{16d}+\frac{5a^4\ln(\csc(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x)
```

```
[Out] -1/6*a^4*cot(d*x+c)*csc(d*x+c)^5/d-5/24*a^4*cot(d*x+c)*csc(d*x+c)^3/d-5/16*
a^4*cot(d*x+c)*csc(d*x+c)/d+5/16/d*a^4*ln(csc(d*x+c)-cot(d*x+c))-4/5/d*a^3*
b/sin(d*x+c)^5-4/3/d*a^3*b/sin(d*x+c)^3-4/d*a^3*b/sin(d*x+c)+4/d*a^3*b*ln(s
```

```
ec(d*x+c)+tan(d*x+c))-3/2/d*a^2*b^2/sin(d*x+c)^4/cos(d*x+c)-15/4/d*a^2*b^2/
sin(d*x+c)^2/cos(d*x+c)+45/4/d*a^2*b^2/cos(d*x+c)+45/4/d*a^2*b^2*ln(csc(d*x
+c)-cot(d*x+c))-4/3/d*a*b^3/sin(d*x+c)^3/cos(d*x+c)^2+10/3/d*a*b^3/sin(d*x+
c)/cos(d*x+c)^2-10/d*a*b^3/sin(d*x+c)+10/d*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+
1/3/d*b^4/sin(d*x+c)^2/cos(d*x+c)^3-5/6/d*b^4/sin(d*x+c)^2/cos(d*x+c)+5/2/d
*b^4/cos(d*x+c)+5/2/d*b^4*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.40, size = 387, normalized size = 0.96

$$5\,a^4\left(\frac{2\left(15\,\cos(dx+c)^5-40\,\cos(dx+c)^3+33\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+40\,b^4\left(\frac{2\left(15\,\mathrm{co}\right.}{\mathrm{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/480*(5*a^4*(2*(15*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 33*cos(d*x + c))/(
cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 15*log(cos(d*x
+ c) + 1) + 15*log(cos(d*x + c) - 1)) + 40*b^4*(2*(15*cos(d*x + c)^4 - 10*c
os(d*x + c)^2 - 2)/(cos(d*x + c)^5 - cos(d*x + c)^3) - 15*log(cos(d*x + c)
+ 1) + 15*log(cos(d*x + c) - 1)) + 180*a^2*b^2*(2*(15*cos(d*x + c)^4 - 25*c
os(d*x + c)^2 + 8)/(cos(d*x + c)^5 - 2*cos(d*x + c)^3 + cos(d*x + c)) - 15*
log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) - 160*a*b^3*(2*(15*sin(d*
x + c)^4 - 10*sin(d*x + c)^2 - 2)/(sin(d*x + c)^5 - sin(d*x + c)^3) - 15*lo
g(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 64*a^3*b*(2*(15*sin(d*x +
 c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 15*log(sin(d*x + c) + 1) + 1
5*log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 4.03, size = 990, normalized size = 2.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^4/sin(c + d*x)^7,x)
```

```
[Out] (a^4*tan(c/2 + (d*x)/2)^6)/(384*d) - (atan(-((10*a*b^3 + 4*a^3*b)*(20*a*b^3
 + 8*a^3*b - 6*tan(c/2 + (d*x)/2)*(10*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)
*((5*a^4)/8 + 5*b^4 + (45*a^2*b^2)/2))*1i + (10*a*b^3 + 4*a^3*b)*(20*a*b^3
+ 8*a^3*b + 6*tan(c/2 + (d*x)/2)*(10*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*
((5*a^4)/8 + 5*b^4 + (45*a^2*b^2)/2))*1i)/(2*tan(c/2 + (d*x)/2)*(400*a^2*b^
6 + 320*a^4*b^4 + 64*a^6*b^2) + (10*a*b^3 + 4*a^3*b)*(20*a*b^3 + 8*a^3*b -
6*tan(c/2 + (d*x)/2)*(10*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((5*a^4)/8 +
 5*b^4 + (45*a^2*b^2)/2)) - (10*a*b^3 + 4*a^3*b)*(20*a*b^3 + 8*a^3*b + 6*ta
n(c/2 + (d*x)/2)*(10*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((5*a^4)/8 + 5*b
^4 + (45*a^2*b^2)/2)) + 100*a*b^7 + 5*a^7*b + 490*a^3*b^5 + (385*a^5*b^3)/2
))*(a*b^3*20i + a^3*b*8i))/d + (tan(c/2 + (d*x)/2)^4*((a^2*(a^2 + 12*b^2))/
128 + a^4/64))/d + (tan(c/2 + (d*x)/2)^2*((a^2*(a^2 + 12*b^2))/16 + (7*a^4)
/128 + b^4/8 + (3*a^2*b^2)/4))/d - (tan(c/2 + (d*x)/2)*((9*a*b^3)/2 + (11*a
^3*b)/4))/d - (tan(c/2 + (d*x)/2)^4*((7*a^4)/2 + 8*b^4 + 78*a^2*b^2) - tan(
c/2 + (d*x)/2)^10*((15*a^4)/2 + 392*b^4 + 864*a^2*b^2) - tan(c/2 + (d*x)/2)
^6*((109*a^4)/6 + (968*b^4)/3 + 1038*a^2*b^2) + tan(c/2 + (d*x)/2)^8*(21*a^
4 + 536*b^4 + 1818*a^2*b^2) + tan(c/2 + (d*x)/2)^2*(a^4 + 6*a^2*b^2) + a^4/
6 - tan(c/2 + (d*x)/2)^11*(32*a*b^3 + 176*a^3*b) + tan(c/2 + (d*x)/2)^3*((3
2*a*b^3)/3 + (208*a^3*b)/15) + tan(c/2 + (d*x)/2)^5*(256*a*b^3 + (624*a^3*b
)/5) - tan(c/2 + (d*x)/2)^7*(1088*a*b^3 + (2368*a^3*b)/5) + tan(c/2 + (d*x)
/2)^9*((2560*a*b^3)/3 + (1528*a^3*b)/3) + (8*a^3*b*tan(c/2 + (d*x)/2))/5)/(
d*(64*tan(c/2 + (d*x)/2)^6 - 192*tan(c/2 + (d*x)/2)^8 + 192*tan(c/2 + (d*x)
/2)^10 - 64*tan(c/2 + (d*x)/2)^12)) - (tan(c/2 + (d*x)/2)^3*((a*b^3)/6 + (7
*a^3*b)/24))/d + (log(tan(c/2 + (d*x)/2))*((5*a^4)/16 + (5*b^4)/2 + (45*a^2
*b^2)/4))/d - (a^3*b*tan(c/2 + (d*x)/2)^5)/(40*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**7*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.51 $\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=274

$$\frac{b\sin^5(c+dx)}{5d\left(a^2+b^2\right)}+\frac{a^2b\sin^3(c+dx)}{3d\left(a^2+b^2\right)^2}-\frac{a\cos^5(c+dx)}{5d\left(a^2+b^2\right)}+\frac{2a\cos^3(c+dx)}{3d\left(a^2+b^2\right)}-\frac{ab^2\cos^3(c+dx)}{3d\left(a^2+b^2\right)^2}-\frac{a\cos(c+dx)}{d\left(a^2+b^2\right)}+\frac{ab^2\cos(c+}{d\left(a^2+b^2\right.}$$

```
[Out] a^5*b*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)/
d+a^3*b^2*cos(d*x+c)/(a^2+b^2)^3/d+a*b^2*cos(d*x+c)/(a^2+b^2)^2/d-a*cos(d*x
+c)/(a^2+b^2)/d-1/3*a*b^2*cos(d*x+c)^3/(a^2+b^2)^2/d+2/3*a*cos(d*x+c)^3/(a^
2+b^2)/d-1/5*a*cos(d*x+c)^5/(a^2+b^2)/d+a^4*b*sin(d*x+c)/(a^2+b^2)^3/d+1/3*
a^2*b*sin(d*x+c)^3/(a^2+b^2)^2/d+1/5*b*sin(d*x+c)^5/(a^2+b^2)/d
```

Rubi [A] time = 0.35, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3518, 3109, 2564, 30, 2633, 3099, 3074, 206, 2638}

$$\frac{a^2b\sin^3(c+dx)}{3d\left(a^2+b^2\right)^2}+\frac{b\sin^5(c+dx)}{5d\left(a^2+b^2\right)}+\frac{a^4b\sin(c+dx)}{d\left(a^2+b^2\right)^3}-\frac{a\cos^5(c+dx)}{5d\left(a^2+b^2\right)}+\frac{2a\cos^3(c+dx)}{3d\left(a^2+b^2\right)}-\frac{ab^2\cos^3(c+dx)}{3d\left(a^2+b^2\right)^2}+\frac{a^3b^2\cos(}{d\left(a^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a^5*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
b^2)^(7/2)*d) + (a^3*b^2*Cos[c + d*x])/((a^2 + b^2)^3*d) + (a*b^2*Cos[c + d
*x])/((a^2 + b^2)^2*d) - (a*Cos[c + d*x])/((a^2 + b^2)*d) - (a*b^2*Cos[c +
d*x]^3)/(3*(a^2 + b^2)^2*d) + (2*a*Cos[c + d*x]^3)/(3*(a^2 + b^2)*d) - (a*C
os[c + d*x]^5)/(5*(a^2 + b^2)*d) + (a^4*b*Sin[c + d*x])/((a^2 + b^2)^3*d) +
 (a^2*b*Sin[c + d*x]^3)/(3*(a^2 + b^2)^2*d) + (b*Sin[c + d*x]^5)/(5*(a^2 +
b^2)*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\cos(c+dx)\sin^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

$$= \frac{a\int \sin^5(c+dx)\,dx}{a^2+b^2} + \frac{b\int \cos(c+dx)\sin^4(c+dx)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\sin^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,d}{a^2+b^2}$$

$$= \frac{a^2b\sin^3(c+dx)}{3\left(a^2+b^2\right)^2 d} - \frac{\left(a^3b\right)\int \frac{\sin^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{\left(a^2+b^2\right)^2} - \frac{\left(ab^2\right)\int \sin^3(c+dx)\,dx}{\left(a^2+b^2\right)^2} - \frac{a\,\mathrm{Sub}}{}$$

$$= -\frac{a\cos(c+dx)}{\left(a^2+b^2\right)d} + \frac{2a\cos^3(c+dx)}{3\left(a^2+b^2\right)d} - \frac{a\cos^5(c+dx)}{5\left(a^2+b^2\right)d} + \frac{a^4b\sin(c+dx)}{\left(a^2+b^2\right)^3 d} + \frac{a^2b\sin^3(c+}{3\left(a^2+b^2\right.}$$

$$= \frac{a^3b^2\cos(c+dx)}{\left(a^2+b^2\right)^3 d} + \frac{ab^2\cos(c+dx)}{\left(a^2+b^2\right)^2 d} - \frac{a\cos(c+dx)}{\left(a^2+b^2\right)d} - \frac{ab^2\cos^3(c+dx)}{3\left(a^2+b^2\right)^2 d} + \frac{2a\cos^3(c}{3\left(a^2+\right.}$$

$$= \frac{a^5b\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2} d} + \frac{a^3b^2\cos(c+dx)}{\left(a^2+b^2\right)^3 d} + \frac{ab^2\cos(c+dx)}{\left(a^2+b^2\right)^2 d} - \frac{a\cos(c+}{\left(a^2+b^2\right.}$$

Mathematica [A] time = 3.19, size = 289, normalized size = 1.05

$$\sqrt{a^2+b^2}\left(-3a^5\cos(5(c+dx))+330a^4b\sin(c+dx)-35a^4b\sin(3(c+dx))+3a^4b\sin(5(c+dx))-6a^3b^2\cos(5(c\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-480*a^5*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 +
 b^2]*(-30*a*(5*a^4 - 4*a^2*b^2 - b^4)*Cos[c + d*x] + 5*a*(5*a^4 + 6*a^2*b^
2 + b^4)*Cos[3*(c + d*x)] - 3*a^5*Cos[5*(c + d*x)] - 6*a^3*b^2*Cos[5*(c + d
*x)] - 3*a*b^4*Cos[5*(c + d*x)] + 330*a^4*b*Sin[c + d*x] + 120*a^2*b^3*Sin[
c + d*x] + 30*b^5*Sin[c + d*x] - 35*a^4*b*Sin[3*(c + d*x)] - 50*a^2*b^3*Sin
[3*(c + d*x)] - 15*b^5*Sin[3*(c + d*x)] + 3*a^4*b*Sin[5*(c + d*x)] + 6*a^2*
b^3*Sin[5*(c + d*x)] + 3*b^5*Sin[5*(c + d*x)]))/(240*(a^2 + b^2)^(7/2)*d)
```

fricas [A] time = 0.49, size = 370, normalized size = 1.35

$$15\sqrt{a^2+b^2}\,a^5b\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)-6\left(a^7+3\,a^5b^2+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(15*sqrt(a^2 + b^2)*a^5*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2
- b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a
*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)
^2 + b^2)) - 6*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^5 + 10*(2
*a^7 + 5*a^5*b^2 + 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 30*(a^7 + a^5*b^2)*c
os(d*x + c) + 2*(23*a^6*b + 34*a^4*b^3 + 14*a^2*b^5 + 3*b^7 + 3*(a^6*b + 3*
a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^4 - (11*a^6*b + 28*a^4*b^3 + 23*a^2
*b^5 + 6*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8)*d)
```

giac [A] time = 2.84, size = 464, normalized size = 1.69

$$\frac{15\,a^5b\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{2\left(15\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+15\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+80\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+20\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+9\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/15*(15*a^5*b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/
abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2*(15*a^4*b*tan(1/2*d*x + 1/2*c)^9 +
15*a^3*b^2*tan(1/2*d*x + 1/2*c)^8 + 80*a^4*b*tan(1/2*d*x + 1/2*c)^7 + 20*a^
2*b^3*tan(1/2*d*x + 1/2*c)^7 + 90*a^3*b^2*tan(1/2*d*x + 1/2*c)^6 + 30*a*b^4
*tan(1/2*d*x + 1/2*c)^6 + 178*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 136*a^2*b^3*ta
n(1/2*d*x + 1/2*c)^5 + 48*b^5*tan(1/2*d*x + 1/2*c)^5 - 80*a^5*tan(1/2*d*x +
 1/2*c)^4 - 10*a*b^4*tan(1/2*d*x + 1/2*c)^4 + 80*a^4*b*tan(1/2*d*x + 1/2*c)
^3 + 20*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 40*a^5*tan(1/2*d*x + 1/2*c)^2 + 30
*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 10*a*b^4*tan(1/2*d*x + 1/2*c)^2 + 15*a^4*
```

```
b*tan(1/2*d*x + 1/2*c) - 8*a^5 + 9*a^3*b^2 + 2*a*b^4)/((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*(tan(1/2*d*x + 1/2*c)^2 + 1)^5))/d
```

maple [A] time = 0.40, size = 361, normalized size = 1.32

$$-\frac{64a^5b\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(32a^6+96a^4b^2+96b^4a^2+32b^6\right)\sqrt{a^2+b^2}}-\frac{2\left(-\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^4b-b^2a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{16}{3}ba^4-\frac{4}{3}a^2b^3\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-6b^2a^3-2ab^4\right)\left(\tan^6\left(\frac{d}{}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d*(-64*a^5*b/(32*a^6+96*a^4*b^2+96*a^2*b^4+32*b^6)/(a^2+b^2)^(1/2)*arctan
h(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-2/(a^4+2*a^2*b^2+b^4)/(
a^2+b^2)*(-tan(1/2*d*x+1/2*c)^9*a^4*b-b^2*a^3*tan(1/2*d*x+1/2*c)^8+(-16/3*b
*a^4-4/3*a^2*b^3)*tan(1/2*d*x+1/2*c)^7+(-6*a^3*b^2-2*a*b^4)*tan(1/2*d*x+1/2
*c)^6+(-178/15*b*a^4-136/15*a^2*b^3-16/5*b^5)*tan(1/2*d*x+1/2*c)^5+(16/3*a^
5+2/3*a*b^4)*tan(1/2*d*x+1/2*c)^4+(-16/3*b*a^4-4/3*a^2*b^3)*tan(1/2*d*x+1/2
*c)^3+(-2*b^2*a^3+8/3*a^5-2/3*a*b^4)*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)*a^4*b+8/15*a^5-3/5*b^2*a^3-2/15*a*b^4)/(1+tan(1/2*d*x+1/2*c)^2)^5)
```

maxima [B] time = 0.80, size = 658, normalized size = 2.40

$$\frac{\frac{15a^5b\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\sqrt{a^2+b^2}}-\frac{2\left(8a^5-9a^3b^2-2ab^4-\frac{15a^4b\sin(dx+c)}{\cos(dx+c)+1}-\frac{15a^3b^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{15a^4b\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{10\left(4a^5-3a^3b^2-ab^4\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{20\left(4\right.}{}\right.}{a^6+3a^4b^2+3a^2b^4+b^6+\frac{5\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\sin(dx}{(\cos(dx+c)+1)^4}}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/15*(15*a^5*b*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2)
)/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(8*a^5 - 9*a^3*b^2 - 2*a*b^4 -
15*a^4*b*sin(d*x + c)/(cos(d*x + c) + 1) - 15*a^3*b^2*sin(d*x + c)^8/(cos(d
*x + c) + 1)^8 - 15*a^4*b*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 10*(4*a^5 -
 3*a^3*b^2 - a*b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 20*(4*a^4*b + a^2
*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*(8*a^5 + a*b^4)*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 - 2*(89*a^4*b + 68*a^2*b^3 + 24*b^5)*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 - 30*(3*a^3*b^2 + a*b^4)*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6 - 20*(4*a^4*b + a^2*b^3)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*
sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*s
in(d*x + c)^10/(cos(d*x + c) + 1)^10))/d
```

mupad [B] time = 6.83, size = 683, normalized size = 2.49

$$\frac{\frac{2\left(-8a^5+9a^3b^2+2ab^4\right)}{15\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}+\frac{8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(4a^4b+a^2b^3\right)}{3\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}+\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(3a^3b^2+ab^4\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-4a^5+3a^3b^2+ab^4\right)}{3\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}+\frac{4\tan\left(\frac{c}{2}\right.}{15}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + b*tan(c + d*x)),x)
```

```
[Out] ((2*(2*a*b^4 - 8*a^5 + 9*a^3*b^2))/(15*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
 + (8*tan(c/2 + (d*x)/2)^3*(4*a^4*b + a^2*b^3))/(3*(a^6 + b^6 + 3*a^2*b^4 +
 3*a^4*b^2)) + (4*tan(c/2 + (d*x)/2)^6*(a*b^4 + 3*a^3*b^2))/(a^6 + b^6 + 3*
a^2*b^4 + 3*a^4*b^2) + (4*tan(c/2 + (d*x)/2)^2*(a*b^4 - 4*a^5 + 3*a^3*b^2))
/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (4*tan(c/2 + (d*x)/2)^5*(89*a^4*
b + 24*b^5 + 68*a^2*b^3))/(15*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (4*tan
(c/2 + (d*x)/2)^4*(a*b^4 + 8*a^5))/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
+ (2*a^3*b^2*tan(c/2 + (d*x)/2)^8)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (8
*b*tan(c/2 + (d*x)/2)^7*(4*a^4 + a^2*b^2))/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)) + (2*a^4*b*tan(c/2 + (d*x)/2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)
+ (2*a^4*b*tan(c/2 + (d*x)/2)^9)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))/(d*(5
*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 +
 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) + (2*a^5*b*atanh((2*a
^6*b + 2*b^7 + 6*a^2*b^5 + 6*a^4*b^3 - 2*a*tan(c/2 + (d*x)/2)*(a^6 + b^6 +
3*a^2*b^4 + 3*a^4*b^2))/(2*(a^2 + b^2)^(7/2))))/(d*(a^2 + b^2)^(7/2))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*tan(d*x+c)),x)
```

```
[Out] Timed out
```

3.52 $\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\, dx$

Optimal. Leaf size=158

$$\frac{\cos^4(c+dx)(a\tan(c+dx)+b)}{4d\left(a^2+b^2\right)}-\frac{\cos^2(c+dx)\left(a\left(5a^2+b^2\right)\tan(c+dx)+4b\left(2a^2+b^2\right)\right)}{8d\left(a^2+b^2\right)^2}+\frac{a^4b\log(a\cos(c+d}{d\left(a^2+\right.}$$

```
[Out] 1/8*a*(3*a^4-6*a^2*b^2-b^4)*x/(a^2+b^2)^3+a^4*b*ln(a*cos(d*x+c)+b*sin(d*x+c
))/(a^2+b^2)^3/d+1/4*cos(d*x+c)^4*(b+a*tan(d*x+c))/(a^2+b^2)/d-1/8*cos(d*x+
c)^2*(4*b*(2*a^2+b^2)+a*(5*a^2+b^2)*tan(d*x+c))/(a^2+b^2)^2/d
```

Rubi [A] time = 0.34, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1647, 801, 635, 203, 260}

$$\frac{\cos^4(c+dx)(a\tan(c+dx)+b)}{4d\left(a^2+b^2\right)}-\frac{\cos^2(c+dx)\left(a\left(5a^2+b^2\right)\tan(c+dx)+4b\left(2a^2+b^2\right)\right)}{8d\left(a^2+b^2\right)^2}+\frac{a^4b\log(a\cos(c+d}{d\left(a^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*(3*a^4 - 6*a^2*b^2 - b^4)*x)/(8*(a^2 + b^2)^3) + (a^4*b*Log[a*Cos[c + d*
x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d) + (Cos[c + d*x]^4*(b + a*Tan[c + d*
x]))/(4*(a^2 + b^2)*d) - (Cos[c + d*x]^2*(4*b*(2*a^2 + b^2) + a*(5*a^2 + b^
2)*Tan[c + d*x]))/(8*(a^2 + b^2)^2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
```

```
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^4}{(a+x)\left(b^2+x^2\right)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^4}{a^2+b^2}-\frac{3ab^4x}{a^2+b^2}-4b^2x^2}{(a+x)\left(b^2+x^2\right)^2}\,dx, x, b\tan(c+dx)\right)}{4bd}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d} - \frac{\cos^2(c+dx)\left(4b\left(2a^2+b^2\right)+a\left(5a^2+b^2\right)\tan(c+d\right.}{8\left(a^2+b^2\right)^2d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d} - \frac{\cos^2(c+dx)\left(4b\left(2a^2+b^2\right)+a\left(5a^2+b^2\right)\tan(c+d\right.}{8\left(a^2+b^2\right)^2d}$$

$$= \frac{a^4b\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3d} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d} - \frac{\cos^2(c+dx)\left(4b\left(2a^2+\right.\right.}{8(}$$

$$= \frac{a^4b\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3d} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d} - \frac{\cos^2(c+dx)\left(4b\left(2a^2+\right.\right.}{8(}$$

$$= \frac{a\left(3a^4-6a^2b^2-b^4\right)x}{8\left(a^2+b^2\right)^3} + \frac{a^4b\log(\cos(c+dx))}{\left(a^2+b^2\right)^3d} + \frac{a^4b\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3d} + \frac{\cos^4(c+\ldots}{}$$

Mathematica [A] time = 2.81, size = 249, normalized size = 1.58

$$-\frac{8a^4\left(-2b^2\log(a+b\tan(c+dx))+\left(a\sqrt{-b^2}+b^2\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+\left(b^2-a\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\,\text{t}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/16*(2*a*b*(5*a^4 + 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]] + 8*b^2*(2*a^4
+ 3*a^2*b^2 + b^4)*Cos[c + d*x]^2 - 4*b^2*(a^2 + b^2)^2*Cos[c + d*x]^4 + 8*
a^4*((b^2 + a*Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*b^2*Log[a +
b*Tan[c + d*x]] + (b^2 - a*Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]]) -
4*a*b*(a^2 + b^2)^2*Cos[c + d*x]^3*Sin[c + d*x] + a*(5*a^4*b + 6*a^2*b^3 +
b^5)*Sin[2*(c + d*x)])/(b*(a^2 + b^2)^3*d)
```

fricas [A] time = 0.46, size = 216, normalized size = 1.37

$$\frac{4\,a^4b\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)+2\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4+\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*a^4*b*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)
^2 + b^2) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^4 + (3*a^5 - 6*a^3*b^2
 - a*b^4)*d*x - 4*(2*a^4*b + 3*a^2*b^3 + b^5)*cos(d*x + c)^2 + (2*(a^5 + 2*
a^3*b^2 + a*b^4)*cos(d*x + c)^3 - (5*a^5 + 6*a^3*b^2 + a*b^4)*cos(d*x + c))
*sin(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

giac [B] time = 3.87, size = 334, normalized size = 2.11

$$\frac{\frac{8\,a^4b^2\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{4\,a^4b\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(3\,a^5-6\,a^3b^2-ab^4\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{6\,a^4b\tan(dx+c)^4-5\,a^5\tan(dx+c)^3-6\,a^3b^2\tan(dx+c)^3-ab\cdots}{\cdots}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(8*a^4*b^2*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5
+ b^7) - 4*a^4*b*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
) + (3*a^5 - 6*a^3*b^2 - a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6) + (6*a^4*b*tan(d*x + c)^4 - 5*a^5*tan(d*x + c)^3 - 6*a^3*b^2*tan(d*x + c
)^3 - a*b^4*tan(d*x + c)^3 + 4*a^4*b*tan(d*x + c)^2 - 12*a^2*b^3*tan(d*x +
c)^2 - 4*b^5*tan(d*x + c)^2 - 3*a^5*tan(d*x + c) - 2*a^3*b^2*tan(d*x + c) +
 a*b^4*tan(d*x + c) - 8*a^2*b^3 - 2*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)*(tan(d*x + c)^2 + 1)^2))/d
```

maple [B] time = 0.36, size = 565, normalized size = 3.58

$$\frac{b\,a^4\ln\left(a+b\tan\left(dx+c\right)\right)}{d\left(a^2+b^2\right)^3}-\frac{5\left(\tan^3\left(dx+c\right)\right)a^5}{8d\left(a^2+b^2\right)^3\left(1+\tan^2\left(dx+c\right)\right)^2}-\frac{3\left(\tan^3\left(dx+c\right)\right)b^2a^3}{4d\left(a^2+b^2\right)^3\left(1+\tan^2\left(dx+c\right)\right)^2}-\frac{\left(\tan\cdots\right.}{8d\left(a^2+b^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d*b*a^4/(a^2+b^2)^3*ln(a+b*tan(d*x+c))-5/8/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)
^2*tan(d*x+c)^3*a^5-3/4/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*b^2*a
^3-1/8/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*a*b^4-1/d/(a^2+b^2)^3/
(1+tan(d*x+c)^2)^2*tan(d*x+c)^2*b*a^4-3/2/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*
tan(d*x+c)^2*a^2*b^3-1/2/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*tan(d*x+c)^2*b^5-
3/8/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a^5-1/4/d/(a^2+b^2)^3/(1+ta
n(d*x+c)^2)^2*tan(d*x+c)*b^2*a^3+1/8/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*tan(d
*x+c)*a*b^4-3/4/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*b*a^4-1/d/(a^2+b^2)^3/(1+t
an(d*x+c)^2)^2*a^2*b^3-1/4/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*b^5+3/8/d/(a^2+
b^2)^3*arctan(tan(d*x+c))*a^5-3/4/d/(a^2+b^2)^3*arctan(tan(d*x+c))*a^3*b^2-
1/8/d/(a^2+b^2)^3*arctan(tan(d*x+c))*a*b^4-1/2/d/(a^2+b^2)^3*ln(1+tan(d*x+c
)^2)*a^4*b
```

maxima [A] time = 0.63, size = 280, normalized size = 1.77

$$\frac{\frac{8\,a^4b\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{4\,a^4b\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(3\,a^5-6\,a^3b^2-ab^4\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{\left(5\,a^3+ab^2\right)\tan(dx+c)^3+6\,a^2b+2\,b^3+4\left(2\,a^2b+b^3\right)\tan(dx+c)\cdots}{\left(a^4+2\,a^2b^2+b^4\right)\tan(dx+c)^4+a^4+2\,a^2b^2+b^4+2\left(a^4+2\,a\cdots\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*(8*a^4*b*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
4*a^4*b*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (3*a^
5 - 6*a^3*b^2 - a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((5*
a^3 + a*b^2)*tan(d*x + c)^3 + 6*a^2*b + 2*b^3 + 4*(2*a^2*b + b^3)*tan(d*x +
 c)^2 + (3*a^3 - a*b^2)*tan(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*tan(d*x + c)
^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*tan(d*x + c)^2))/d
```

mupad [B] time = 4.27, size = 313, normalized size = 1.98

$$\frac{a^4\,b\,\ln\left(a+b\tan\left(c+d\,x\right)\right)}{d\,\left(a^2+b^2\right)^3}-\frac{\ln\left(\tan\left(c+d\,x\right)-\mathrm{i}\right)\,\left(a\,b-a^2\,3\mathrm{i}\right)}{16\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)}-\frac{\ln\left(\tan\left(c+d\,x\right)+1\mathrm{i}\right)\,\left(-3\,a^2+a\,b\,1\mathrm{i}\right)}{16\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}-\frac{3\,a^2\,b-}{4\,\left(a^4+2\,a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b*tan(c + d*x)),x)
```

```
[Out] (a^4*b*log(a + b*tan(c + d*x)))/(d*(a^2 + b^2)^3) - (log(tan(c + d*x) - 1i)
*(a*b - a^2*3i))/(16*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) - (log(tan(c +
d*x) + 1i)*(a*b*1i - 3*a^2))/(16*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) - (
(3*a^2*b + b^3)/(4*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c + d*x)^3*(a*b^2 + 5*a^
3))/(8*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c + d*x)^2*(2*a^2*b + b^3))/(2*(a^4
+ b^4 + 2*a^2*b^2)) + (a*tan(c + d*x)*(3*a^2 - b^2))/(8*(a^4 + b^4 + 2*a^2*
b^2)))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*tan(d*x+c)),x)
```

```
[Out] Timed out
```

3.53 $\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=168

$$\frac{b\sin^3(c+dx)}{3d\left(a^2+b^2\right)}+\frac{a^2b\sin(c+dx)}{d\left(a^2+b^2\right)^2}+\frac{a\cos^3(c+dx)}{3d\left(a^2+b^2\right)}-\frac{a\cos(c+dx)}{d\left(a^2+b^2\right)}+\frac{ab^2\cos(c+dx)}{d\left(a^2+b^2\right)^2}+\frac{a^3b\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{5/2}}$$

```
[Out] a^3*b*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)/
d+a*b^2*cos(d*x+c)/(a^2+b^2)^2/d-a*cos(d*x+c)/(a^2+b^2)/d+1/3*a*cos(d*x+c)^
3/(a^2+b^2)/d+a^2*b*sin(d*x+c)/(a^2+b^2)^2/d+1/3*b*sin(d*x+c)^3/(a^2+b^2)/d
```

Rubi [A] time = 0.22, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3518, 3109, 2564, 30, 2633, 3099, 3074, 206, 2638}

$$\frac{b\sin^3(c+dx)}{3d\left(a^2+b^2\right)}+\frac{a^2b\sin(c+dx)}{d\left(a^2+b^2\right)^2}+\frac{a\cos^3(c+dx)}{3d\left(a^2+b^2\right)}-\frac{a\cos(c+dx)}{d\left(a^2+b^2\right)}+\frac{ab^2\cos(c+dx)}{d\left(a^2+b^2\right)^2}+\frac{a^3b\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a^3*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
b^2)^(5/2)*d) + (a*b^2*Cos[c + d*x])/((a^2 + b^2)^2*d) - (a*Cos[c + d*x])/(
(a^2 + b^2)*d) + (a*Cos[c + d*x]^3)/(3*(a^2 + b^2)*d) + (a^2*b*Sin[c + d*x]
)/((a^2 + b^2)^2*d) + (b*Sin[c + d*x]^3)/(3*(a^2 + b^2)*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx &= \int \frac{\cos(c+dx)\sin^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx \\
&= \frac{a\int \sin^3(c+dx)\,dx}{a^2+b^2} + \frac{b\int \cos(c+dx)\sin^2(c+dx)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\sin^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2} \\
&= \frac{a^2b\sin(c+dx)}{\left(a^2+b^2\right)^2 d} - \frac{\left(a^3b\right)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{\left(a^2+b^2\right)^2} - \frac{\left(ab^2\right)\int \sin(c+dx)\,dx}{\left(a^2+b^2\right)^2} - \frac{a\,\mathrm{Subst}\left(\int\right.}{} \\
&= \frac{ab^2\cos(c+dx)}{\left(a^2+b^2\right)^2 d} - \frac{a\cos(c+dx)}{\left(a^2+b^2\right)d} + \frac{a\cos^3(c+dx)}{3\left(a^2+b^2\right)d} + \frac{a^2b\sin(c+dx)}{\left(a^2+b^2\right)^2 d} + \frac{b\sin^3(c+dx)}{3\left(a^2+b^2\right)d} + \\
&= \frac{a^3b\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}d} + \frac{ab^2\cos(c+dx)}{\left(a^2+b^2\right)^2 d} - \frac{a\cos(c+dx)}{\left(a^2+b^2\right)d} + \frac{a\cos^3(c+dx)}{3\left(a^2+b^2\right)d} +
\end{aligned}$$

Mathematica [A] time = 0.89, size = 139, normalized size = 0.83

$$\frac{\sqrt{a^2+b^2}\left(\left(3ab^2-9a^3\right)\cos(c+dx)+a\left(a^2+b^2\right)\cos(3(c+dx))-2b\sin(c+dx)\left(\left(a^2+b^2\right)\cos(2(c+dx))-7a^2\right.\right.}{12d\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-24*a^3*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 +
b^2]*((-9*a^3 + 3*a*b^2)*Cos[c + d*x] + a*(a^2 + b^2)*Cos[3*(c + d*x)] - 2*
b*(-7*a^2 - b^2 + (a^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(12*(a^2 + b
^2)^(5/2)*d)
```

fricas [A] time = 0.46, size = 261, normalized size = 1.55

$$\frac{3\,\sqrt{a^2+b^2}\,a^3 b\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)+2\left(a^5+2\,a^3b^2+\cdots\right.}{6\left(a^6+3\,a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + b^2)*a^3*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*s
in(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
 + b^2)) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)^3 - 6*(a^5 + a^3*b^2)*c
os(d*x + c) + 2*(4*a^4*b + 5*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*cos(
d*x + c)^2)*sin(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

giac [A] time = 0.98, size = 241, normalized size = 1.43

$$\frac{\frac{3\,a^3 b\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}+\frac{2\left(3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+10\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^3\,\mathrm{ta}\cdots\right.}{\left(a^4+2\,a^2b^2+b^4\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3*(3*a^3*b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/ab
s(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 +
b^4)*sqrt(a^2 + b^2)) + 2*(3*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 3*a*b^2*tan(1/2
*d*x + 1/2*c)^4 + 10*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 4*b^3*tan(1/2*d*x + 1/2
*c)^3 - 6*a^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^2*b*tan(1/2*d*x + 1/2*c) - 2*a^3
 + a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 + 1)^3))/d
```

maple [A] time = 0.36, size = 205, normalized size = 1.22

$$\frac{-\frac{16a^3 b\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(8a^4+16a^2b^2+8b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2b-b^2a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{10}{3}a^2b-\frac{4}{3}b^3\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a^2b\tan\left(\frac{dx}{2}+\cdots\right.\right.}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d*(-16*a^3*b/(8*a^4+16*a^2*b^2+8*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*ta
n(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-2/(a^4+2*a^2*b^2+b^4)*(-tan(1/2*d*x+
1/2*c)^5*a^2*b-b^2*a*tan(1/2*d*x+1/2*c)^4+(-10/3*a^2*b-4/3*b^3)*tan(1/2*d*x
+1/2*c)^3+2*a^3*tan(1/2*d*x+1/2*c)^2-a^2*b*tan(1/2*d*x+1/2*c)+2/3*a^3-1/3*b
^2*a)/(1+tan(1/2*d*x+1/2*c)^2)^3)
```

maxima [B] time = 0.64, size = 364, normalized size = 2.17

$$\frac{\frac{3\,a^3 b \log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(2\,a^3-ab^2-\frac{3\,a^2b\sin(dx+c)}{\cos(dx+c)+1}+\frac{6\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{3\,ab^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3\,a^2b\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{2\left(5\,a^2b+2\,b^3\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4+2\,a^2b^2+b^4+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^3*b*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/
(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^
2 + b^4)*sqrt(a^2 + b^2)) - 2*(2*a^3 - a*b^2 - 3*a^2*b*sin(d*x + c)/(cos(d*
x + c) + 1) + 6*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 3*a*b^2*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 3*a^2*b*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
2*(5*a^2*b + 2*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^4 + 2*a^2*b^2 +
 b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*(a
^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^4 + 2*a^2*b^
2 + b^4)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6))/d
```

mupad [B] time = 6.50, size = 324, normalized size = 1.93

$$\frac{\frac{\frac{2\,a\,b^2}{3}-\frac{4\,a^3}{3}}{a^4+2\,a^2\,b^2+b^4}-\frac{4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^4+2\,a^2\,b^2+b^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{20\,a^2\,b}{3}+\frac{8\,b^3}{3}\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a^4+2\,a^2\,b^2+b^4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{2\,a^3\,b\,\mathrm{ata}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + b*tan(c + d*x)),x)
```

```
[Out] (((2*a*b^2)/3 - (4*a^3)/3)/(a^4 + b^4 + 2*a^2*b^2) - (4*a^3*tan(c/2 + (d*x)
/2)^2)/(a^4 + b^4 + 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^3*((20*a^2*b)/3 + (8*b
^3)/3))/(a^4 + b^4 + 2*a^2*b^2) + (2*a^2*b*tan(c/2 + (d*x)/2))/(a^4 + b^4 +
 2*a^2*b^2) + (2*a*b^2*tan(c/2 + (d*x)/2)^4)/(a^4 + b^4 + 2*a^2*b^2) + (2*a
^2*b*tan(c/2 + (d*x)/2)^5)/(a^4 + b^4 + 2*a^2*b^2))/(d*(3*tan(c/2 + (d*x)/2
)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1)) + (2*a^3*b*atanh(
(a^4*b + b^5 + 2*a^2*b^3 - a*tan(c/2 + (d*x)/2)*(a^4 + b^4 + 2*a^2*b^2))/(a
^2 + b^2)^(5/2)))/(d*(a^2 + b^2)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**3/(a + b*tan(c + d*x)), x)
```

3.54 $\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=94

$$-\frac{\cos^2(c+dx)(a\tan(c+dx)+b)}{2d\left(a^2+b^2\right)}+\frac{a^2b\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{ax\left(a^2-b^2\right)}{2\left(a^2+b^2\right)^2}$$

```
[Out] 1/2*a*(a^2-b^2)*x/(a^2+b^2)^2+a^2*b*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)
^2/d-1/2*cos(d*x+c)^2*(b+a*tan(d*x+c))/(a^2+b^2)/d
```

Rubi [A] time = 0.16, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1647, 801, 635, 203, 260}

$$-\frac{\cos^2(c+dx)(a\tan(c+dx)+b)}{2d\left(a^2+b^2\right)}+\frac{a^2b\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{ax\left(a^2-b^2\right)}{2\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*(a^2 - b^2)*x)/(2*(a^2 + b^2)^2) + (a^2*b*Log[a*Cos[c + d*x] + b*Sin[c +
 d*x]])/((a^2 + b^2)^2*d) - (Cos[c + d*x]^2*(b + a*Tan[c + d*x]))/(2*(a^2 +
 b^2)*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
```

```
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^2}{(a+x)\left(b^2+x^2\right)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^2}{a^2+b^2}+\frac{ab^2x}{a^2+b^2}}{(a+x)\left(b^2+x^2\right)}\,dx, x, b\tan(c+dx)\right)}{2bd}$$

$$= -\frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d} - \frac{\text{Subst}\left(\int \left(-\frac{2a^2b^2}{\left(a^2+b^2\right)^2(a+x)} - \frac{ab^2\left(a^2-b^2-2ax\right)}{\left(a^2+b^2\right)^2\left(b^2+x^2\right)}\right)dx, x, b\,\text{ta}$$

$$= \frac{a^2b\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^2 d} - \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d} + \frac{(ab)\,\text{Subst}\left(\int \frac{a^2-b^2-2ax}{b^2+x^2}\right.}{2\left(a^2+\right.}$$

$$= \frac{a^2b\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^2 d} - \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d} - \frac{\left(a^2b\right)\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx\right.}{\left(a^2+b^2\right.}$$

$$= \frac{a\left(a^2-b^2\right)x}{2\left(a^2+b^2\right)^2} + \frac{a^2b\log(\cos(c+dx))}{\left(a^2+b^2\right)^2 d} + \frac{a^2b\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^2 d} - \frac{\cos^2(c+dx)(b+a}{2\left(a^2+b\right.}$$

Mathematica [A] time = 0.77, size = 170, normalized size = 1.81

$$-\frac{2b^2\left(a^2+b^2\right)\cos^2(c+dx) + 2ab\left(a^2+b^2\right)\tan^{-1}(\tan(c+dx)) + a\left(b\left(a^2+b^2\right)\sin(2(c+dx)) + 2a\left(-2b^2\log(a+\right.\right.}{4bd\left(a^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/4*(2*a*b*(a^2 + b^2)*ArcTan[Tan[c + d*x]] + 2*b^2*(a^2 + b^2)*Cos[c + d*
x]^2 + a*(2*a*((b^2 + a*Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*b^
2*Log[a + b*Tan[c + d*x]] + (b^2 - a*Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c +
 d*x]]) + b*(a^2 + b^2)*Sin[2*(c + d*x)]))/(b*(a^2 + b^2)^2*d)
```

fricas [A] time = 0.46, size = 122, normalized size = 1.30

$$\frac{a^2b\log\left(2\,ab\cos(dx+c)\sin(dx+c) + \left(a^2-b^2\right)\cos(dx+c)^2 + b^2\right) + \left(a^3-ab^2\right)dx - \left(a^2b+b^3\right)\cos(dx+c)^2 - \left(\right.}{2\left(a^4+2\,a^2b^2+b^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*b*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
 + b^2) + (a^3 - a*b^2)*d*x - (a^2*b + b^3)*cos(d*x + c)^2 - (a^3 + a*b^2)*
cos(d*x + c)*sin(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d)
```

giac [B] time = 0.62, size = 184, normalized size = 1.96

$$\frac{\frac{2\,a^{2}b^{2}\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^{4}b+2\,a^{2}b^{3}+b^{5}}-\frac{a^{2}b\log\left(\tan\left(dx+c\right)^{2}+1\right)}{a^{4}+2\,a^{2}b^{2}+b^{4}}+\frac{\left(a^{3}-ab^{2}\right)\left(dx+c\right)}{a^{4}+2\,a^{2}b^{2}+b^{4}}+\frac{a^{2}b\tan\left(dx+c\right)^{2}-a^{3}\tan\left(dx+c\right)-ab^{2}\tan\left(dx+c\right)-b^{3}}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\left(\tan\left(dx+c\right)^{2}+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*b^2*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) - a^2
*b*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 - a*b^2)*(d*x + c
)/(a^4 + 2*a^2*b^2 + b^4) + (a^2*b*tan(d*x + c)^2 - a^3*tan(d*x + c) - a*b^
2*tan(d*x + c) - b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(d*x + c)^2 + 1)))/d
```

maple [B] time = 0.36, size = 238, normalized size = 2.53

$$\frac{b\,a^{2}\ln\left(a+b\tan\left(dx+c\right)\right)}{d\left(a^{2}+b^{2}\right)^{2}}-\frac{\tan\left(dx+c\right)a^{3}}{2d\left(a^{2}+b^{2}\right)^{2}\left(1+\tan^{2}\left(dx+c\right)\right)}-\frac{\tan\left(dx+c\right)b^{2}a}{2d\left(a^{2}+b^{2}\right)^{2}\left(1+\tan^{2}\left(dx+c\right)\right)}-\frac{}{2d\left(a^{2}+b^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d*b*a^2/(a^2+b^2)^2*ln(a+b*tan(d*x+c))-1/2/d/(a^2+b^2)^2/(1+tan(d*x+c)^2)
*tan(d*x+c)*a^3-1/2/d/(a^2+b^2)^2/(1+tan(d*x+c)^2)*tan(d*x+c)*b^2*a-1/2/d/(
a^2+b^2)^2/(1+tan(d*x+c)^2)*a^2*b-1/2/d/(a^2+b^2)^2/(1+tan(d*x+c)^2)*b^3-1/
2/d/(a^2+b^2)^2*ln(1+tan(d*x+c)^2)*a^2*b+1/2/d/(a^2+b^2)^2*arctan(tan(d*x+c
))*a^3-1/2/d/(a^2+b^2)^2*arctan(tan(d*x+c))*a*b^2
```

maxima [A] time = 0.58, size = 144, normalized size = 1.53

$$\frac{\frac{2\,a^{2}b\log\left(b\tan\left(dx+c\right)+a\right)}{a^{4}+2\,a^{2}b^{2}+b^{4}}-\frac{a^{2}b\log\left(\tan\left(dx+c\right)^{2}+1\right)}{a^{4}+2\,a^{2}b^{2}+b^{4}}+\frac{\left(a^{3}-ab^{2}\right)\left(dx+c\right)}{a^{4}+2\,a^{2}b^{2}+b^{4}}-\frac{a\tan\left(dx+c\right)+b}{\left(a^{2}+b^{2}\right)\tan\left(dx+c\right)^{2}+a^{2}+b^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2*b*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) - a^2*b*log(ta
n(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 - a*b^2)*(d*x + c)/(a^4 +
2*a^2*b^2 + b^4) - (a*tan(d*x + c) + b)/((a^2 + b^2)*tan(d*x + c)^2 + a^2 +
 b^2))/d
```

mupad [B] time = 3.87, size = 147, normalized size = 1.56

$$\frac{a^{2}\,b\,\ln\left(a+b\tan\left(c+d\,x\right)\right)}{d\left(a^{2}+b^{2}\right)^{2}}-\frac{a\,\ln\left(\tan\left(c+d\,x\right)-\mathrm{i}\right)}{4\,d\left(-a^{2}\,1\mathrm{i}+2\,a\,b+b^{2}\,1\mathrm{i}\right)}-\frac{\cos\left(c+d\,x\right)^{2}\left(\frac{b}{2\left(a^{2}+b^{2}\right)}+\frac{a\tan\left(c+d\,x\right)}{2\left(a^{2}+b^{2}\right)}\right)}{d}-\frac{a\,\ln\left(\tan\left(c+\right.\right.}{4\,d\left(-a^{2}+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b*tan(c + d*x)),x)
```

```
[Out] (a^2*b*log(a + b*tan(c + d*x)))/(d*(a^2 + b^2)^2) - (a*log(tan(c + d*x) + 1
i)*1i)/(4*d*(a*b*2i - a^2 + b^2)) - (a*log(tan(c + d*x) - 1i))/(4*d*(2*a*b
```

```
- a^2*1i + b^2*1i)) - (cos(c + d*x)^2*(b/(2*(a^2 + b^2)) + (a*tan(c + d*x))
/(2*(a^2 + b^2))))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2/(a + b*tan(c + d*x)), x)
```

3.55 $\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=90

$$\frac{b\sin(c+dx)}{d\left(a^2+b^2\right)}-\frac{a\cos(c+dx)}{d\left(a^2+b^2\right)}+\frac{ab\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

```
[Out] a*b*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d-
a*cos(d*x+c)/(a^2+b^2)/d+b*sin(d*x+c)/(a^2+b^2)/d
```

Rubi [A] time = 0.11, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3518, 3109, 2637, 2638, 3074, 206}

$$\frac{b\sin(c+dx)}{d\left(a^2+b^2\right)}-\frac{a\cos(c+dx)}{d\left(a^2+b^2\right)}+\frac{ab\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 + b^
2)^(3/2)*d) - (a*Cos[c + d*x])/((a^2 + b^2)*d) + (b*Sin[c + d*x])/((a^2 + b
^2)*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx &= \int \frac{\cos(c+dx)\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx\\ &= \frac{a\int \sin(c+dx)\,dx}{a^2+b^2}+\frac{b\int \cos(c+dx)\,dx}{a^2+b^2}-\frac{(ab)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}\\ &= -\frac{a\cos(c+dx)}{\left(a^2+b^2\right)d}+\frac{b\sin(c+dx)}{\left(a^2+b^2\right)d}+\frac{(ab)\,\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(c+dx)-a\sin(c+\right.}{\left(a^2+b^2\right)d}\\ &= \frac{ab\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d}-\frac{a\cos(c+dx)}{\left(a^2+b^2\right)d}+\frac{b\sin(c+dx)}{\left(a^2+b^2\right)d}\end{aligned}$$

Mathematica [A] time = 0.37, size = 79, normalized size = 0.88

$$\frac{\sqrt{a^2+b^2}\,(b\sin(c+dx)-a\cos(c+dx))-2ab\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-2*a*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2
]*(-(a*Cos[c + d*x]) + b*Sin[c + d*x]))/((a^2 + b^2)^(3/2)*d)
```

fricas [B] time = 0.48, size = 185, normalized size = 2.06

$$\frac{\sqrt{a^2+b^2}\,ab\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)-2\left(a^3+ab^2\right)\cos\left(dx+\right.}{2\left(a^4+2\,a^2b^2+b^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*a*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)
*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d
*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b
^2)) - 2*(a^3 + a*b^2)*cos(d*x + c) + 2*(a^2*b + b^3)*sin(d*x + c))/((a^4 +
 2*a^2*b^2 + b^4)*d)
```

giac [A] time = 0.61, size = 118, normalized size = 1.31

$$\frac{\frac{ab\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}{\left(a^2+b^2\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*ta
n(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(b*tan
(1/2*d*x + 1/2*c) - a)/((a^2 + b^2)*(tan(1/2*d*x + 1/2*c)^2 + 1)))/d
```

maple [A] time = 0.32, size = 100, normalized size = 1.11

$$\frac{-\frac{4ab\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}-\frac{2\left(-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)}{\left(a^2+b^2\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d*(-4*a*b/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*
c)-2*b)/(a^2+b^2)^(1/2))-2/(a^2+b^2)*(-tan(1/2*d*x+1/2*c)*b+a)/(1+tan(1/2*d
*x+1/2*c)^2))
```

maxima [A] time = 0.79, size = 141, normalized size = 1.57

$$\frac{\frac{ab\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(a-\frac{b\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2+b^2+\frac{\left(a^2+b^2\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*b*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*s
in(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a
 - b*sin(d*x + c)/(cos(d*x + c) + 1))/(a^2 + b^2 + (a^2 + b^2)*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2))/d
```

mupad [B] time = 3.91, size = 110, normalized size = 1.22

$$\frac{2\,a\,b\,\mathrm{atanh}\left(\frac{a^2\,b+b^3-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2+b^2\right)}{{\left(a^2+b^2\right)}^{3/2}}\right)}{d\,{\left(a^2+b^2\right)}^{3/2}}-\frac{\frac{2\,a}{a^2+b^2}-\frac{2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2+b^2}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b*tan(c + d*x)),x)
```

```
[Out] (2*a*b*atanh((a^2*b + b^3 - a*tan(c/2 + (d*x)/2)*(a^2 + b^2))/(a^2 + b^2)^(
3/2)))/(d*(a^2 + b^2)^(3/2)) - ((2*a)/(a^2 + b^2) - (2*b*tan(c/2 + (d*x)/2)
)/(a^2 + b^2))/(d*(tan(c/2 + (d*x)/2)^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*tan(c + d*x)), x)
```

3.56 $\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=66

$$\frac{b\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] -arctanh(cos(d*x+c))/a/d+b*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1
/2))/a/d/(a^2+b^2)^(1/2)
```

Rubi **[A]** time = 0.13, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3518, 3110, 3770, 3074, 206}

$$\frac{b\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + (b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x
])/Sqrt[a^2 + b^2]])/(a*Sqrt[a^2 + b^2]*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx &= \int \frac{\cot(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx \\
&= \int \left(\frac{\csc(c+dx)}{a} - \frac{b}{a(a\cos(c+dx)+b\sin(c+dx))}\right) dx \\
&= \frac{\int \csc(c+dx)\,dx}{a} - \frac{b\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{ad} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{b\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 75, normalized size = 1.14

$$\frac{-\frac{2b\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] ((-2*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
- Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]])/(a*d)
```

fricas [B] time = 0.48, size = 183, normalized size = 2.77

$$\frac{\sqrt{a^2+b^2}\,b\log\left(\frac{2ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2a^2-b^2-2\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right) - \left(a^2+b^2\right)\log\left(\frac{1}{2}\cos\right.}{2\left(a^3+ab^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*c
os(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x
 + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2
)) - (a^2 + b^2)*log(1/2*cos(d*x + c) + 1/2) + (a^2 + b^2)*log(-1/2*cos(d*x
 + c) + 1/2))/((a^3 + a*b^2)*d)
```

giac [A] time = 1.62, size = 94, normalized size = 1.42

$$\frac{\frac{b\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a} + \frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(
1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) + log(abs(
tan(1/2*d*x + 1/2*c)))/a)/d
```

maple [A] time = 0.36, size = 65, normalized size = 0.98

$$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{2b\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{da\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d/a*ln(tan(1/2*d*x+1/2*c))-2/d/a*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1
/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))
```

maxima [A] time = 0.47, size = 107, normalized size = 1.62

$$\frac{\frac{b\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin
(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) + log(
sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 4.09, size = 174, normalized size = 2.64

$$\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{a\,d}-\frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{a^2+b^2}\,\left(1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\,a^2+2\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\,a\,b+4\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\,b^2\right)}{b^3\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\,4\mathrm{i}+a\,b^2\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\,1\mathrm{i}+a^2\,b\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\,3\mathrm{i}+a\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(a^2+b^2\right)\,1\mathrm{i}}\right)}{a\,d\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + b*tan(c + d*x))),x)
```

```
[Out] log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(a*d) - (2*b*atanh(((a^2 + b^2)^
(1/2)*(a^2*sin(c/2 + (d*x)/2)*1i + b^2*sin(c/2 + (d*x)/2)*4i + a*b*cos(c/2
+ (d*x)/2)*2i))/(b^3*sin(c/2 + (d*x)/2)*4i + a*b^2*cos(c/2 + (d*x)/2)*1i +
a^2*b*sin(c/2 + (d*x)/2)*3i + a*cos(c/2 + (d*x)/2)*(a^2 + b^2)*1i)))/(a*d*(
a^2 + b^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*tan(c + d*x)), x)
```

3.57 $\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=50

$$-\frac{b\log(\tan(c+dx))}{a^2d}+\frac{b\log(a+b\tan(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

```
[Out] -cot(d*x+c)/a/d-b*ln(tan(d*x+c))/a^2/d+b*ln(a+b*tan(d*x+c))/a^2/d
```

Rubi [A] time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 44}

$$-\frac{b\log(\tan(c+dx))}{a^2d}+\frac{b\log(a+b\tan(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(Cot[c + d*x]/(a*d)) - (b*Log[Tan[c + d*x]])/(a^2*d) + (b*Log[a + b*Tan[c
+ d*x]])/(a^2*d)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx &= \frac{b\,\text{Subst}\left(\int \frac{1}{x^2(a+x)}\,dx, x, b\tan(c+dx)\right)}{d}\\
&= \frac{b\,\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{1}{a^2x}+\frac{1}{a^2(a+x)}\right)dx, x, b\tan(c+dx)\right)}{d}\\
&= -\frac{\cot(c+dx)}{ad}-\frac{b\log(\tan(c+dx))}{a^2d}+\frac{b\log(a+b\tan(c+dx))}{a^2d}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 47, normalized size = 0.94

$$\frac{b(\log(a\cos(c+dx)+b\sin(c+dx))-\log(\sin(c+dx)))-a\cot(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-(a*Cot[c + d*x]) + b*(-Log[Sin[c + d*x]] + Log[a*Cos[c + d*x] + b*Sin[c +
 d*x]]))/(a^2*d)
```

fricas [A] time = 0.44, size = 95, normalized size = 1.90

$$\frac{b\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+b^{2}\right)\sin\left(dx+c\right)-b\log\left(-\frac{1}{4}\cos\left(dx+c\right)^{2}+\frac{1}{4}\right)\sin}{2\,a^{2}d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b
^2)*sin(d*x + c) - b*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 2*a*cos(
d*x + c))/(a^2*d*sin(d*x + c))
```

giac [A] time = 0.70, size = 60, normalized size = 1.20

$$\frac{\frac{b\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^{2}}-\frac{b\log\left(|\tan\left(dx+c\right)|\right)}{a^{2}}+\frac{b\tan\left(dx+c\right)-a}{a^{2}\tan\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (b*log(abs(b*tan(d*x + c) + a))/a^2 - b*log(abs(tan(d*x + c)))/a^2 + (b*tan
(d*x + c) - a)/(a^2*tan(d*x + c)))/d
```

maple [A] time = 0.35, size = 53, normalized size = 1.06

$$\frac{b\ln\left(a+b\tan\left(dx+c\right)\right)}{a^{2}d}-\frac{1}{da\tan\left(dx+c\right)}-\frac{b\ln\left(\tan\left(dx+c\right)\right)}{a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*tan(d*x+c)),x)
```

```
[Out] b*ln(a+b*tan(d*x+c))/a^2/d-1/d/a/tan(d*x+c)-b*ln(tan(d*x+c))/a^2/d
```

maxima [A] time = 0.58, size = 47, normalized size = 0.94

$$\frac{\frac{b\log\left(b\tan\left(dx+c\right)+a\right)}{a^{2}}-\frac{b\log\left(\tan\left(dx+c\right)\right)}{a^{2}}-\frac{1}{a\tan\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b*log(b*tan(d*x + c) + a)/a^2 - b*log(tan(d*x + c))/a^2 - 1/(a*tan(d*x + c
)))/d
```

mupad [B] time = 3.74, size = 39, normalized size = 0.78

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\tan\left(c+d\,x\right)}{a}+1\right)}{a^{2}\,d}-\frac{\cot\left(c+d\,x\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b*tan(c + d*x))),x)
```

```
[Out] (2*b*atanh((2*b*tan(c + d*x))/a + 1))/(a^2*d) - cot(c + d*x)/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^{2}\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*tan(c + d*x)), x)
```

3.58 $\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=122

$$-\frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}+\frac{b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot(c+$$

```
[Out] -1/2*arctanh(cos(d*x+c))/a/d-b^2*arctanh(cos(d*x+c))/a^3/d+b*csc(d*x+c)/a^2
/d-1/2*cot(d*x+c)*csc(d*x+c)/a/d+b*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2
+b^2)^(1/2))*(a^2+b^2)^(1/2)/a^3/d
```

Rubi [A] time = 0.31, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3518, 3110, 3768, 3770, 2621, 321, 207, 2622, 3104, 3074, 206}

$$-\frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot(c+$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] -ArcTanh[Cos[c + d*x]]/(2*a*d) - (b^2*ArcTanh[Cos[c + d*x]])/(a^3*d) + (b*S
qrt[a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/
(a^3*d) + (b*Csc[c + d*x])/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
```

```
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\cot(c+dx)\csc^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$= \int \left(\frac{\csc^3(c+dx)}{a} - \frac{b\csc^2(c+dx)\sec(c+dx)}{a^2} + \frac{b^2\csc(c+dx)\sec^2(c+dx)}{a^3} - \frac{}{a^3(a\cos}\right.$$
$$= \frac{\int \csc^3(c+dx)\,dx}{a} - \frac{b\int \csc^2(c+dx)\sec(c+dx)\,dx}{a^2} + \frac{b^2\int \csc(c+dx)\sec^2(c+dx)\,dx}{a^3}$$
$$= -\frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{b^2\sec(c+dx)}{a^3d} + \frac{\int \csc(c+dx)\,dx}{2a} + \frac{b\int \sec(c+dx)\,dx}{a^2} - $$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad} + \frac{b\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{b\csc(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+d}{2ad}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{b\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+}{\sqrt{a^2+b^2}}\right.}{a^3d}$$

Mathematica [A] time = 0.81, size = 179, normalized size = 1.47

$$\frac{-16b\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right) + a^2\left(-\csc^2\left(\frac{1}{2}(c+dx)\right)\right) + a^2\sec^2\left(\frac{1}{2}(c+dx)\right) + 4a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-16*b*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] +
 4*a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 - 4*a^2*Log[Cos[(c + d*x)/
2]] - 8*b^2*Log[Cos[(c + d*x)/2]] + 4*a^2*Log[Sin[(c + d*x)/2]] + 8*b^2*Log
[Sin[(c + d*x)/2]] + a^2*Sec[(c + d*x)/2]^2 + 4*a*b*Tan[(c + d*x)/2])/(8*a^
3*d)
```

fricas [B] time = 0.53, size = 270, normalized size = 2.21

$$\frac{2\,a^2\cos(dx+c) - 4\,ab\sin(dx+c) + 2\left(b\cos(dx+c)^2 - b\right)\sqrt{a^2+b^2}\,\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*cos(d*x + c) - 4*a*b*sin(d*x + c) + 2*(b*cos(d*x + c)^2 - b)*sqr
t(a^2 + b^2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c
)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2
*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - ((a^2
 + 2*b^2)*cos(d*x + c)^2 - a^2 - 2*b^2)*log(1/2*cos(d*x + c) + 1/2) + ((a^2
 + 2*b^2)*cos(d*x + c)^2 - a^2 - 2*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(a^3*
d*cos(d*x + c)^2 - a^3*d)
```

giac [A] time = 2.07, size = 209, normalized size = 1.71

$$\frac{\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2} + \frac{4\left(a^2+2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3} + \frac{8\left(a^2b+b^3\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a^3} - \frac{6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*((a*tan(1/2*d*x + 1/2*c)^2 + 4*b*tan(1/2*d*x + 1/2*c))/a^2 + 4*(a^2 + 2
*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 8*(a^2*b + b^3)*log(abs(2*a*tan(
1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) -
2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3) - (6*a^2*tan(1/2*d*x + 1/2*
c)^2 + 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 4*a*b*tan(1/2*d*x + 1/2*c) + a^2)/(a
^3*tan(1/2*d*x + 1/2*c)^2))/d
```

maple [A] time = 0.41, size = 162, normalized size = 1.33

$$\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{2d\,a^2}-\frac{1}{8da\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2}{d\,a^3}+\frac{b}{2d\,a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/8/d/a*tan(1/2*d*x+1/2*c)^2+1/2/d/a^2*tan(1/2*d*x+1/2*c)*b-1/8/d/a/tan(1/2
*d*x+1/2*c)^2+1/2/d/a*ln(tan(1/2*d*x+1/2*c))+1/d/a^3*ln(tan(1/2*d*x+1/2*c))
*b^2+1/2/d*b/a^2/tan(1/2*d*x+1/2*c)-2/d*b*(a^2+b^2)^(1/2)/a^3*arctanh(1/2*(
2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))
```

maxima [A] time = 0.66, size = 215, normalized size = 1.76

$$\frac{\frac{\frac{4\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^2}+\frac{4\left(a^2+2\,b^2\right)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{\left(a-\frac{4\,b\sin(dx+c)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)^2}{a^2\sin(dx+c)^2}+\frac{8\left(a^2b+b^3\right)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*((4*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2)/a^2 + 4*(a^2 + 2*b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - (a
 - 4*b*sin(d*x + c)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)^2/(a^2*sin(d*x +
 c)^2) + 8*(a^2*b + b^3)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(
a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqr
t(a^2 + b^2)*a^3))/d
```

mupad [B] time = 4.45, size = 764, normalized size = 6.26

$$\frac{b^2\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2\left(\frac{a^3\,d}{2}-\frac{a^3\,d\,\cos(2\,c+2\,d\,x)}{2}\right)}-\frac{a^2\left(\frac{\cos(c+d\,x)}{2}-\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4}+\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(2\,c+2\,d\,x)}{4}\right)}{\frac{a^3\,d}{2}-\frac{a^3\,d\,\cos(2\,c+2\,d\,x)}{2}}+\frac{a\,b\,\sin\left(c+d\,x\right)}{\frac{a^3\,d}{2}-\frac{a^3\,d\,\cos(2\,c+2\,d\,x)}{2}}-\frac{b^2\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + b*tan(c + d*x))),x)
```

```
[Out] (b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(2*((a^3*d)/2 - (a^3*d*cos
(2*c + 2*d*x))/2)) - (a^2*(cos(c + d*x)/2 - log(sin(c/2 + (d*x)/2)/cos(c/2
+ (d*x)/2))/4 + (log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x
```

```
))/4))/((a^3*d)/2 - (a^3*d*cos(2*c + 2*d*x))/2) + (a*b*sin(c + d*x))/((a^3*
d)/2 - (a^3*d*cos(2*c + 2*d*x))/2) - (b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2))*cos(2*c + 2*d*x))/(2*((a^3*d)/2 - (a^3*d*cos(2*c + 2*d*x))/2)) +
(b*atan((a^4*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*1i + b^4*sin(c/2 + (d*x)/
2)*(a^2 + b^2)^(1/2)*8i + a*b^3*cos(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*4i + a
^3*b*cos(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*3i + a^2*b^2*sin(c/2 + (d*x)/2)*(
a^2 + b^2)^(1/2)*8i)/(a^5*cos(c/2 + (d*x)/2) + 8*b^5*sin(c/2 + (d*x)/2) + 4
*a*b^4*cos(c/2 + (d*x)/2) + 4*a^4*b*sin(c/2 + (d*x)/2) + 5*a^3*b^2*cos(c/2
+ (d*x)/2) + 12*a^2*b^3*sin(c/2 + (d*x)/2)))*(a^2 + b^2)^(1/2)*1i)/((a^3*d)
/2 - (a^3*d*cos(2*c + 2*d*x))/2) - (b*cos(2*c + 2*d*x)*atan((a^4*sin(c/2 +
(d*x)/2)*(a^2 + b^2)^(1/2)*1i + b^4*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*8i
 + a*b^3*cos(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*4i + a^3*b*cos(c/2 + (d*x)/2)
*(a^2 + b^2)^(1/2)*3i + a^2*b^2*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*8i)/(a
^5*cos(c/2 + (d*x)/2) + 8*b^5*sin(c/2 + (d*x)/2) + 4*a*b^4*cos(c/2 + (d*x)/
2) + 4*a^4*b*sin(c/2 + (d*x)/2) + 5*a^3*b^2*cos(c/2 + (d*x)/2) + 12*a^2*b^3
*sin(c/2 + (d*x)/2)))*(a^2 + b^2)^(1/2)*1i)/((a^3*d)/2 - (a^3*d*cos(2*c + 2
*d*x))/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**3/(a + b*tan(c + d*x)), x)
```

3.59 $\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=108

$$\frac{b\cot^2(c+dx)}{2a^2d}-\frac{b\left(a^2+b^2\right)\log(\tan(c+dx))}{a^4d}+\frac{b\left(a^2+b^2\right)\log(a+b\tan(c+dx))}{a^4d}-\frac{\left(a^2+b^2\right)\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3ad}$$

```
[Out] -(a^2+b^2)*cot(d*x+c)/a^3/d+1/2*b*cot(d*x+c)^2/a^2/d-1/3*cot(d*x+c)^3/a/d-b
*(a^2+b^2)*ln(tan(d*x+c))/a^4/d+b*(a^2+b^2)*ln(a+b*tan(d*x+c))/a^4/d
```

Rubi [A] time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 894}

$$-\frac{\left(a^2+b^2\right)\cot(c+dx)}{a^3d}-\frac{b\left(a^2+b^2\right)\log(\tan(c+dx))}{a^4d}+\frac{b\left(a^2+b^2\right)\log(a+b\tan(c+dx))}{a^4d}+\frac{b\cot^2(c+dx)}{2a^2d}-\frac{\cot^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(((a^2 + b^2)*Cot[c + d*x])/(a^3*d)) + (b*Cot[c + d*x]^2)/(2*a^2*d) - Cot[
c + d*x]^3/(3*a*d) - (b*(a^2 + b^2)*Log[Tan[c + d*x]])/(a^4*d) + (b*(a^2 +
b^2)*Log[a + b*Tan[c + d*x]])/(a^4*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx &= \frac{b\,\text{Subst}\left(\int \frac{b^2+x^2}{x^4(a+x)}\,dx,x,b\tan(c+dx)\right)}{d}\\
&= \frac{b\,\text{Subst}\left(\int \left(\frac{b^2}{ax^4}-\frac{b^2}{a^2x^3}+\frac{a^2+b^2}{a^3x^2}+\frac{-a^2-b^2}{a^4x}+\frac{a^2+b^2}{a^4(a+x)}\right)dx,x,b\tan(c+dx)\right)}{d}\\
&= -\frac{\left(a^2+b^2\right)\cot(c+dx)}{a^3d}+\frac{b\cot^2(c+dx)}{2a^2d}-\frac{\cot^3(c+dx)}{3ad}-\frac{b\left(a^2+b^2\right)\log(\tan(c+dx))}{a^4d}
\end{aligned}$$

Mathematica [A] time = 0.49, size = 95, normalized size = 0.88

$$\frac{-2\cot(c+dx)\left(a^3\csc^2(c+dx)+2a^3+3ab^2\right)-6b\left(a^2+b^2\right)(\log(\sin(c+dx))-\log(a\cos(c+dx)+b\sin(c+$$
$$6a^4d$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] (3*a^2*b*Csc[c + d*x]^2 - 2*Cot[c + d*x]*(2*a^3 + 3*a*b^2 + a^3*Csc[c + d*x
]^2) - 6*b*(a^2 + b^2)*(Log[Sin[c + d*x]] - Log[a*Cos[c + d*x] + b*Sin[c +
d*x]]))/(6*a^4*d)
```

fricas [A] time = 0.46, size = 208, normalized size = 1.93

$$-\frac{2\left(2a^3+3ab^2\right)\cos\left(dx+c\right)^3+3a^2b\sin\left(dx+c\right)+3\left(a^2b+b^3-\left(a^2b+b^3\right)\cos\left(dx+c\right)^2\right)\log\left(2ab\cos\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3 + 3*a^2*b*sin(d*x + c) + 3*(a^2*b
+ b^3 - (a^2*b + b^3)*cos(d*x + c)^2)*log(2*a*b*cos(d*x + c)*sin(d*x + c) +
 (a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c) - 3*(a^2*b + b^3 - (a^2*b +
 b^3)*cos(d*x + c)^2)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 6*(a^3
+ a*b^2)*cos(d*x + c))/((a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))
```

giac [A] time = 2.93, size = 144, normalized size = 1.33

$$-\frac{\frac{6\left(a^2b+b^3\right)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{a^4}-\frac{6\left(a^2b^2+b^4\right)\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^4b}-\frac{11a^2b\tan\left(dx+c\right)^3+11b^3\tan\left(dx+c\right)^3-6a^3\tan\left(dx+c\right)^2-6ab^2\tan\left(dx+c\right)^2+3a^2b\tan\left(dx+c\right)\cdots}{a^4\tan\left(dx+c\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(a^2*b + b^3)*log(abs(tan(d*x + c)))/a^4 - 6*(a^2*b^2 + b^4)*log(ab
s(b*tan(d*x + c) + a))/(a^4*b) - (11*a^2*b*tan(d*x + c)^3 + 11*b^3*tan(d*x
+ c)^3 - 6*a^3*tan(d*x + c)^2 - 6*a*b^2*tan(d*x + c)^2 + 3*a^2*b*tan(d*x +
c) - 2*a^3)/(a^4*tan(d*x + c)^3))/d
```

maple [A] time = 0.39, size = 144, normalized size = 1.33

$$\frac{b\ln\left(a+b\tan\left(dx+c\right)\right)}{a^2d}+\frac{b^3\ln\left(a+b\tan\left(dx+c\right)\right)}{d\,a^4}-\frac{1}{3da\tan\left(dx+c\right)^3}-\frac{1}{da\tan\left(dx+c\right)}-\frac{b^2}{d\,a^3\tan\left(dx+c\right)}+\frac{}{2d\,a^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*tan(d*x+c)),x)
```

```
[Out] b*ln(a+b*tan(d*x+c))/a^2/d+1/d/a^4*b^3*ln(a+b*tan(d*x+c))-1/3/d/a/tan(d*x+c
)^3-1/d/a/tan(d*x+c)-1/d/a^3/tan(d*x+c)*b^2+1/2/d*b/a^2/tan(d*x+c)^2-b*ln(t
an(d*x+c))/a^2/d-1/d/a^4*b^3*ln(tan(d*x+c))
```

maxima [A] time = 0.49, size = 97, normalized size = 0.90

$$\frac{\frac{6\left(a^2b+b^3\right)\log\left(b\tan\left(dx+c\right)+a\right)}{a^4}-\frac{6\left(a^2b+b^3\right)\log\left(\tan\left(dx+c\right)\right)}{a^4}+\frac{3ab\tan\left(dx+c\right)-6\left(a^2+b^2\right)\tan\left(dx+c\right)^2-2a^2}{a^3\tan\left(dx+c\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(a^2*b + b^3)*log(b*tan(d*x + c) + a)/a^4 - 6*(a^2*b + b^3)*log(tan(
d*x + c))/a^4 + (3*a*b*tan(d*x + c) - 6*(a^2 + b^2)*tan(d*x + c)^2 - 2*a^2)
/(a^3*tan(d*x + c)^3))/d
```

mupad [B] time = 3.84, size = 102, normalized size = 0.94

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{b\,\left(a^2+b^2\right)\,\left(a+2\,b\,\tan(c+d\,x)\right)}{a\,\left(a^2\,b+b^3\right)}\right)\,\left(a^2+b^2\right)}{a^4\,d}-\frac{\frac{1}{3\,a}+\frac{\tan(c+d\,x)^2\,\left(a^2+b^2\right)}{a^3}-\frac{b\,\tan(c+d\,x)}{2\,a^2}}{d\,\tan\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + b*tan(c + d*x))),x)
```

```
[Out] (2*b*atanh((b*(a^2 + b^2)*(a + 2*b*tan(c + d*x)))/(a*(a^2*b + b^3)))*(a^2 +
 b^2))/(a^4*d) - (1/(3*a) + (tan(c + d*x)^2*(a^2 + b^2))/a^3 - (b*tan(c + d
*x))/(2*a^2))/(d*tan(c + d*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*tan(c + d*x)), x)
```

3.60 $\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=169

$$\frac{b\cot^4(c+dx)}{4a^2d}-\frac{b\left(a^2+b^2\right)^2\log(\tan(c+dx))}{a^6d}+\frac{b\left(a^2+b^2\right)^2\log(a+b\tan(c+dx))}{a^6d}-\frac{\left(a^2+b^2\right)^2\cot(c+dx)}{a^5d}+\frac{b\left(2a^2\right.}{}$$

```
[Out] -(a^2+b^2)^2*cot(d*x+c)/a^5/d+1/2*b*(2*a^2+b^2)*cot(d*x+c)^2/a^4/d-1/3*(2*a
^2+b^2)*cot(d*x+c)^3/a^3/d+1/4*b*cot(d*x+c)^4/a^2/d-1/5*cot(d*x+c)^5/a/d-b*
(a^2+b^2)^2*ln(tan(d*x+c))/a^6/d+b*(a^2+b^2)^2*ln(a+b*tan(d*x+c))/a^6/d
```

Rubi [A] time = 0.15, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 894}

$$-\frac{\left(2a^2+b^2\right)\cot^3(c+dx)}{3a^3d}+\frac{b\left(2a^2+b^2\right)\cot^2(c+dx)}{2a^4d}-\frac{\left(a^2+b^2\right)^2\cot(c+dx)}{a^5d}-\frac{b\left(a^2+b^2\right)^2\log(\tan(c+dx))}{a^6d}+\frac{b\left(a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(((a^2 + b^2)^2*Cot[c + d*x])/(a^5*d)) + (b*(2*a^2 + b^2)*Cot[c + d*x]^2)/
(2*a^4*d) - ((2*a^2 + b^2)*Cot[c + d*x]^3)/(3*a^3*d) + (b*Cot[c + d*x]^4)/(
4*a^2*d) - Cot[c + d*x]^5/(5*a*d) - (b*(a^2 + b^2)^2*Log[Tan[c + d*x]])/(a^
6*d) + (b*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]])/(a^6*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx &= \frac{b\,\text{Subst}\left(\int \frac{\left(b^2+x^2\right)^2}{x^6(a+x)}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\,\text{Subst}\left(\int \left(\frac{b^4}{ax^6}-\frac{b^4}{a^2x^5}+\frac{2a^2b^2+b^4}{a^3x^4}+\frac{b^2\left(-2a^2-b^2\right)}{a^4x^3}+\frac{\left(a^2+b^2\right)^2}{a^5x^2}-\frac{\left(a^2+b^2\right)^2}{a^6x}+\frac{\left(a^2+b^2\right)^2}{a^6(a+x)}\right)dx,x,b\,\text{ta}\right.}{d}\\ &= -\frac{\left(a^2+b^2\right)^2\cot(c+dx)}{a^5d}+\frac{b\left(2a^2+b^2\right)\cot^2(c+dx)}{2a^4d}-\frac{\left(2a^2+b^2\right)\cot^3(c+dx)}{3a^3d}+\frac{b\cot^4}{4}\end{aligned}$$

Mathematica [A] time = 2.10, size = 150, normalized size = 0.89

$$\frac{15b\left(a^4\csc^4(c+dx)+2a^2\left(a^2+b^2\right)\csc^2(c+dx)-4\left(a^2+b^2\right)^2(\log(\sin(c+dx))-\log(a\cos(c+dx)+b\sin(c+\right.}{60a^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-4*Cot[c + d*x]*(8*a^5 + 25*a^3*b^2 + 15*a*b^4 + a^3*(4*a^2 + 5*b^2)*Csc[c
 + d*x]^2 + 3*a^5*Csc[c + d*x]^4) + 15*b*(2*a^2*(a^2 + b^2)*Csc[c + d*x]^2
+ a^4*Csc[c + d*x]^4 - 4*(a^2 + b^2)^2*(Log[Sin[c + d*x]] - Log[a*Cos[c + d
*x] + b*Sin[c + d*x]])))/(60*a^6*d)
```

fricas [B] time = 0.47, size = 385, normalized size = 2.28

$$-\frac{4\left(8\,a^{5}+25\,a^{3}b^{2}+15\,ab^{4}\right)\cos\left(dx+c\right)^{5}-20\left(4\,a^{5}+11\,a^{3}b^{2}+6\,ab^{4}\right)\cos\left(dx+c\right)^{3}-30\left(a^{4}b+2\,a^{2}b^{3}+b^{5}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(4*(8*a^5 + 25*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 - 20*(4*a^5 + 11*a^
3*b^2 + 6*a*b^4)*cos(d*x + c)^3 - 30*(a^4*b + 2*a^2*b^3 + b^5 + (a^4*b + 2*
a^2*b^3 + b^5)*cos(d*x + c)^4 - 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^2)
*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)*si
n(d*x + c) + 30*(a^4*b + 2*a^2*b^3 + b^5 + (a^4*b + 2*a^2*b^3 + b^5)*cos(d*
x + c)^4 - 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^2)*log(-1/4*cos(d*x + c
)^2 + 1/4)*sin(d*x + c) + 60*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c) - 15*(3
*a^4*b + 2*a^2*b^3 - 2*(a^4*b + a^2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/((a^
6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 + a^6*d)*sin(d*x + c))
```

giac [A] time = 2.93, size = 251, normalized size = 1.49

$$-\frac{\frac{60\left(a^{4}b+2\,a^{2}b^{3}+b^{5}\right)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{a^{6}}-\frac{60\left(a^{4}b^{2}+2\,a^{2}b^{4}+b^{6}\right)\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^{6}b}-\frac{137\,a^{4}b\tan\left(dx+c\right)^{5}+274\,a^{2}b^{3}\tan\left(dx+c\right)^{5}+137\,b^{5}\tan\left(dx+c\right)}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(60*(a^4*b + 2*a^2*b^3 + b^5)*log(abs(tan(d*x + c)))/a^6 - 60*(a^4*b^
2 + 2*a^2*b^4 + b^6)*log(abs(b*tan(d*x + c) + a))/(a^6*b) - (137*a^4*b*tan(
d*x + c)^5 + 274*a^2*b^3*tan(d*x + c)^5 + 137*b^5*tan(d*x + c)^5 - 60*a^5*t
an(d*x + c)^4 - 120*a^3*b^2*tan(d*x + c)^4 - 60*a*b^4*tan(d*x + c)^4 + 60*a
^4*b*tan(d*x + c)^3 + 30*a^2*b^3*tan(d*x + c)^3 - 40*a^5*tan(d*x + c)^2 - 2
0*a^3*b^2*tan(d*x + c)^2 + 15*a^4*b*tan(d*x + c) - 12*a^5)/(a^6*tan(d*x + c
)^5))/d
```

maple [A] time = 0.41, size = 273, normalized size = 1.62

$$\frac{b\ln\left(a+b\tan\left(dx+c\right)\right)}{a^{2}d}+\frac{2b^{3}\ln\left(a+b\tan\left(dx+c\right)\right)}{d\,a^{4}}+\frac{b^{5}\ln\left(a+b\tan\left(dx+c\right)\right)}{d\,a^{6}}-\frac{1}{5da\tan\left(dx+c\right)^{5}}-\frac{2}{3da\tan\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+b*tan(d*x+c)),x)
```

```
[Out] b*ln(a+b*tan(d*x+c))/a^2/d+2/d/a^4*b^3*ln(a+b*tan(d*x+c))+1/d/a^6*b^5*ln(a+
b*tan(d*x+c))-1/5/d/a/tan(d*x+c)^5-2/3/d/a/tan(d*x+c)^3-1/3/d/a^3/tan(d*x+c
)^3*b^2-1/d/a/tan(d*x+c)-2/d/a^3/tan(d*x+c)*b^2-1/d/a^5/tan(d*x+c)*b^4+1/4/
d*b/a^2/tan(d*x+c)^4+1/d*b/a^2/tan(d*x+c)^2+1/2/d/a^4*b^3/tan(d*x+c)^2-b*ln
(tan(d*x+c))/a^2/d-2/d/a^4*b^3*ln(tan(d*x+c))-1/d/a^6*b^5*ln(tan(d*x+c))
```

maxima [A] time = 0.54, size = 168, normalized size = 0.99

$$\frac{\frac{60\left(a^4b+2\,a^2b^3+b^5\right)\log(b\tan(dx+c)+a)}{a^6}-\frac{60\left(a^4b+2\,a^2b^3+b^5\right)\log(\tan(dx+c))}{a^6}+\frac{15\,a^3b\tan(dx+c)-60\left(a^4+2\,a^2b^2+b^4\right)\tan(dx+c)^4-12\,a^4+30\left(2\,a^3b+\cdots\right.}{a^5\tan(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*(a^4*b + 2*a^2*b^3 + b^5)*log(b*tan(d*x + c) + a)/a^6 - 60*(a^4*b
+ 2*a^2*b^3 + b^5)*log(tan(d*x + c))/a^6 + (15*a^3*b*tan(d*x + c) - 60*(a^4
 + 2*a^2*b^2 + b^4)*tan(d*x + c)^4 - 12*a^4 + 30*(2*a^3*b + a*b^3)*tan(d*x
+ c)^3 - 20*(2*a^4 + a^2*b^2)*tan(d*x + c)^2)/(a^5*tan(d*x + c)^5))/d
```

mupad [B] time = 4.33, size = 167, normalized size = 0.99

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{b\left(a^2+b^2\right)^2(a+2\,b\tan(c+d\,x))}{a\left(a^4\,b+2\,a^2\,b^3+b^5\right)}\right)\left(a^2+b^2\right)^2}{a^6\,d}-\frac{\frac{1}{5\,a}+\frac{\tan(c+d\,x)^2\left(2\,a^2+b^2\right)}{3\,a^3}+\frac{\tan(c+d\,x)^4\left(a^4+2\,a^2\,b^2+b^4\right)}{a^5}-\frac{b\tan(c+d\,x)}{4\,a^2}-\frac{b\tan\cdots}{\cdots}}{d\tan\left(c+d\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a + b*tan(c + d*x))),x)
```

```
[Out] (2*b*atanh((b*(a^2 + b^2)^2*(a + 2*b*tan(c + d*x)))/(a*(a^4*b + b^5 + 2*a^2
*b^3)))*(a^2 + b^2)^2)/(a^6*d) - (1/(5*a) + (tan(c + d*x)^2*(2*a^2 + b^2))/
(3*a^3) + (tan(c + d*x)^4*(a^4 + b^4 + 2*a^2*b^2))/a^5 - (b*tan(c + d*x))/(
4*a^2) - (b*tan(c + d*x)^3*(2*a^2 + b^2))/(2*a^4))/(d*tan(c + d*x)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^6\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**6/(a + b*tan(c + d*x)), x)
```

3.61 $\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=297

$$-\frac{\cos^6(c+dx)\left(\left(a^2-b^2\right)\tan(c+dx)+2ab\right)}{6d\left(a^2+b^2\right)^2}-\frac{a^6b}{d\left(a^2+b^2\right)^4(a+b\tan(c+dx))}+\frac{2a^5b\left(a^2-3b^2\right)\log(a\cos(c+d}{d\left(a^2+b^2\right)^5}$$

```
[Out] 1/16*(5*a^8-80*a^6*b^2+50*a^4*b^4+8*a^2*b^6+b^8)*x/(a^2+b^2)^5+2*a^5*b*(a^2
-3*b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^5/d-a^6*b/(a^2+b^2)^4/d/(a+
b*tan(d*x+c))-1/6*cos(d*x+c)^6*(2*a*b+(a^2-b^2)*tan(d*x+c))/(a^2+b^2)^2/d+1
/24*cos(d*x+c)^4*(12*a*b*(3*a^2+b^2)+(13*a^4-18*a^2*b^2-7*b^4)*tan(d*x+c))/
(a^2+b^2)^3/d-1/16*cos(d*x+c)^2*(48*a^5*b+(11*a^6-43*a^4*b^2-7*a^2*b^4-b^6)
*tan(d*x+c))/(a^2+b^2)^4/d
```

Rubi [A] time = 0.91, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$-\frac{a^6b}{d\left(a^2+b^2\right)^4(a+b\tan(c+dx))}-\frac{\cos^6(c+dx)\left(\left(a^2-b^2\right)\tan(c+dx)+2ab\right)}{6d\left(a^2+b^2\right)^2}+\frac{\cos^4(c+dx)\left(\left(-18a^2b^2+13a^4\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((5*a^8 - 80*a^6*b^2 + 50*a^4*b^4 + 8*a^2*b^6 + b^8)*x)/(16*(a^2 + b^2)^5)
+ (2*a^5*b*(a^2 - 3*b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)
^5*d) - (a^6*b)/((a^2 + b^2)^4*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^6*(2
*a*b + (a^2 - b^2)*Tan[c + d*x]))/(6*(a^2 + b^2)^2*d) + (Cos[c + d*x]^4*(12
*a*b*(3*a^2 + b^2) + (13*a^4 - 18*a^2*b^2 - 7*b^4)*Tan[c + d*x]))/(24*(a^2
+ b^2)^3*d) - (Cos[c + d*x]^2*(48*a^5*b + (11*a^6 - 43*a^4*b^2 - 7*a^2*b^4
- b^6)*Tan[c + d*x]))/(16*(a^2 + b^2)^4*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^6}{(a+x)^2\left(b^2+x^2\right)^4}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^6(c+dx)\left(2ab+\left(a^2-b^2\right)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^2 d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^6\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2}+\frac{2ab^6\left(5a^2+b^2\right)x}{\left(a^2+b^2\right)^2}+\frac{b^4\left(6a^4}{\ldots}}{(a+x)^2\left(b^2+x^2\right)^3} \ldots\right)}{6b\ldots}$$

$$= -\frac{\cos^6(c+dx)\left(2ab+\left(a^2-b^2\right)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^2 d} + \frac{\cos^4(c+dx)\left(12ab\left(3a^2+b^2\right)+\left(13a\ldots\right.\right.}{24\left(a^2+b^2\ldots\right.}$$

$$= -\frac{\cos^6(c+dx)\left(2ab+\left(a^2-b^2\right)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^2 d} + \frac{\cos^4(c+dx)\left(12ab\left(3a^2+b^2\right)+\left(13a\ldots\right.\right.}{24\left(a^2+b^2\ldots\right.}$$

$$= -\frac{\cos^6(c+dx)\left(2ab+\left(a^2-b^2\right)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^2 d} + \frac{\cos^4(c+dx)\left(12ab\left(3a^2+b^2\right)+\left(13a\ldots\right.\right.}{24\left(a^2+b^2\ldots\right.}$$

$$= \frac{2a^5b\left(a^2-3b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} - \frac{a^6b}{\left(a^2+b^2\right)^4 d(a+b\tan(c+dx))} - \frac{\cos^6(c+\ldots}{\ldots}$$

$$= \frac{2a^5b\left(a^2-3b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} - \frac{a^6b}{\left(a^2+b^2\right)^4 d(a+b\tan(c+dx))} - \frac{\cos^6(c+\ldots}{\ldots}$$

$$= \frac{\left(5a^8-80a^6b^2+50a^4b^4+8a^2b^6+b^8\right)x}{16\left(a^2+b^2\right)^5} + \frac{2a^5b\left(a^2-3b^2\right)\log(\cos(c+dx))}{\left(a^2+b^2\right)^5 d} + \frac{2a^5b\ldots}{\ldots}$$

Mathematica [B] time = 6.55, size = 603, normalized size = 2.03

$$b\left(-\frac{a\cos^6(c+dx)}{3(a^2+b^2)^2}+\frac{a(3a^2+b^2)\cos^4(c+dx)}{2(a^2+b^2)^3}-\frac{(a^2-b^2)\sin(c+dx)\cos^5(c+dx)}{6b(a^2+b^2)^2}-\frac{5(a^2-b^2)\left(3b^2\left(\frac{\tan^{-1}(\tan(c+dx))}{b^3}+\frac{\sin(c+dx)\cos(c+dx)}{b^3}\right)+\frac{2\sin(c+dx)\cos}{b}\right.}{48(a^2+b^2)^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b*(-1/2*((3*a^6 - 6*a^4*b^2 - 4*a^2*b^4 - b^6)*ArcTan[Tan[c + d*x]])/(b*(a
^2 + b^2)^4) - (3*a^5*Cos[c + d*x]^2)/(a^2 + b^2)^4 + (a*(3*a^2 + b^2)*Cos[
c + d*x]^4)/(2*(a^2 + b^2)^3) - (a*Cos[c + d*x]^6)/(3*(a^2 + b^2)^2) - (a^5
*(2*a^2 - 6*b^2 - (a^3 - 7*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*
x]])/(2*(a^2 + b^2)^5) + (2*a^5*(a^2 - 3*b^2)*Log[a + b*Tan[c + d*x]])/(a^2
 + b^2)^5 - (a^5*(2*a^2 - 6*b^2 + (a^3 - 7*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2
] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^5) - ((3*a^6 - 6*a^4*b^2 - 4*a^2*b^4 -
b^6)*Cos[c + d*x]*Sin[c + d*x])/(2*b*(a^2 + b^2)^4) + ((3*a^4 - 3*a^2*b^2 -
 2*b^4)*Cos[c + d*x]^3*Sin[c + d*x])/(4*b*(a^2 + b^2)^3) - ((a^2 - b^2)*Cos
[c + d*x]^5*Sin[c + d*x])/(6*b*(a^2 + b^2)^2) + (3*(3*a^4 - 3*a^2*b^2 - 2*b
^4)*(ArcTan[Tan[c + d*x]]/b + (Cos[c + d*x]*Sin[c + d*x])/b))/(8*(a^2 + b^2
)^3) - (5*(a^2 - b^2)*((2*Cos[c + d*x]^3*Sin[c + d*x])/b + 3*b^2*(ArcTan[Ta
n[c + d*x]]/b^3 + (Cos[c + d*x]*Sin[c + d*x])/b^3)))/(48*(a^2 + b^2)^2) - a
^6/((a^2 + b^2)^4*(a + b*Tan[c + d*x]))))/d
```

fricas [B] time = 0.54, size = 619, normalized size = 2.08

$$-\frac{8\left(a^8b+4a^6b^3+6a^4b^5+4a^2b^7+b^9\right)\cos(dx+c)^7-2\left(19a^8b+68a^6b^3+90a^4b^5+52a^2b^7+11b^9\right)\cos(d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/48*(8*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^7 -
 2*(19*a^8*b + 68*a^6*b^3 + 90*a^4*b^5 + 52*a^2*b^7 + 11*b^9)*cos(d*x + c)^
5 + (85*a^8*b + 224*a^6*b^3 + 210*a^4*b^5 + 88*a^2*b^7 + 17*b^9)*cos(d*x +
c)^3 - (17*a^8*b + 72*a^6*b^3 + 120*a^4*b^5 + 20*a^2*b^7 + 3*b^9 + 3*(5*a^9
 - 80*a^7*b^2 + 50*a^5*b^4 + 8*a^3*b^6 + a*b^8)*d*x)*cos(d*x + c) - 48*((a^
8*b - 3*a^6*b^3)*cos(d*x + c) + (a^7*b^2 - 3*a^5*b^4)*sin(d*x + c))*log(2*a
*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (98*a^7*
b^2 + 24*a^5*b^4 - 30*a^3*b^6 - 4*a*b^8 - 8*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 +
4*a^3*b^6 + a*b^8)*cos(d*x + c)^6 + 2*(13*a^9 + 44*a^7*b^2 + 54*a^5*b^4 + 2
8*a^3*b^6 + 5*a*b^8)*cos(d*x + c)^4 + 3*(5*a^8*b - 80*a^6*b^3 + 50*a^4*b^5
+ 8*a^2*b^7 + b^9)*d*x - 3*(11*a^9 + 16*a^7*b^2 - 2*a^5*b^4 - 8*a^3*b^6 - a
*b^8)*cos(d*x + c)^2)*sin(d*x + c))/((a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^
5*b^6 + 5*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^10*b + 5*a^8*b^3 + 10*a^6*b
^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11)*d*sin(d*x + c))
```

giac [B] time = 1.53, size = 735, normalized size = 2.47

$$\frac{3\left(5a^8-80a^6b^2+50a^4b^4+8a^2b^6+b^8\right)(dx+c)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}}-\frac{48\left(a^7b-3a^5b^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}}+\frac{96\left(a^7b^2-3a^5b^4\right)\log(|b\tan(dx+c)+a|)}{a^{10}b+5a^8b^3+10a^6b^5+10a^4b^7+5a^2b^9+b^{11}}-\frac{48}{(a^{10}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*a^8 - 80*a^6*b^2 + 50*a^4*b^4 + 8*a^2*b^6 + b^8)*(d*x + c)/(a^10
 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - 48*(a^7*b - 3*
a^5*b^3)*log(tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^
6 + 5*a^2*b^8 + b^10) + 96*(a^7*b^2 - 3*a^5*b^4)*log(abs(b*tan(d*x + c) + a
))/(a^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11) - 48*(
2*a^7*b^2*tan(d*x + c) - 6*a^5*b^4*tan(d*x + c) + 3*a^8*b - 5*a^6*b^3)/((a^
10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*(b*tan(d*x + c
) + a)) + (88*a^7*b*tan(d*x + c)^6 - 264*a^5*b^3*tan(d*x + c)^6 - 33*a^8*ta
n(d*x + c)^5 + 96*a^6*b^2*tan(d*x + c)^5 + 150*a^4*b^4*tan(d*x + c)^5 + 24*
a^2*b^6*tan(d*x + c)^5 + 3*b^8*tan(d*x + c)^5 + 120*a^7*b*tan(d*x + c)^4 -
936*a^5*b^3*tan(d*x + c)^4 - 40*a^8*tan(d*x + c)^3 + 208*a^6*b^2*tan(d*x +
c)^3 + 240*a^4*b^4*tan(d*x + c)^3 - 16*a^2*b^6*tan(d*x + c)^3 - 8*b^8*tan(d
*x + c)^3 + 48*a^7*b*tan(d*x + c)^2 - 912*a^5*b^3*tan(d*x + c)^2 + 120*a^3*
b^5*tan(d*x + c)^2 + 24*a*b^7*tan(d*x + c)^2 - 15*a^8*tan(d*x + c) + 96*a^6
*b^2*tan(d*x + c) + 90*a^4*b^4*tan(d*x + c) - 24*a^2*b^6*tan(d*x + c) - 3*b
^8*tan(d*x + c) - 288*a^5*b^3 + 72*a^3*b^5 + 8*a*b^7)/((a^10 + 5*a^8*b^2 +
10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*(tan(d*x + c)^2 + 1)^3))/d
```

maple [B] time = 0.50, size = 1211, normalized size = 4.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x)
```

```
[Out] -a^6*b/(a^2+b^2)^4/d/(a+b*tan(d*x+c))+5/16/d/(a^2+b^2)^5*arctan(tan(d*x+c))
*a^8+1/16/d/(a^2+b^2)^5*arctan(tan(d*x+c))*b^8-1/d/(a^2+b^2)^5*ln(1+tan(d*x
+c)^2)*a^7*b-5/d/(a^2+b^2)^5*arctan(tan(d*x+c))*a^6*b^2-11/16/d/(a^2+b^2)^5
/(1+tan(d*x+c)^2)^3*tan(d*x+c)^5*a^8+1/16/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*
tan(d*x+c)^5*b^8-5/6/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^3*a^8-1/6/
d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^3*b^8-5/16/d/(a^2+b^2)^5/(1+tan
(d*x+c)^2)^3*tan(d*x+c)*a^8-1/16/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c
)*b^8+3/2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*a^3*b^5-1/2/d/(a^2+b^2)^5/(1+tan
(d*x+c)^2)^3*b^3*a^5+1/2/d/(a^2+b^2)^5*arctan(tan(d*x+c))*a^2*b^6-11/6/d/(a
^2+b^2)^5/(1+tan(d*x+c)^2)^3*a^7*b+1/6/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*a*b
^7+2/d*b*a^7/(a^2+b^2)^5*ln(a+b*tan(d*x+c))-6/d*b^3*a^5/(a^2+b^2)^5*ln(a+b*
tan(d*x+c))+25/8/d/(a^2+b^2)^5*arctan(tan(d*x+c))*a^4*b^4+3/d/(a^2+b^2)^5*l
n(1+tan(d*x+c)^2)*a^5*b^3-1/3/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^3
*a^2*b^6+1/2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^5*a^2*b^6+5/2/d/(a
^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^2*a^3*b^5+1/2/d/(a^2+b^2)^5/(1+tan(
d*x+c)^2)^3*tan(d*x+c)^2*a*b^7-5/2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x
+c)^2*b^3*a^5+13/3/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^3*a^6*b^2+5/
d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^3*a^4*b^4-9/2/d/(a^2+b^2)^5/(1+
tan(d*x+c)^2)^3*tan(d*x+c)^2*a^7*b+25/8/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*ta
n(d*x+c)^5*a^4*b^4-3/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^4*a^5*b^3+
2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^5*a^6*b^2-1/2/d/(a^2+b^2)^5/(
1+tan(d*x+c)^2)^3*tan(d*x+c)*a^2*b^6+2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan
(d*x+c)*a^6*b^2+15/8/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)*a^4*b^4-3/
d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^4*a^7*b
```

maxima [B] time = 0.96, size = 799, normalized size = 2.69

$$\frac{3\left(5a^8-80a^6b^2+50a^4b^4+8a^2b^6+b^8\right)(dx+c)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}}+\frac{96\left(a^7b-3a^5b^3\right)\log\left(b\tan(dx+c)+a\right)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}}-\frac{48\left(a^7b-3a^5b^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}}-\frac{}{a^9+4a^7b^2+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(5*a^8 - 80*a^6*b^2 + 50*a^4*b^4 + 8*a^2*b^6 + b^8)*(d*x + c)/(a^10
 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + 96*(a^7*b - 3*
a^5*b^3)*log(b*tan(d*x + c) + a)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^
6 + 5*a^2*b^8 + b^10) - 48*(a^7*b - 3*a^5*b^3)*log(tan(d*x + c)^2 + 1)/(a^1
0 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - (136*a^6*b -
64*a^4*b^3 - 8*a^2*b^5 + 3*(27*a^6*b - 43*a^4*b^3 - 7*a^2*b^5 - b^7)*tan(d*
x + c)^6 + 3*(11*a^7 + 5*a^5*b^2 - 7*a^3*b^4 - a*b^6)*tan(d*x + c)^5 + 8*(4
1*a^6*b - 31*a^4*b^3 + a^2*b^5 + b^7)*tan(d*x + c)^4 + 8*(5*a^7 - 4*a^5*b^2
 - 11*a^3*b^4 - 2*a*b^6)*tan(d*x + c)^3 + 3*(125*a^6*b - 69*a^4*b^3 - a^2*b
^5 + b^7)*tan(d*x + c)^2 + (15*a^7 - 23*a^5*b^2 - 43*a^3*b^4 - 5*a*b^6)*tan
(d*x + c))/(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8 + (a^8*b + 4*a^
6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*tan(d*x + c)^7 + (a^9 + 4*a^7*b^2 + 6*
a^5*b^4 + 4*a^3*b^6 + a*b^8)*tan(d*x + c)^6 + 3*(a^8*b + 4*a^6*b^3 + 6*a^4*
b^5 + 4*a^2*b^7 + b^9)*tan(d*x + c)^5 + 3*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*
a^3*b^6 + a*b^8)*tan(d*x + c)^4 + 3*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*
b^7 + b^9)*tan(d*x + c)^3 + 3*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*
b^8)*tan(d*x + c)^2 + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*tan
(d*x + c)))/d
```

mupad [B] time = 5.54, size = 757, normalized size = 2.55

$$\frac{\ln\left(a+b\tan\left(c+d\,x\right)\right)\left(\frac{2\,a\,b}{{\left(a^2+b^2\right)}^2}-\frac{12\,a\,b^3}{{\left(a^2+b^2\right)}^3}+\frac{18\,a\,b^5}{{\left(a^2+b^2\right)}^4}-\frac{8\,a\,b^7}{{\left(a^2+b^2\right)}^5}\right)}{d}+\frac{\frac{{\mathrm{tan}\left(c+d\,x\right)}^3\,\left(-5\,a^5+9\,a^3\,b^2+2\,a\,b^4\right)}{6\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{{\mathrm{tan}\left(c+d\,x\right)}^5\,\left(-11\,a^5+\dots\right.}{16\,\left(a^6+3\,a^4\,b^2+3\dots\right.}}{d\,\left(\dots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + b*tan(c + d*x))^2,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((2*a*b)/(a^2 + b^2)^2 - (12*a*b^3)/(a^2 + b^2)^3
+ (18*a*b^5)/(a^2 + b^2)^4 - (8*a*b^7)/(a^2 + b^2)^5))/d + ((tan(c + d*x)^3
*(2*a*b^4 - 5*a^5 + 9*a^3*b^2))/(6*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (
tan(c + d*x)^5*(a*b^4 - 11*a^5 + 6*a^3*b^2))/(16*(a^6 + b^6 + 3*a^2*b^4 + 3
*a^4*b^2)) + (tan(c + d*x)^6*(b^7 - 27*a^6*b + 7*a^2*b^5 + 43*a^4*b^3))/(16
*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (tan(c + d*x)*(5*a*b^4
- 15*a^5 + 38*a^3*b^2))/(48*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a*(a*b^
5 - 17*a^5*b + 8*a^3*b^3))/(6*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^
2)) - (tan(c + d*x)^4*(41*a^6*b + b^7 + a^2*b^5 - 31*a^4*b^3))/(6*(a^2 + b^
2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (tan(c + d*x)^2*(125*a^6*b + b^7
- a^2*b^5 - 69*a^4*b^3))/(16*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2
)))/(d*(a + b*tan(c + d*x) + 3*a*tan(c + d*x)^2 + 3*a*tan(c + d*x)^4 + a*ta
n(c + d*x)^6 + 3*b*tan(c + d*x)^3 + 3*b*tan(c + d*x)^5 + b*tan(c + d*x)^7))
 + (log(tan(c + d*x) + 1i)*(a*b^2*5i - 7*a^2*b + a^3*5i + b^3))/(32*d*(5*a*
b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)) - (log(tan(c + d
*x) - 1i)*(a*b^2*5i + 7*a^2*b + a^3*5i - b^3))/(32*d*(5*a*b^4 + a^4*b*5i +
a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.62 $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2} dx$

Optimal. Leaf size=217

$$\frac{\cos^4(c+dx)\left(\left(a^2-b^2\right)\tan(c+dx)+2ab\right)}{4d\left(a^2+b^2\right)^2}-\frac{a^4b}{d\left(a^2+b^2\right)^3(a+b\tan(c+dx))}+\frac{2a^3b\left(a^2-2b^2\right)\log(a\cos(c+dx)+}{d\left(a^2+b^2\right)^4}$$

```
[Out] 1/8*(3*a^6-33*a^4*b^2+13*a^2*b^4+b^6)*x/(a^2+b^2)^4+2*a^3*b*(a^2-2*b^2)*ln(
a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^4/d-a^4*b/(a^2+b^2)^3/d/(a+b*tan(d*x+c
))+1/4*cos(d*x+c)^4*(2*a*b+(a^2-b^2)*tan(d*x+c))/(a^2+b^2)^2/d-1/8*cos(d*x+
c)^2*(16*a^3*b+(5*a^4-12*a^2*b^2-b^4)*tan(d*x+c))/(a^2+b^2)^3/d
```

Rubi [A] time = 0.56, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$-\frac{a^4b}{d\left(a^2+b^2\right)^3(a+b\tan(c+dx))}+\frac{\cos^4(c+dx)\left(\left(a^2-b^2\right)\tan(c+dx)+2ab\right)}{4d\left(a^2+b^2\right)^2}-\frac{\cos^2(c+dx)\left(\left(-12a^2b^2+5a^4-b^4\right.\right.}{8d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((3*a^6 - 33*a^4*b^2 + 13*a^2*b^4 + b^6)*x)/(8*(a^2 + b^2)^4) + (2*a^3*b*(a
^2 - 2*b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) - (a^4*
b)/((a^2 + b^2)^3*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^4*(2*a*b + (a^2 -
 b^2)*Tan[c + d*x]))/(4*(a^2 + b^2)^2*d) - (Cos[c + d*x]^2*(16*a^3*b + (5*a
^4 - 12*a^2*b^2 - b^4)*Tan[c + d*x]))/(8*(a^2 + b^2)^3*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
```

```
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{x^4}{(a+x)^2\left(b^2+x^2\right)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^4(c+dx)\left(2ab+\left(a^2-b^2\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^2 d} - \frac{\mathrm{Subst}\left(\int \frac{\frac{a^2b^4\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2} - \frac{2ab^4\left(3a^2+b^2\right)x}{\left(a^2+b^2\right)^2} - \frac{b^2(4a^4}{}}{(a+x)^2\left(b^2+x^2\right)^2}}{4bd}$$

$$= \frac{\cos^4(c+dx)\left(2ab+\left(a^2-b^2\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^2 d} - \frac{\cos^2(c+dx)\left(16a^3b+\left(5a^4-12a^2b\right.\right.}{8\left(a^2+b^2\right)^3 d}$$

$$= \frac{\cos^4(c+dx)\left(2ab+\left(a^2-b^2\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^2 d} - \frac{\cos^2(c+dx)\left(16a^3b+\left(5a^4-12a^2b\right.\right.}{8\left(a^2+b^2\right)^3 d}$$

$$= \frac{2a^3b\left(a^2-2b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^4 d} - \frac{a^4b}{\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))} + \frac{\cos^4(}{}$$

$$= \frac{2a^3b\left(a^2-2b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^4 d} - \frac{a^4b}{\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))} + \frac{\cos^4(}{}$$

$$= \frac{\left(3a^6-33a^4b^2+13a^2b^4+b^6\right)x}{8\left(a^2+b^2\right)^4} + \frac{2a^3b\left(a^2-2b^2\right)\log(\cos(c+dx))}{\left(a^2+b^2\right)^4 d} + \frac{2a^3b\left(a^2-\right.}{}$$

Mathematica [A] time = 3.99, size = 373, normalized size = 1.72

$$\frac{b\left(4a\left(a^2+b^2\right)^2\cos^4(c+dx) + \frac{3\left(a^2-b^2\right)\left(a^2+b^2\right)^2\left(\sin(2(c+dx))+2\tan^{-1}(\tan(c+dx))\right)}{2b} + \frac{2\left(a^2-b^2\right)\left(a^2+b^2\right)^2\sin(c+dx)\cos^3(c+dx)}{b} - \frac{8}{a+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b*((4*(a^2 + b^2)*(-2*a^4 + 3*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]])/b - 16*
a^3*(a^2 + b^2)*Cos[c + d*x]^2 + 4*a*(a^2 + b^2)^2*Cos[c + d*x]^4 - 4*a^3*(
```

```
2*a^2 - 4*b^2 + (-a^3 + 5*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x
]] + 16*a^3*(a^2 - 2*b^2)*Log[a + b*Tan[c + d*x]] - 4*a^3*(2*a^2 - 4*b^2 +
(a^3 - 5*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (2*(a^2 - b^
2)*(a^2 + b^2)^2*Cos[c + d*x]^3*Sin[c + d*x])/b + (2*(a^2 + b^2)*(-2*a^4 +
3*a^2*b^2 + b^4)*Sin[2*(c + d*x)])/b + (3*(a^2 - b^2)*(a^2 + b^2)^2*(2*ArcT
an[Tan[c + d*x]] + Sin[2*(c + d*x)]))/(2*b) - (8*a^4*(a^2 + b^2))/(a + b*Ta
n[c + d*x])))/(8*(a^2 + b^2)^4*d)
```

fricas [B] time = 0.52, size = 444, normalized size = 2.05

$$\frac{4\left(a^6b + 3\,a^4b^3 + 3\,a^2b^5 + b^7\right)\cos\left(dx + c\right)^5 - 6\left(3\,a^6b + 7\,a^4b^3 + 5\,a^2b^5 + b^7\right)\cos\left(dx + c\right)^3 + \left(3\,a^6b + 8\,a^4b^3 + 23\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^5 - 6*(3*a^6*b +
 7*a^4*b^3 + 5*a^2*b^5 + b^7)*cos(d*x + c)^3 + (3*a^6*b + 8*a^4*b^3 + 23*a^
2*b^5 + 2*b^7 + 2*(3*a^7 - 33*a^5*b^2 + 13*a^3*b^4 + a*b^6)*d*x)*cos(d*x +
c) + 16*((a^6*b - 2*a^4*b^3)*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4)*sin(d*x +
 c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2
) + (29*a^5*b^2 + 10*a^3*b^4 - 3*a*b^6 + 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a
*b^6)*cos(d*x + c)^4 + 2*(3*a^6*b - 33*a^4*b^3 + 13*a^2*b^5 + b^7)*d*x - 2*
(5*a^7 + 9*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^9
 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*d*cos(d*x + c) + (a^8*b + 4*a
^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*d*sin(d*x + c))
```

giac [B] time = 1.45, size = 513, normalized size = 2.36

$$\frac{\frac{\left(3\,a^6 - 33\,a^4b^2 + 13\,a^2b^4 + b^6\right)(dx+c)}{a^8 + 4\,a^6b^2 + 6\,a^4b^4 + 4\,a^2b^6 + b^8} - \frac{8\left(a^5b - 2\,a^3b^3\right)\log\left(\tan(dx+c)^2 + 1\right)}{a^8 + 4\,a^6b^2 + 6\,a^4b^4 + 4\,a^2b^6 + b^8} + \frac{16\left(a^5b^2 - 2\,a^3b^4\right)\log(|b\tan(dx+c) + a|)}{a^8b + 4\,a^6b^3 + 6\,a^4b^5 + 4\,a^2b^7 + b^9} - \frac{8\left(2\,a^5b^2\tan(dx+c) - 4\,a^3b^4\tan(\right.}{\left(a^8 + 4\,a^6b^2 + 6\,a^4b^4 + 4\,a^2b^6 + b\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*((3*a^6 - 33*a^4*b^2 + 13*a^2*b^4 + b^6)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6
*a^4*b^4 + 4*a^2*b^6 + b^8) - 8*(a^5*b - 2*a^3*b^3)*log(tan(d*x + c)^2 + 1)
/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 16*(a^5*b^2 - 2*a^3*b^4)
*log(abs(b*tan(d*x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 +
b^9) - 8*(2*a^5*b^2*tan(d*x + c) - 4*a^3*b^4*tan(d*x + c) + 3*a^6*b - 3*a^4
*b^3)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*x + c) + a)
) + (12*a^5*b*tan(d*x + c)^4 - 24*a^3*b^3*tan(d*x + c)^4 - 5*a^6*tan(d*x +
c)^3 + 7*a^4*b^2*tan(d*x + c)^3 + 13*a^2*b^4*tan(d*x + c)^3 + b^6*tan(d*x +
 c)^3 + 8*a^5*b*tan(d*x + c)^2 - 64*a^3*b^3*tan(d*x + c)^2 - 3*a^6*tan(d*x
+ c) + 9*a^4*b^2*tan(d*x + c) + 11*a^2*b^4*tan(d*x + c) - b^6*tan(d*x + c)
- 32*a^3*b^3 + 4*a*b^5)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(t
an(d*x + c)^2 + 1)^2))/d
```

maple [B] time = 0.48, size = 724, normalized size = 3.34

$$-\frac{a^4b}{\left(a^2+b^2\right)^3 d\left(a+b\tan\left(dx+c\right)\right)}+\frac{2b\,a^5\ln\left(a+b\tan\left(dx+c\right)\right)}{d\left(a^2+b^2\right)^4}-\frac{4b^3a^3\ln\left(a+b\tan\left(dx+c\right)\right)}{d\left(a^2+b^2\right)^4}-\frac{5\left(\tan^3\left(dx\right.\right.}{8d\left(a^2+b^2\right)^4\left(1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x)
```

```
[Out] -a^4*b/(a^2+b^2)^3/d/(a+b*tan(d*x+c))+2/d*b*a^5/(a^2+b^2)^4*ln(a+b*tan(d*x+
c))-4/d*b^3*a^3/(a^2+b^2)^4*ln(a+b*tan(d*x+c))-5/8/d/(a^2+b^2)^4/(1+tan(d*x
+c)^2)^2*tan(d*x+c)^3*a^6+7/8/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3
*a^4*b^2+13/8/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*b^4*a^2+1/8/d/(
a^2+b^2)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*b^6-2/d/(a^2+b^2)^4/(1+tan(d*x+c
)^2)^2*tan(d*x+c)^2*a^5*b-2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)^2*a
^3*b^3-3/8/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a^6+9/8/d/(a^2+b^2)^
4/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a^4*b^2+11/8/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)
^2*tan(d*x+c)*b^4*a^2-1/8/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)*b^6-3
/2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*a^5*b-1/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^
2*b^3*a^3+1/2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*a*b^5-1/d/(a^2+b^2)^4*ln(1+t
an(d*x+c)^2)*a^5*b+2/d/(a^2+b^2)^4*ln(1+tan(d*x+c)^2)*a^3*b^3+3/8/d/(a^2+b^
2)^4*arctan(tan(d*x+c))*a^6-33/8/d/(a^2+b^2)^4*arctan(tan(d*x+c))*a^4*b^2+1
3/8/d/(a^2+b^2)^4*arctan(tan(d*x+c))*b^4*a^2+1/8/d/(a^2+b^2)^4*arctan(tan(d
*x+c))*b^6
```

maxima [B] time = 0.92, size = 507, normalized size = 2.34

$$\frac{\frac{(3a^6-33a^4b^2+13a^2b^4+b^6)(dx+c)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}+\frac{16(a^5b-2a^3b^3)\log(b\tan(dx+c)+a)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}-\frac{8(a^5b-2a^3b^3)\log(\tan(dx+c)^2+1)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}-\frac{\cdots}{a^7+3a^5b^2+3a^3b^4+ab^6+(a^6b\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^6 - 33*a^4*b^2 + 13*a^2*b^4 + b^6)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6
*a^4*b^4 + 4*a^2*b^6 + b^8) + 16*(a^5*b - 2*a^3*b^3)*log(b*tan(d*x + c) + a
)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 8*(a^5*b - 2*a^3*b^3)*l
og(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - (2
0*a^4*b - 4*a^2*b^3 + (13*a^4*b - 12*a^2*b^3 - b^5)*tan(d*x + c)^4 + (5*a^5
 + 4*a^3*b^2 - a*b^4)*tan(d*x + c)^3 + (35*a^4*b - 12*a^2*b^3 + b^5)*tan(d*
x + c)^2 + 3*(a^5 - a*b^4)*tan(d*x + c))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b
^6 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(d*x + c)^5 + (a^7 + 3*a^5*b^
2 + 3*a^3*b^4 + a*b^6)*tan(d*x + c)^4 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
b^7)*tan(d*x + c)^3 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*tan(d*x + c)^
2 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(d*x + c)))/d
```

mupad [B] time = 4.82, size = 481, normalized size = 2.22

$$\frac{\frac{\tan(c+d\,x)^3\,(a\,b^2-5\,a^3)}{8\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan(c+d\,x)^4\,(-13\,a^4\,b+12\,a^2\,b^3+b^5)}{8\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}+\frac{3\tan(c+d\,x)\,(a\,b^2-a^3)}{8\,(a^4+2\,a^2\,b^2+b^4)}-\frac{\tan(c+d\,x)^2\,(35\,a^4\,b-12\,a^2\,b^3+b^5)}{8\,(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)}+\frac{a\,(a\,b^3-5\cdots}{2\,(a^2+b^2)\,(a^4+\cdots}}{d\,\left(b\tan(c+d\,x)^5+a\tan(c+d\,x)^4+2\,b\tan(c+d\,x)^3+2\,a\tan(c+d\,x)^2+b\tan(c+d\,x)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b*tan(c + d*x))^2,x)
```

```
[Out] ((tan(c + d*x)^3*(a*b^2 - 5*a^3))/(8*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c + d*
x)^4*(b^5 - 13*a^4*b + 12*a^2*b^3))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
 + (3*tan(c + d*x)*(a*b^2 - a^3))/(8*(a^4 + b^4 + 2*a^2*b^2)) - (tan(c + d*
x)^2*(35*a^4*b + b^5 - 12*a^2*b^3))/(8*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2))
 + (a*(a*b^3 - 5*a^3*b))/(2*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a + b
*tan(c + d*x) + 2*a*tan(c + d*x)^2 + a*tan(c + d*x)^4 + 2*b*tan(c + d*x)^3
+ b*tan(c + d*x)^5)) + (log(a + b*tan(c + d*x))*((2*a*b)/(a^2 + b^2)^2 - (8
*a*b^3)/(a^2 + b^2)^3 + (6*a*b^5)/(a^2 + b^2)^4))/d + (log(tan(c + d*x) - 1
i)*(3*a^2 - a*b*4i + b^2))/(16*d*(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2
*b^2*6i)) - (log(tan(c + d*x) + 1i)*(a*b*4i + 3*a^2 + b^2))/(16*d*(4*a^3*b
- 4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2*6i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.63 $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=148

$$-\frac{a^2b}{d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{\cos^2(c+dx)\left(\left(a^2-b^2\right)\tan(c+dx)+2ab\right)}{2d\left(a^2+b^2\right)^2}+\frac{2ab\left(a^2-b^2\right)\log(a\cos(c+dx)}{d\left(a^2+b^2\right)^3}$$

```
[Out] 1/2*(a^4-6*a^2*b^2+b^4)*x/(a^2+b^2)^3+2*a*b*(a^2-b^2)*ln(a*cos(d*x+c)+b*sin
(d*x+c))/(a^2+b^2)^3/d-a^2*b/(a^2+b^2)^2/d/(a+b*tan(d*x+c))-1/2*cos(d*x+c)^
2*(2*a*b+(a^2-b^2)*tan(d*x+c))/(a^2+b^2)^2/d
```

Rubi [A] time = 0.29, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$-\frac{a^2b}{d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{\cos^2(c+dx)\left(\left(a^2-b^2\right)\tan(c+dx)+2ab\right)}{2d\left(a^2+b^2\right)^2}+\frac{2ab\left(a^2-b^2\right)\log(a\cos(c+dx)}{d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^4 - 6*a^2*b^2 + b^4)*x)/(2*(a^2 + b^2)^3) + (2*a*b*(a^2 - b^2)*Log[a*Co
s[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d) - (a^2*b)/((a^2 + b^2)^2*d*
(a + b*Tan[c + d*x])) - (Cos[c + d*x]^2*(2*a*b + (a^2 - b^2)*Tan[c + d*x]))
/(2*(a^2 + b^2)^2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
```

```
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^2}{(a+x)^2\left(b^2+x^2\right)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^2(c+dx)\left(2ab+\left(a^2-b^2\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^2 d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^2\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2}+\frac{2ab^2x}{a^2+b^2}+\frac{b^2\left(a^2-b^2\right)x^2}{\left(a^2+b^2\right)^2}}{(a+x)^2\left(b^2+x^2\right)}\,d\right.}{2bd}$$

$$= -\frac{\cos^2(c+dx)\left(2ab+\left(a^2-b^2\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^2 d} - \frac{\text{Subst}\left(\int\left(-\frac{2a^2b^2}{\left(a^2+b^2\right)^2(a+x)^2}+\frac{4ab^2\left(-a^2+b\right.}{\left(a^2+b^2\right)^3(a+}\right.\right.}{}$$

$$= \frac{2ab\left(a^2-b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3 d} - \frac{a^2b}{\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx}{}$$

$$= \frac{2ab\left(a^2-b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3 d} - \frac{a^2b}{\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx}{}$$

$$= \frac{\left(a^4-6a^2b^2+b^4\right)x}{2\left(a^2+b^2\right)^3} + \frac{2ab\left(a^2-b^2\right)\log(\cos(c+dx))}{\left(a^2+b^2\right)^3 d} + \frac{2ab\left(a^2-b^2\right)\log(a+b\tan(}{\left(a^2+b^2\right)^3 d}$$

Mathematica [A] time = 3.39, size = 246, normalized size = 1.66

$$-\frac{b\left(\frac{(a-b)(a+b)\left(a^2+b^2\right)\sin(2(c+dx))}{2b}+2a\left(a^2+b^2\right)\cos^2(c+dx)+\frac{\left(a^2-b^2\right)\left(a^2+b^2\right)\tan^{-1}(\tan(c+dx))}{b}+\frac{2a^2\left(a^2+b^2\right)}{a+b\tan(c+dx)}+a\left(\frac{3ab^2-a^3}{\sqrt{-b^2}}\right.\right.+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/2*(b*(((a^2 - b^2)*(a^2 + b^2)*ArcTan[Tan[c + d*x]])/b + 2*a*(a^2 + b^2)
*Cos[c + d*x]^2 + a*(2*a^2 - 2*b^2 + (-a^3 + 3*a*b^2)/Sqrt[-b^2])*Log[Sqrt[
-b^2] - b*Tan[c + d*x]] - 4*a*(a - b)*(a + b)*Log[a + b*Tan[c + d*x]] + a*(
2*a^2 - 2*b^2 + (a^3 - 3*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]
] + ((a - b)*(a + b)*(a^2 + b^2)*Sin[2*(c + d*x)])/(2*b) + (2*a^2*(a^2 + b^
2))/(a + b*Tan[c + d*x])))/((a^2 + b^2)^3*d)
```

fricas [B] time = 0.47, size = 292, normalized size = 1.97

$$-\frac{\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^3+\left(a^2b^3-b^5-\left(a^5-6\,a^3b^2+ab^4\right)dx\right)\cos\left(dx+c\right)-2\left(\left(a^4b-a^2b^3\right)\cos\left(dx+\right.\right.}{2\left(\left(a^7+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 + (a^2*b^3 - b^5 - (a^5 - 6*
a^3*b^2 + a*b^4)*d*x)*cos(d*x + c) - 2*((a^4*b - a^2*b^3)*cos(d*x + c) + (a
^3*b^2 - a*b^4)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
b^2)*cos(d*x + c)^2 + b^2) - (3*a^3*b^2 + a*b^4 + (a^4*b - 6*a^2*b^3 + b^5)
*d*x - (a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 + 3*a^
5*b^2 + 3*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5
+ b^7)*d*sin(d*x + c))
```

giac [A] time = 0.89, size = 263, normalized size = 1.78

$$\frac{\frac{(a^4-6a^2b^2+b^4)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{2(a^3b-ab^3)\log(\tan(dx+c)^2+1)}{a^6+3a^4b^2+3a^2b^4+b^6} + \frac{4(a^3b^2-ab^4)\log(|b\tan(dx+c)+a|)}{a^6b+3a^4b^3+3a^2b^5+b^7} - \frac{3a^2b\tan(dx+c)^2-b^3\tan(dx+c)^2+a^3\tan(dx+c}{(a^4+2a^2b^2+b^4)(b\tan(dx+c)^3+a\tan(dx+c}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*((a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
- 2*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6) + 4*(a^3*b^2 - a*b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3
+ 3*a^2*b^5 + b^7) - (3*a^2*b*tan(d*x + c)^2 - b^3*tan(d*x + c)^2 + a^3*tan
(d*x + c) + a*b^2*tan(d*x + c) + 4*a^2*b)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(d
*x + c)^3 + a*tan(d*x + c)^2 + b*tan(d*x + c) + a)))/d
```

maple [B] time = 0.46, size = 352, normalized size = 2.38

$$-\frac{a^2b}{(a^2+b^2)^2 d(a+b\tan(dx+c))} + \frac{2a^3b\ln(a+b\tan(dx+c))}{d(a^2+b^2)^3} - \frac{2ab^3\ln(a+b\tan(dx+c))}{d(a^2+b^2)^3} - \frac{\tan(dx}{2d(a^2+b^2)^3(1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x)
```

```
[Out] -a^2*b/(a^2+b^2)^2/d/(a+b*tan(d*x+c))+2/d*a^3*b/(a^2+b^2)^3*ln(a+b*tan(d*x+
c))-2/d*a*b^3/(a^2+b^2)^3*ln(a+b*tan(d*x+c))-1/2/d/(a^2+b^2)^3/(1+tan(d*x+c
)^2)*tan(d*x+c)*a^4+1/2/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)*tan(d*x+c)*b^4-1/d/(
a^2+b^2)^3/(1+tan(d*x+c)^2)*a^3*b-1/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)*a*b^3-1/
d/(a^2+b^2)^3*ln(1+tan(d*x+c)^2)*a^3*b+1/d/(a^2+b^2)^3*ln(1+tan(d*x+c)^2)*a
*b^3-3/d/(a^2+b^2)^3*arctan(tan(d*x+c))*a^2*b^2+1/2/d/(a^2+b^2)^3*arctan(ta
n(d*x+c))*b^4+1/2/d/(a^2+b^2)^3*arctan(tan(d*x+c))*a^4
```

maxima [B] time = 0.81, size = 293, normalized size = 1.98

$$\frac{\frac{(a^4-6a^2b^2+b^4)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6} + \frac{4(a^3b-ab^3)\log(b\tan(dx+c)+a)}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{2(a^3b-ab^3)\log(\tan(dx+c)^2+1)}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{4a^2b+(3a^2b-b^3)\tan}{a^5+2a^3b^2+ab^4+(a^4b+2a^2b^3+b^5)\tan(dx+c)^3+(}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
+ 4*(a^3*b - a*b^3)*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6) - 2*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6) - (4*a^2*b + (3*a^2*b - b^3)*tan(d*x + c)^2 + (a^3 + a*b^2)*tan(d
*x + c))/(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*x + c)^
```

```
3 + (a^5 + 2*a^3*b^2 + a*b^4)*tan(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*ta
n(d*x + c)))/d
```

mupad [B] time = 4.06, size = 255, normalized size = 1.72

$$\frac{\ln(a+b\tan(c+d\,x))\left(\frac{2\,a\,b}{\left(a^2+b^2\right)^2}-\frac{4\,a\,b^3}{\left(a^2+b^2\right)^3}\right)}{d}-\frac{\frac{\tan(c+d\,x)^2\left(3\,a^2\,b-b^3\right)}{2\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{a\tan(c+d\,x)}{2\left(a^2+b^2\right)}+\frac{2\,a^2\,b}{\left(a^2+b^2\right)^2}}{d\left(b\tan(c+d\,x)^3+a\tan(c+d\,x)^2+b\tan(c+d\,x)+a\right)}+\frac{\ln(\tan}{4\,d\,\left(-a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b*tan(c + d*x))^2,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((2*a*b)/(a^2 + b^2)^2 - (4*a*b^3)/(a^2 + b^2)^3))
/d - ((tan(c + d*x)^2*(3*a^2*b - b^3))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (a*tan
(c + d*x))/(2*(a^2 + b^2)) + (2*a^2*b)/(a^2 + b^2)^2)/(d*(a + b*tan(c + d*x
) + a*tan(c + d*x)^2 + b*tan(c + d*x)^3)) + (log(tan(c + d*x) + 1i)*(a + b*
1i))/(4*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) + (log(tan(c + d*x) - 1i)*(a
*1i + b))/(4*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.64 $\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=72

$$-\frac{2b\log(\tan(c+dx))}{a^3d}+\frac{2b\log(a+b\tan(c+dx))}{a^3d}-\frac{b}{a^2d(a+b\tan(c+dx))}-\frac{\cot(c+dx)}{a^2d}$$

```
[Out] -cot(d*x+c)/a^2/d-2*b*ln(tan(d*x+c))/a^3/d+2*b*ln(a+b*tan(d*x+c))/a^3/d-b/a
^2/d/(a+b*tan(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 44}

$$-\frac{b}{a^2d(a+b\tan(c+dx))}-\frac{2b\log(\tan(c+dx))}{a^3d}+\frac{2b\log(a+b\tan(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(Cot[c + d*x]/(a^2*d)) - (2*b*Log[Tan[c + d*x]])/(a^3*d) + (2*b*Log[a + b*
Tan[c + d*x]])/(a^3*d) - b/(a^2*d*(a + b*Tan[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= \frac{b\,\text{Subst}\left(\int \frac{1}{x^2(a+x)^2}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\,\text{Subst}\left(\int \left(\frac{1}{a^2x^2}-\frac{2}{a^3x}+\frac{1}{a^2(a+x)^2}+\frac{2}{a^3(a+x)}\right)dx,x,b\tan(c+dx)\right)}{d}\\ &= -\frac{\cot(c+dx)}{a^2d}-\frac{2b\log(\tan(c+dx))}{a^3d}+\frac{2b\log(a+b\tan(c+dx))}{a^3d}-\frac{b}{a^2d(a+b\tan(\end{aligned}$$

Mathematica [A] time = 0.40, size = 109, normalized size = 1.51

$$\frac{-a^2\cot^2(c+dx)+b^2(2\log(a\cos(c+dx)+b\sin(c+dx))-2\log(\sin(c+dx))+1)-ab\cot(c+dx)(-2\log(a}{a^3d(a\cot(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-(a^2*Cot[c + d*x]^2) - a*b*Cot[c + d*x]*(1 + 2*Log[Sin[c + d*x]] - 2*Log[
a*Cos[c + d*x] + b*Sin[c + d*x]]) + b^2*(1 - 2*Log[Sin[c + d*x]] + 2*Log[a*
Cos[c + d*x] + b*Sin[c + d*x]]))/(a^3*d*(b + a*Cot[c + d*x]))
```

fricas [B] time = 0.46, size = 293, normalized size = 4.07

$$-\frac{a^2b^2 - \left(a^4 + 2\,a^2b^2\right)\cos\left(dx+c\right)^2 - \left(a^3b + 2\,ab^3\right)\cos\left(dx+c\right)\sin\left(dx+c\right) + \left(a^2b^2 + b^4 - \left(a^2b^2 + b^4\right)\cos\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*b^2 - (a^4 + 2*a^2*b^2)*cos(d*x + c)^2 - (a^3*b + 2*a*b^3)*cos(d*x +
c)*sin(d*x + c) + (a^2*b^2 + b^4 - (a^2*b^2 + b^4)*cos(d*x + c)^2 + (a^3*b
+ a*b^3)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (
a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*b^2 + b^4 - (a^2*b^2 + b^4)*cos(d*x
 + c)^2 + (a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(-1/4*cos(d*x + c)^
2 + 1/4))/((a^5*b + a^3*b^3)*d*cos(d*x + c)^2 - (a^6 + a^4*b^2)*d*cos(d*x +
 c)*sin(d*x + c) - (a^5*b + a^3*b^3)*d)
```

giac [A] time = 0.89, size = 74, normalized size = 1.03

$$\frac{\frac{2\,b\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^3} - \frac{2\,b\log\left(\left|\tan\left(dx+c\right)\right|\right)}{a^3} - \frac{2\,b\tan\left(dx+c\right)+a}{\left(b\tan\left(dx+c\right)^2+a\tan\left(dx+c\right)\right)a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*b*log(abs(b*tan(d*x + c) + a))/a^3 - 2*b*log(abs(tan(d*x + c)))/a^3 - (2
*b*tan(d*x + c) + a)/((b*tan(d*x + c)^2 + a*tan(d*x + c))*a^2))/d
```

maple [A] time = 0.47, size = 75, normalized size = 1.04

$$-\frac{b}{a^2d\left(a+b\tan\left(dx+c\right)\right)} + \frac{2b\ln\left(a+b\tan\left(dx+c\right)\right)}{a^3d} - \frac{1}{d\,a^2\tan\left(dx+c\right)} - \frac{2b\ln\left(\tan\left(dx+c\right)\right)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x)
```

```
[Out] -b/a^2/d/(a+b*tan(d*x+c))+2*b*ln(a+b*tan(d*x+c))/a^3/d-1/d/a^2/tan(d*x+c)-2
*b*ln(tan(d*x+c))/a^3/d
```

maxima [A] time = 0.57, size = 74, normalized size = 1.03

$$-\frac{\frac{2\,b\tan\left(dx+c\right)+a}{a^2b\tan\left(dx+c\right)^2+a^3\tan\left(dx+c\right)} - \frac{2\,b\log\left(b\tan\left(dx+c\right)+a\right)}{a^3} + \frac{2\,b\log\left(\tan\left(dx+c\right)\right)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((2*b*tan(d*x + c) + a)/(a^2*b*tan(d*x + c)^2 + a^3*tan(d*x + c)) - 2*b*lo
g(b*tan(d*x + c) + a)/a^3 + 2*b*log(tan(d*x + c))/a^3)/d
```

mupad [B] time = 3.84, size = 79, normalized size = 1.10

$$\frac{2\,b\,\ln\left(\frac{a+b\tan\left(c+d\,x\right)}{\tan\left(c+d\,x\right)}\right)}{a^3\,d} - \frac{2\,b}{a^2\,d\,\left(a+b\tan\left(c+d\,x\right)\right)} - \frac{1}{a\,d\tan\left(c+d\,x\right)\,\left(a+b\tan\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b*tan(c + d*x))^2),x)
```

```
[Out] (2*b*log((a + b*tan(c + d*x))/tan(c + d*x)))/(a^3*d) - (2*b)/(a^2*d*(a + b*
tan(c + d*x))) - 1/(a*d*tan(c + d*x)*(a + b*tan(c + d*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*tan(c + d*x))**2, x)
```

3.65 $\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=140

$$\frac{b\cot^2(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{2b\left(a^2+2b^2\right)\log(\tan(c+dx))}{a^5d}+\frac{2b\left(a^2+2b^2\right)\log(a+b\tan(c+dx))}{a^5d}-\frac{b\left(a^2+\right.}{a^4d(a+b\tan}$$

```
[Out] -(a^2+3*b^2)*cot(d*x+c)/a^4/d+b*cot(d*x+c)^2/a^3/d-1/3*cot(d*x+c)^3/a^2/d-2
*b*(a^2+2*b^2)*ln(tan(d*x+c))/a^5/d+2*b*(a^2+2*b^2)*ln(a+b*tan(d*x+c))/a^5/
d-b*(a^2+b^2)/a^4/d/(a+b*tan(d*x+c))
```

Rubi [A] time = 0.12, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 894}

$$-\frac{b\left(a^2+b^2\right)}{a^4d(a+b\tan(c+dx))}-\frac{\left(a^2+3b^2\right)\cot(c+dx)}{a^4d}-\frac{2b\left(a^2+2b^2\right)\log(\tan(c+dx))}{a^5d}+\frac{2b\left(a^2+2b^2\right)\log(a+b\tan(c+\right.}{a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 + 3*b^2)*Cot[c + d*x])/(a^4*d)) + (b*Cot[c + d*x]^2)/(a^3*d) - Cot[
c + d*x]^3/(3*a^2*d) - (2*b*(a^2 + 2*b^2)*Log[Tan[c + d*x]])/(a^5*d) + (2*b
*(a^2 + 2*b^2)*Log[a + b*Tan[c + d*x]])/(a^5*d) - (b*(a^2 + b^2))/(a^4*d*(a
 + b*Tan[c + d*x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= \frac{b\,\text{Subst}\left(\int \frac{b^2+x^2}{x^4(a+x)^2}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\,\text{Subst}\left(\int \left(\frac{b^2}{a^2x^4}-\frac{2b^2}{a^3x^3}+\frac{a^2+3b^2}{a^4x^2}-\frac{2\left(a^2+2b^2\right)}{a^5x}+\frac{a^2+b^2}{a^4(a+x)^2}+\frac{2\left(a^2+2b^2\right)}{a^5(a+x)}\right)dx,x,b\tan(c+dx\right.}{d}\\ &= -\frac{\left(a^2+3b^2\right)\cot(c+dx)}{a^4d}+\frac{b\cot^2(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{2b\left(a^2+2b^2\right)\log(\tan(c\right.}{a^5d}\end{aligned}$$

Mathematica [A] time = 2.66, size = 244, normalized size = 1.74

$$\frac{3b^2\left(-2\left(a^2+2b^2\right)\log(\sin(c+dx))+2a^2\log(a\cos(c+dx)+b\sin(c+dx))+a^2\csc^2(c+dx)+a^2+4b^2\log(a\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-(Cot[c + d*x]^2*(2*a^4 + 9*a^2*b^2 + a^4*Csc[c + d*x]^2)) + 3*b^2*(a^2 +
b^2 + a^2*Csc[c + d*x]^2 - 2*(a^2 + 2*b^2)*Log[Sin[c + d*x]] + 2*a^2*Log[a*
Cos[c + d*x] + b*Sin[c + d*x]] + 4*b^2*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]
) + a*b*Cot[c + d*x]*(-2*a^2 - 9*b^2 + 2*a^2*Csc[c + d*x]^2 - 6*(a^2 + 2*b^
2)*Log[Sin[c + d*x]] + 6*a^2*Log[a*Cos[c + d*x] + b*Sin[c + d*x]] + 12*b^2*
Log[a*Cos[c + d*x] + b*Sin[c + d*x]]))/(3*a^5*d*(b + a*Cot[c + d*x]))
```

fricas [B] time = 0.50, size = 442, normalized size = 3.16

$$\frac{2\left(a^{4}+6\,a^{2}b^{2}\right)\cos\left(dx+c\right)^{4}+6\,a^{2}b^{2}-3\left(a^{4}+6\,a^{2}b^{2}\right)\cos\left(dx+c\right)^{2}+3\left(\left(a^{2}b^{2}+2\,b^{4}\right)\cos\left(dx+c\right)^{4}+a^{2}b^{2}+2\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*(a^4 + 6*a^2*b^2)*cos(d*x + c)^4 + 6*a^2*b^2 - 3*(a^4 + 6*a^2*b^2)*c
os(d*x + c)^2 + 3*((a^2*b^2 + 2*b^4)*cos(d*x + c)^4 + a^2*b^2 + 2*b^4 - 2*(
a^2*b^2 + 2*b^4)*cos(d*x + c)^2 - ((a^3*b + 2*a*b^3)*cos(d*x + c)^3 - (a^3*
b + 2*a*b^3)*cos(d*x + c))*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c
) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 3*((a^2*b^2 + 2*b^4)*cos(d*x + c)^4
 + a^2*b^2 + 2*b^4 - 2*(a^2*b^2 + 2*b^4)*cos(d*x + c)^2 - ((a^3*b + 2*a*b^3
)*cos(d*x + c)^3 - (a^3*b + 2*a*b^3)*cos(d*x + c))*sin(d*x + c))*log(-1/4*c
os(d*x + c)^2 + 1/4) - 2*(6*a*b^3*cos(d*x + c) - (a^3*b + 6*a*b^3)*cos(d*x
+ c)^3)*sin(d*x + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*b*d*cos(d*x + c)^2 +
a^5*b*d - (a^6*d*cos(d*x + c)^3 - a^6*d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 0.80, size = 203, normalized size = 1.45

$$-\frac{\frac{6\left(a^{2}b+2\,b^{3}\right)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{a^{5}}-\frac{6\left(a^{2}b^{2}+2\,b^{4}\right)\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^{5}b}+\frac{3\left(2\,a^{2}b^{2}\tan\left(dx+c\right)+4\,b^{4}\tan\left(dx+c\right)+3\,a^{3}b+5\,ab^{3}\right)}{\left(b\tan\left(dx+c\right)+a\right)a^{5}}-\frac{11\,a^{2}b\tan\left(dx+c\right)^{3}\cdots}{\cdots}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(6*(a^2*b + 2*b^3)*log(abs(tan(d*x + c)))/a^5 - 6*(a^2*b^2 + 2*b^4)*lo
g(abs(b*tan(d*x + c) + a))/(a^5*b) + 3*(2*a^2*b^2*tan(d*x + c) + 4*b^4*tan(
d*x + c) + 3*a^3*b + 5*a*b^3)/((b*tan(d*x + c) + a)*a^5) - (11*a^2*b*tan(d*
x + c)^3 + 22*b^3*tan(d*x + c)^3 - 3*a^3*tan(d*x + c)^2 - 9*a*b^2*tan(d*x +
 c)^2 + 3*a^2*b*tan(d*x + c) - a^3)/(a^5*tan(d*x + c)^3))/d
```

maple [A] time = 0.51, size = 189, normalized size = 1.35

$$-\frac{b}{a^{2}d\left(a+b\tan\left(dx+c\right)\right)}-\frac{b^{3}}{d\,a^{4}\left(a+b\tan\left(dx+c\right)\right)}+\frac{2b\ln\left(a+b\tan\left(dx+c\right)\right)}{a^{3}d}+\frac{4b^{3}\ln\left(a+b\tan\left(dx+c\right)\right)}{d\,a^{5}}-\frac{\cdots}{3d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x)
```

```
[Out] -b/a^2/d/(a+b*tan(d*x+c))-1/d*b^3/a^4/(a+b*tan(d*x+c))+2*b*ln(a+b*tan(d*x+c
))/a^3/d+4/d*b^3/a^5*ln(a+b*tan(d*x+c))-1/3/d/a^2/tan(d*x+c)^3-1/d/a^2/tan(
d*x+c)-3/d/a^4/tan(d*x+c)*b^2+1/d/a^3*b/tan(d*x+c)^2-2*b*ln(tan(d*x+c))/a^3
/d-4/d*b^3/a^5*ln(tan(d*x+c))
```

maxima [A] time = 0.42, size = 144, normalized size = 1.03

$$\frac{\frac{2\,a^2b\tan(dx+c)-6\left(a^2b+2\,b^3\right)\tan(dx+c)^3-a^3-3\left(a^3+2\,ab^2\right)\tan(dx+c)^2}{a^4b\tan(dx+c)^4+a^5\tan(dx+c)^3}+\frac{6\left(a^2b+2\,b^3\right)\log(b\tan(dx+c)+a)}{a^5}-\frac{6\left(a^2b+2\,b^3\right)\log(\tan(dx+c))}{a^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((2*a^2*b*tan(d*x + c) - 6*(a^2*b + 2*b^3)*tan(d*x + c)^3 - a^3 - 3*(a^
3 + 2*a*b^2)*tan(d*x + c)^2)/(a^4*b*tan(d*x + c)^4 + a^5*tan(d*x + c)^3) +
6*(a^2*b + 2*b^3)*log(b*tan(d*x + c) + a)/a^5 - 6*(a^2*b + 2*b^3)*log(tan(d
*x + c))/a^5)/d
```

mupad [B] time = 3.91, size = 150, normalized size = 1.07

$$\frac{4\,b\,\mathrm{atanh}\left(\frac{2\,b\,\left(a^2+2\,b^2\right)\,(a+2\,b\tan(c+d\,x))}{a\,\left(2\,a^2\,b+4\,b^3\right)}\right)\,\left(a^2+2\,b^2\right)}{a^5\,d}-\frac{\frac{1}{3\,a}+\frac{\tan(c+d\,x)^2\,\left(a^2+2\,b^2\right)}{a^3}-\frac{2\,b\tan(c+d\,x)}{3\,a^2}+\frac{2\,b\tan(c+d\,x)^3\,\left(a^2+2\,b^2\right)}{a^4}}{d\,\left(b\tan\left(c+d\,x\right)^4+a\tan\left(c+d\,x\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + b*tan(c + d*x))^2),x)
```

```
[Out] (4*b*atanh((2*b*(a^2 + 2*b^2)*(a + 2*b*tan(c + d*x)))/(a*(2*a^2*b + 4*b^3))
)*(a^2 + 2*b^2))/(a^5*d) - (1/(3*a) + (tan(c + d*x)^2*(a^2 + 2*b^2))/a^3 -
(2*b*tan(c + d*x))/(3*a^2) + (2*b*tan(c + d*x)^3*(a^2 + 2*b^2))/a^4)/(d*(a*
tan(c + d*x)^3 + b*tan(c + d*x)^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*tan(c + d*x))**2, x)
```

3.66 $\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=219

$$\frac{b\cot^4(c+dx)}{2a^3d}-\frac{\cot^5(c+dx)}{5a^2d}-\frac{2b\left(a^2+b^2\right)\left(a^2+3b^2\right)\log(\tan(c+dx))}{a^7d}+\frac{2b\left(a^2+b^2\right)\left(a^2+3b^2\right)\log(a+b\tan(\cdots}{a^7d}$$

```
[Out] -(a^2+b^2)*(a^2+5*b^2)*cot(d*x+c)/a^6/d+2*b*(a^2+b^2)*cot(d*x+c)^2/a^5/d-1/
3*(2*a^2+3*b^2)*cot(d*x+c)^3/a^4/d+1/2*b*cot(d*x+c)^4/a^3/d-1/5*cot(d*x+c)^
5/a^2/d-2*b*(a^2+b^2)*(a^2+3*b^2)*ln(tan(d*x+c))/a^7/d+2*b*(a^2+b^2)*(a^2+3
*b^2)*ln(a+b*tan(d*x+c))/a^7/d-b*(a^2+b^2)^2/a^6/d/(a+b*tan(d*x+c))
```

Rubi [A] time = 0.20, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 894}

$$-\frac{b\left(a^2+b^2\right)^2}{a^6d(a+b\tan(c+dx))}-\frac{\left(2a^2+3b^2\right)\cot^3(c+dx)}{3a^4d}+\frac{2b\left(a^2+b^2\right)\cot^2(c+dx)}{a^5d}-\frac{\left(a^2+b^2\right)\left(a^2+5b^2\right)\cot(c+dx\cdots}{a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 + b^2)*(a^2 + 5*b^2)*Cot[c + d*x])/(a^6*d)) + (2*b*(a^2 + b^2)*Cot[
c + d*x]^2)/(a^5*d) - ((2*a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*a^4*d) + (b*Cot[c
 + d*x]^4)/(2*a^3*d) - Cot[c + d*x]^5/(5*a^2*d) - (2*b*(a^2 + b^2)*(a^2 + 3
*b^2)*Log[Tan[c + d*x]])/(a^7*d) + (2*b*(a^2 + b^2)*(a^2 + 3*b^2)*Log[a + b
*Tan[c + d*x]])/(a^7*d) - (b*(a^2 + b^2)^2)/(a^6*d*(a + b*Tan[c + d*x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= \frac{b\,\text{Subst}\left(\int \frac{\left(b^2+x^2\right)^2}{x^6(a+x)^2}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\,\text{Subst}\left(\int \left(\frac{b^4}{a^2x^6}-\frac{2b^4}{a^3x^5}+\frac{2a^2b^2+3b^4}{a^4x^4}-\frac{4b^2\left(a^2+b^2\right)}{a^5x^3}+\frac{a^4+6a^2b^2+5b^4}{a^6x^2}-\frac{2\left(a^4+4a^2b^2+3b^4\right)}{a^7x}+\cdots\right.\right.}{d}\\ &= -\frac{\left(a^2+b^2\right)\left(a^2+5b^2\right)\cot(c+dx)}{a^6d}+\frac{2b\left(a^2+b^2\right)\cot^2(c+dx)}{a^5d}-\frac{\left(2a^2+3b^2\right)\cot^3\cdots}{3a^4d}\end{aligned}$$

Mathematica [B] time = 6.25, size = 589, normalized size = 2.69

$$\frac{b\csc^4(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{2a^3d(a+b\tan(c+dx))^2}-\frac{\csc^5(c+dx)\sec(c+dx)(a\cos(c+dx)+b\sin(c+dx)}{5a^2d(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/5*(Csc[c + d*x]^5*Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(a^2
*d*(a + b*Tan[c + d*x])^2) + ((-8*a^4*Cos[c + d*x] - 75*a^2*b^2*Cos[c + d*x
] - 75*b^4*Cos[c + d*x])*Csc[c + d*x]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Si
n[c + d*x])^2)/(15*a^6*d*(a + b*Tan[c + d*x])^2) + (b*(a^2 + 2*b^2)*Csc[c +
 d*x]^2*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(a^5*d*(a + b*T
an[c + d*x])^2) + ((-4*a^2*Cos[c + d*x] - 15*b^2*Cos[c + d*x])*Csc[c + d*x]
^3*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(15*a^4*d*(a + b*Tan
[c + d*x])^2) + (b*Csc[c + d*x]^4*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^2)/(2*a^3*d*(a + b*Tan[c + d*x])^2) - (2*(a^4*b + 4*a^2*b^3 + 3*b^5
)*Log[Sin[c + d*x]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(a^
7*d*(a + b*Tan[c + d*x])^2) + (2*(a^4*b + 4*a^2*b^3 + 3*b^5)*Log[a*Cos[c +
d*x] + b*Sin[c + d*x]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/
(a^7*d*(a + b*Tan[c + d*x])^2) + (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c
+ d*x])*(a^4*b^2*Sin[c + d*x] + 2*a^2*b^4*Sin[c + d*x] + b^6*Sin[c + d*x]))
/(a^7*d*(a + b*Tan[c + d*x])^2)
```

fricas [B] time = 0.53, size = 787, normalized size = 3.59

$$\frac{4\left(4a^6+45a^4b^2+45a^2b^4\right)\cos\left(dx+c\right)^6-75a^4b^2-90a^2b^4-10\left(4a^6+45a^4b^2+45a^2b^4\right)\cos\left(dx+c\right)^4+15\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(4*(4*a^6 + 45*a^4*b^2 + 45*a^2*b^4)*cos(d*x + c)^6 - 75*a^4*b^2 - 90*
a^2*b^4 - 10*(4*a^6 + 45*a^4*b^2 + 45*a^2*b^4)*cos(d*x + c)^4 + 15*(2*a^6 +
 23*a^4*b^2 + 24*a^2*b^4)*cos(d*x + c)^2 + 30*((a^4*b^2 + 4*a^2*b^4 + 3*b^6
)*cos(d*x + c)^6 - a^4*b^2 - 4*a^2*b^4 - 3*b^6 - 3*(a^4*b^2 + 4*a^2*b^4 + 3
*b^6)*cos(d*x + c)^4 + 3*(a^4*b^2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^2 - ((a
^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^5 - 2*(a^5*b + 4*a^3*b^3 + 3*a*b^5
)*cos(d*x + c)^3 + (a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c))*sin(d*x + c)
)*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) -
 30*((a^4*b^2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^6 - a^4*b^2 - 4*a^2*b^4 - 3
*b^6 - 3*(a^4*b^2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^4 + 3*(a^4*b^2 + 4*a^2*
b^4 + 3*b^6)*cos(d*x + c)^2 - ((a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^5
 - 2*(a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 + (a^5*b + 4*a^3*b^3 + 3*
a*b^5)*cos(d*x + c))*sin(d*x + c))*log(-1/4*cos(d*x + c)^2 + 1/4) + (4*(4*a
^5*b + 45*a^3*b^3 + 45*a*b^5)*cos(d*x + c)^5 - 10*(a^5*b + 33*a^3*b^3 + 36*
a*b^5)*cos(d*x + c)^3 - 15*(a^5*b - 10*a^3*b^3 - 12*a*b^5)*cos(d*x + c))*si
n(d*x + c))/(a^7*b*d*cos(d*x + c)^6 - 3*a^7*b*d*cos(d*x + c)^4 + 3*a^7*b*d*
cos(d*x + c)^2 - a^7*b*d - (a^8*d*cos(d*x + c)^5 - 2*a^8*d*cos(d*x + c)^3 +
 a^8*d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 1.01, size = 332, normalized size = 1.52

$$-\frac{\frac{60\left(a^4b+4a^2b^3+3b^5\right)\log\left(\left|\tan(dx+c)\right|\right)}{a^7}-\frac{60\left(a^4b^2+4a^2b^4+3b^6\right)\log\left(\left|b\tan(dx+c)+a\right|\right)}{a^7b}+\frac{30\left(2a^4b^2\tan(dx+c)+8a^2b^4\tan(dx+c)+6b^6\tan(dx+c)+3\right.}{(b\tan(dx+c)+a)a^7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/30*(60*(a^4*b + 4*a^2*b^3 + 3*b^5)*log(abs(tan(d*x + c)))/a^7 - 60*(a^4*
b^2 + 4*a^2*b^4 + 3*b^6)*log(abs(b*tan(d*x + c) + a))/(a^7*b) + 30*(2*a^4*b
^2*tan(d*x + c) + 8*a^2*b^4*tan(d*x + c) + 6*b^6*tan(d*x + c) + 3*a^5*b + 1
0*a^3*b^3 + 7*a*b^5)/((b*tan(d*x + c) + a)*a^7) - (137*a^4*b*tan(d*x + c)^5
 + 548*a^2*b^3*tan(d*x + c)^5 + 411*b^5*tan(d*x + c)^5 - 30*a^5*tan(d*x + c
)^4 - 180*a^3*b^2*tan(d*x + c)^4 - 150*a*b^4*tan(d*x + c)^4 + 60*a^4*b*tan(
d*x + c)^3 + 60*a^2*b^3*tan(d*x + c)^3 - 20*a^5*tan(d*x + c)^2 - 30*a^3*b^2
*tan(d*x + c)^2 + 15*a^4*b*tan(d*x + c) - 6*a^5)/(a^7*tan(d*x + c)^5))/d
```

maple [A] time = 0.52, size = 343, normalized size = 1.57

$$-\frac{b}{a^2 d\left(a+b\tan\left(dx+c\right)\right)}-\frac{2b^3}{d\,a^4\left(a+b\tan\left(dx+c\right)\right)}-\frac{b^5}{d\,a^6\left(a+b\tan\left(dx+c\right)\right)}+\frac{2b\ln\left(a+b\tan\left(dx+c\right)\right)}{a^3 d}+\frac{8b^3\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x)
```

```
[Out] -b/a^2/d/(a+b*tan(d*x+c))-2/d*b^3/a^4/(a+b*tan(d*x+c))-1/d*b^5/a^6/(a+b*tan
(d*x+c))+2*b*ln(a+b*tan(d*x+c))/a^3/d+8/d*b^3/a^5*ln(a+b*tan(d*x+c))+6/d*b^
5/a^7*ln(a+b*tan(d*x+c))-1/5/d/a^2/tan(d*x+c)^5-2/3/d/a^2/tan(d*x+c)^3-1/d/
a^4/tan(d*x+c)^3*b^2-1/d/a^2/tan(d*x+c)-6/d/a^4/tan(d*x+c)*b^2-5/d/a^6/tan(
d*x+c)*b^4+1/2/d/a^3*b/tan(d*x+c)^4+2/d/a^3*b/tan(d*x+c)^2+2/d*b^3/a^5/tan(
d*x+c)^2-2*b*ln(tan(d*x+c))/a^3/d-8/d*b^3/a^5*ln(tan(d*x+c))-6/d*b^5/a^7*ln
(tan(d*x+c))
```

maxima [A] time = 0.60, size = 225, normalized size = 1.03

$$\frac{\frac{9\,a^4 b\tan(dx+c)-60\left(a^4 b+4\,a^2 b^3+3\,b^5\right)\tan(dx+c)^5-6\,a^5-30\left(a^5+4\,a^3 b^2+3\,ab^4\right)\tan(dx+c)^4+10\left(4\,a^4 b+3\,a^2 b^3\right)\tan(dx+c)^3-5\left(4\,a^5+3\,a^3 b^2\right)\tan(dx+}{a^6 b\tan(dx+c)^6+a^7\tan(dx+c)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*((9*a^4*b*tan(d*x + c) - 60*(a^4*b + 4*a^2*b^3 + 3*b^5)*tan(d*x + c)^5
 - 6*a^5 - 30*(a^5 + 4*a^3*b^2 + 3*a*b^4)*tan(d*x + c)^4 + 10*(4*a^4*b + 3*
a^2*b^3)*tan(d*x + c)^3 - 5*(4*a^5 + 3*a^3*b^2)*tan(d*x + c)^2)/(a^6*b*tan(
d*x + c)^6 + a^7*tan(d*x + c)^5) + 60*(a^4*b + 4*a^2*b^3 + 3*b^5)*log(b*tan
(d*x + c) + a)/a^7 - 60*(a^4*b + 4*a^2*b^3 + 3*b^5)*log(tan(d*x + c))/a^7)/
d
```

mupad [B] time = 5.01, size = 237, normalized size = 1.08

$$\frac{4\,b\,\mathrm{atanh}\left(\frac{2\,b\left(a^2+3\,b^2\right)\left(a^2+b^2\right)\left(a+2\,b\tan(c+d\,x)\right)}{a\left(2\,a^4\,b+8\,a^2\,b^3+6\,b^5\right)}\right)\left(a^2+3\,b^2\right)\left(a^2+b^2\right)}{a^7\,d}-\frac{\frac{1}{5\,a}+\frac{\tan(c+d\,x)^4\left(a^4+4\,a^2\,b^2+3\,b^4\right)}{a^5}+\frac{\tan(c+d\,x)^2\left(4\,a^2\right.}{6\,a^3}}{d\left(b\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a + b*tan(c + d*x))^2),x)
```

```
[Out] (4*b*atanh((2*b*(a^2 + 3*b^2)*(a^2 + b^2)*(a + 2*b*tan(c + d*x)))/(a*(2*a^4
*b + 6*b^5 + 8*a^2*b^3)))*(a^2 + 3*b^2)*(a^2 + b^2))/(a^7*d) - (1/(5*a) + (
tan(c + d*x)^4*(a^4 + 3*b^4 + 4*a^2*b^2))/a^5 + (tan(c + d*x)^2*(4*a^2 + 3*
b^2))/(6*a^3) - (3*b*tan(c + d*x))/(10*a^2) + (2*b*tan(c + d*x)^5*(a^4 + 3*
b^4 + 4*a^2*b^2))/a^6 - (b*tan(c + d*x)^3*(4*a^2 + 3*b^2))/(3*a^4))/(d*(a*t
an(c + d*x)^5 + b*tan(c + d*x)^6))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+b*tan(d*x+c))**2,x)
[Out] Integral(csc(c + d*x)**6/(a + b*tan(c + d*x))**2, x)
```

3.67 $\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=382

$$-\frac{\cos^6(c+dx)\left(a\left(a^2-3b^2\right)\tan(c+dx)+b\left(3a^2-b^2\right)\right)}{6d\left(a^2+b^2\right)^3}-\frac{a^6b}{2d\left(a^2+b^2\right)^4(a+b\tan(c+dx))^2}-\frac{2a^5b\left(a^2-\right.}{d\left(a^2+b^2\right)^5(a+b\right.}$$

```
[Out] 1/16*a*(5*a^8-180*a^6*b^2+390*a^4*b^4-68*a^2*b^6-3*b^8)*x/(a^2+b^2)^6+a^4*b
*(3*a^4-22*a^2*b^2+15*b^4)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^6/d-1/2*
a^6*b/(a^2+b^2)^4/d/(a+b*tan(d*x+c))^2-2*a^5*b*(a^2-3*b^2)/(a^2+b^2)^5/d/(a
+b*tan(d*x+c))-1/6*cos(d*x+c)^6*(b*(3*a^2-b^2)+a*(a^2-3*b^2)*tan(d*x+c))/(a
^2+b^2)^3/d+1/24*cos(d*x+c)^4*(6*b*(9*a^4-4*a^2*b^2-b^4)+a*(13*a^4-62*a^2*b
^2-3*b^4)*tan(d*x+c))/(a^2+b^2)^4/d-1/16*a*cos(d*x+c)^2*(24*a^3*b*(3*a^2-5*
b^2)+(11*a^6-119*a^4*b^2+65*a^2*b^4+3*b^6)*tan(d*x+c))/(a^2+b^2)^5/d
```

Rubi [A] time = 1.43, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$-\frac{a^6b}{2d\left(a^2+b^2\right)^4(a+b\tan(c+dx))^2}-\frac{2a^5b\left(a^2-3b^2\right)}{d\left(a^2+b^2\right)^5(a+b\tan(c+dx))}-\frac{a\cos^2(c+dx)\left(\left(-119a^4b^2+65a^2b^4+11\right.\right.}{16d\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(5*a^8 - 180*a^6*b^2 + 390*a^4*b^4 - 68*a^2*b^6 - 3*b^8)*x)/(16*(a^2 + b
^2)^6) + (a^4*b*(3*a^4 - 22*a^2*b^2 + 15*b^4)*Log[a*Cos[c + d*x] + b*Sin[c
+ d*x]])/((a^2 + b^2)^6*d) - (a^6*b)/(2*(a^2 + b^2)^4*d*(a + b*Tan[c + d*x]
)^2) - (2*a^5*b*(a^2 - 3*b^2))/((a^2 + b^2)^5*d*(a + b*Tan[c + d*x])) - (Co
s[c + d*x]^6*(b*(3*a^2 - b^2) + a*(a^2 - 3*b^2)*Tan[c + d*x]))/(6*(a^2 + b^
2)^3*d) + (Cos[c + d*x]^4*(6*b*(9*a^4 - 4*a^2*b^2 - b^4) + a*(13*a^4 - 62*a
^2*b^2 - 3*b^4)*Tan[c + d*x]))/(24*(a^2 + b^2)^4*d) - (a*Cos[c + d*x]^2*(24
*a^3*b*(3*a^2 - 5*b^2) + (11*a^6 - 119*a^4*b^2 + 65*a^2*b^4 + 3*b^6)*Tan[c
+ d*x]))/(16*(a^2 + b^2)^5*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
```

```
d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^6}{(a+x)^3\left(b^2+x^2\right)^4}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^6(c+dx)\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^3 d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^4b^6\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^3}+\frac{3a^3b^6}{\left(a^2 \cdots\right.}}{\cdots}\right.}{\cdots}$$

$$= -\frac{\cos^6(c+dx)\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^3 d} + \frac{\cos^4(c+dx)\left(6b\left(9a^4-4a \cdots\right.\right.}{\cdots}$$

$$= -\frac{\cos^6(c+dx)\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^3 d} + \frac{\cos^4(c+dx)\left(6b\left(9a^4-4a \cdots\right.\right.}{\cdots}$$

$$= -\frac{\cos^6(c+dx)\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^3 d} + \frac{\cos^4(c+dx)\left(6b\left(9a^4-4a \cdots\right.\right.}{\cdots}$$

$$= \frac{a^4b\left(3a^4-22a^2b^2+15b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^6 d} - \frac{a^6b}{2\left(a^2+b^2\right)^4 d(a+b\tan(c+dx)) \cdots}$$

$$= \frac{a^4b\left(3a^4-22a^2b^2+15b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^6 d} - \frac{a^6b}{2\left(a^2+b^2\right)^4 d(a+b\tan(c+dx)) \cdots}$$

$$= \frac{a\left(5a^8-180a^6b^2+390a^4b^4-68a^2b^6-3b^8\right)x}{16\left(a^2+b^2\right)^6} + \frac{a^4b\left(3a^4-22a^2b^2+15b^4\right)\log(\cos \cdots}{\left(a^2+b^2\right)^6 d}$$

Mathematica [A] time = 6.69, size = 683, normalized size = 1.79

$$b\left(-\frac{(3a^2-b^2)\cos^6(c+dx)}{6(a^2+b^2)^3}-\frac{a(a^2-3b^2)\sin(c+dx)\cos^5(c+dx)}{6b(a^2+b^2)^3}-\frac{5a(a^2-3b^2)\left(3b^2\left(\frac{\tan^{-1}(\tan(c+dx))}{b^3}+\frac{\sin(c+dx)\cos(c+dx)}{b^3}\right)+\frac{2\sin(c+dx)\cos^3(c+dx)}{b}\right)}{48(a^2+b^2)^3}-$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b*((-3*a^5*(a^2 - 7*b^2)*ArcTan[Tan[c + d*x]])/(2*b*(a^2 + b^2)^5) - (3*a^
4*(3*a^2 - 5*b^2)*Cos[c + d*x]^2)/(2*(a^2 + b^2)^5) + ((9*a^4 - 4*a^2*b^2 -
 b^4)*Cos[c + d*x]^4)/(4*(a^2 + b^2)^4) - ((3*a^2 - b^2)*Cos[c + d*x]^6)/(6
*(a^2 + b^2)^3) - (a^4*(3*a^4 - 22*a^2*b^2 + 15*b^4 - (a^5 - 18*a^3*b^2 + 2
1*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(2*(a^2 + b^2)^6) +
(a^4*(3*a^4 - 22*a^2*b^2 + 15*b^4)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^6 -
 (a^4*(3*a^4 - 22*a^2*b^2 + 15*b^4 + (a^5 - 18*a^3*b^2 + 21*a*b^4)/Sqrt[-b^
2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^6) - (3*a^5*(a^2 - 7*b
^2)*Cos[c + d*x]*Sin[c + d*x])/(2*b*(a^2 + b^2)^5) + (3*a*(a^4 - 4*a^2*b^2
- b^4)*Cos[c + d*x]^3*Sin[c + d*x])/(4*b*(a^2 + b^2)^4) - (a*(a^2 - 3*b^2)*
Cos[c + d*x]^5*Sin[c + d*x])/(6*b*(a^2 + b^2)^3) + (9*a*(a^4 - 4*a^2*b^2 -
b^4)*(ArcTan[Tan[c + d*x]]/b + (Cos[c + d*x]*Sin[c + d*x])/b))/(8*(a^2 + b^
2)^4) - (5*a*(a^2 - 3*b^2)*((2*Cos[c + d*x]^3*Sin[c + d*x])/b + 3*b^2*(ArcT
an[Tan[c + d*x]]/b^3 + (Cos[c + d*x]*Sin[c + d*x])/b^3)))/(48*(a^2 + b^2)^3
) - a^6/(2*(a^2 + b^2)^4*(a + b*Tan[c + d*x])^2) - (2*a^5*(a^2 - 3*b^2))/((
a^2 + b^2)^5*(a + b*Tan[c + d*x]))))/d
```

fricas [B] time = 0.63, size = 932, normalized size = 2.44

$$195\,a^8b^3-427\,a^6b^5-165\,a^4b^7+27\,a^2b^9+2\,b^{11}-8\left(a^{10}b+5\,a^8b^3+10\,a^6b^5+10\,a^4b^7+5\,a^2b^9+b^{11}\right)\cos(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(195*a^8*b^3 - 427*a^6*b^5 - 165*a^4*b^7 + 27*a^2*b^9 + 2*b^11 - 8*(a^
10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11)*cos(d*x + c)
^8 + 20*(2*a^10*b + 9*a^8*b^3 + 16*a^6*b^5 + 14*a^4*b^7 + 6*a^2*b^9 + b^11)
*cos(d*x + c)^6 - 2*(49*a^10*b + 162*a^8*b^3 + 198*a^6*b^5 + 112*a^4*b^7 +
33*a^2*b^9 + 6*b^11)*cos(d*x + c)^4 + 3*(5*a^9*b^2 - 180*a^7*b^4 + 390*a^5*
b^6 - 68*a^3*b^8 - 3*a*b^10)*d*x + (9*a^10*b - 46*a^8*b^3 + 994*a^6*b^5 + 1
44*a^4*b^7 - 43*a^2*b^9 - 2*b^11 + 3*(5*a^11 - 185*a^9*b^2 + 570*a^7*b^4 -
458*a^5*b^6 + 65*a^3*b^8 + 3*a*b^10)*d*x)*cos(d*x + c)^2 + 24*(3*a^8*b^3 -
22*a^6*b^5 + 15*a^4*b^7 + (3*a^10*b - 25*a^8*b^3 + 37*a^6*b^5 - 15*a^4*b^7)
*cos(d*x + c)^2 + 2*(3*a^9*b^2 - 22*a^7*b^4 + 15*a^5*b^6)*cos(d*x + c)*sin(
d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2) - (8*(a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^5*b^6 + 5*a^3*b^8 + a*b^1
0)*cos(d*x + c)^7 - 2*(13*a^11 + 55*a^9*b^2 + 90*a^7*b^4 + 70*a^5*b^6 + 25*
a^3*b^8 + 3*a*b^10)*cos(d*x + c)^5 + (33*a^11 + 49*a^9*b^2 - 54*a^7*b^4 - 1
26*a^5*b^6 - 59*a^3*b^8 - 3*a*b^10)*cos(d*x + c)^3 - (261*a^9*b^2 - 338*a^7
*b^4 + 120*a^5*b^6 - 150*a^3*b^8 - 5*a*b^10 + 6*(5*a^10*b - 180*a^8*b^3 + 3
90*a^6*b^5 - 68*a^4*b^7 - 3*a^2*b^9)*d*x)*cos(d*x + c))*sin(d*x + c))/((a^1
4 + 5*a^12*b^2 + 9*a^10*b^4 + 5*a^8*b^6 - 5*a^6*b^8 - 9*a^4*b^10 - 5*a^2*b^
12 - b^14)*d*cos(d*x + c)^2 + 2*(a^13*b + 6*a^11*b^3 + 15*a^9*b^5 + 20*a^7*
b^7 + 15*a^5*b^9 + 6*a^3*b^11 + a*b^13)*d*cos(d*x + c)*sin(d*x + c) + (a^12
*b^2 + 6*a^10*b^4 + 15*a^8*b^6 + 20*a^6*b^8 + 15*a^4*b^10 + 6*a^2*b^12 + b^
14)*d)
```

giac [B] time = 6.86, size = 923, normalized size = 2.42

$$\frac{3\left(5a^9-180a^7b^2+390a^5b^4-68a^3b^6-3ab^8\right)(dx+c)}{a^{12}+6a^{10}b^2+15a^8b^4+20a^6b^6+15a^4b^8+6a^2b^{10}+b^{12}}-\frac{24\left(3a^8b-22a^6b^3+15a^4b^5\right)\log\left(\tan(dx+c)^2+1\right)}{a^{12}+6a^{10}b^2+15a^8b^4+20a^6b^6+15a^4b^8+6a^2b^{10}+b^{12}}+\frac{48\left(3a^8b^2-22a^6b^4+15a^4b^6\right)\log(|b}{a^{12}b+6a^{10}b^3+15a^8b^5+20a^6b^7+15a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*a^9 - 180*a^7*b^2 + 390*a^5*b^4 - 68*a^3*b^6 - 3*a*b^8)*(d*x + c
)/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
b^12) - 24*(3*a^8*b - 22*a^6*b^3 + 15*a^4*b^5)*log(tan(d*x + c)^2 + 1)/(a^1
2 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)
+ 48*(3*a^8*b^2 - 22*a^6*b^4 + 15*a^4*b^6)*log(abs(b*tan(d*x + c) + a))/(a^
12*b + 6*a^10*b^3 + 15*a^8*b^5 + 20*a^6*b^7 + 15*a^4*b^9 + 6*a^2*b^11 + b^1
3) - 24*(9*a^8*b^3*tan(d*x + c)^2 - 66*a^6*b^5*tan(d*x + c)^2 + 45*a^4*b^7*
tan(d*x + c)^2 + 22*a^9*b^2*tan(d*x + c) - 140*a^7*b^4*tan(d*x + c) + 78*a^
5*b^6*tan(d*x + c) + 14*a^10*b - 72*a^8*b^3 + 34*a^6*b^5)/((a^12 + 6*a^10*b
^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*(b*tan(d*x +
 c) + a)^2) + (132*a^8*b*tan(d*x + c)^6 - 968*a^6*b^3*tan(d*x + c)^6 + 660*
a^4*b^5*tan(d*x + c)^6 - 33*a^9*tan(d*x + c)^5 + 324*a^7*b^2*tan(d*x + c)^5
 + 162*a^5*b^4*tan(d*x + c)^5 - 204*a^3*b^6*tan(d*x + c)^5 - 9*a*b^8*tan(d*
x + c)^5 + 180*a^8*b*tan(d*x + c)^4 - 2760*a^6*b^3*tan(d*x + c)^4 + 2340*a^
4*b^5*tan(d*x + c)^4 - 40*a^9*tan(d*x + c)^3 + 576*a^7*b^2*tan(d*x + c)^3 +
 96*a^5*b^4*tan(d*x + c)^3 - 544*a^3*b^6*tan(d*x + c)^3 - 24*a*b^8*tan(d*x
+ c)^3 + 72*a^8*b*tan(d*x + c)^2 - 2448*a^6*b^3*tan(d*x + c)^2 + 2700*a^4*b
^5*tan(d*x + c)^2 - 72*a^2*b^7*tan(d*x + c)^2 - 12*b^9*tan(d*x + c)^2 - 15*
a^9*tan(d*x + c) + 252*a^7*b^2*tan(d*x + c) - 18*a^5*b^4*tan(d*x + c) - 276
*a^3*b^6*tan(d*x + c) + 9*a*b^8*tan(d*x + c) - 720*a^6*b^3 + 972*a^4*b^5 -
72*a^2*b^7 - 4*b^9)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*
b^8 + 6*a^2*b^10 + b^12)*(tan(d*x + c)^2 + 1)^3))/d
```

maple [B] time = 0.50, size = 1449, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x)
```

```
[Out] -1/2*a^6*b/(a^2+b^2)^4/d/(a+b*tan(d*x+c))^2-3/2/d/(a^2+b^2)^6*ln(1+tan(d*x+
c)^2)*a^8*b+11/d/(a^2+b^2)^6*ln(1+tan(d*x+c)^2)*a^6*b^3-15/2/d/(a^2+b^2)^6*
ln(1+tan(d*x+c)^2)*a^4*b^5+3/d*a^8*b/(a^2+b^2)^6*ln(a+b*tan(d*x+c))-22/d*a^
6*b^3/(a^2+b^2)^6*ln(a+b*tan(d*x+c))+15/d*a^4*b^5/(a^2+b^2)^6*ln(a+b*tan(d*
x+c))-2/d*b*a^7/(a^2+b^2)^5/(a+b*tan(d*x+c))+6/d*b^3*a^5/(a^2+b^2)^5/(a+b*t
an(d*x+c))-3/2/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*a^2*b^7+13/2/d/(a^2+b^2)^6/
(1+tan(d*x+c)^2)^3*a^4*b^5+31/6/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*a^6*b^3-11
/4/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*a^8*b-11/16/d/(a^2+b^2)^6/(1+tan(d*x+c)
^2)^3*tan(d*x+c)^5*a^9-5/6/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^3*a^
9-1/4/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^2*b^9-5/16/d/(a^2+b^2)^6/
(1+tan(d*x+c)^2)^3*tan(d*x+c)*a^9-45/4/d/(a^2+b^2)^6*arctan(tan(d*x+c))*a^7
*b^2+195/8/d/(a^2+b^2)^6*arctan(tan(d*x+c))*a^5*b^4-17/4/d/(a^2+b^2)^6*arct
an(tan(d*x+c))*a^3*b^6-3/16/d/(a^2+b^2)^6*arctan(tan(d*x+c))*a*b^8+5/16/d/(
a^2+b^2)^6*arctan(tan(d*x+c))*a^9-1/12/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*b^9
-23/4/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)*a^3*b^6+3/16/d/(a^2+b^2)^
6/(1+tan(d*x+c)^2)^3*tan(d*x+c)*a*b^8+15/2/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3
*tan(d*x+c)^4*a^4*b^5+12/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^3*a^7*
b^2+2/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^3*a^5*b^4+27/8/d/(a^2+b^2
)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^5*a^5*b^4-17/4/d/(a^2+b^2)^6/(1+tan(d*x+c
)^2)^3*tan(d*x+c)^5*a^3*b^6+19/2/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c
```

```
)^2*a^6*b^3+15/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^2*a^4*b^5-3/2/d/
(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^2*a^2*b^7-3/8/d/(a^2+b^2)^6/(1+ta
n(d*x+c)^2)^3*tan(d*x+c)*a^5*b^4+21/4/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(
d*x+c)*a^7*b^2-34/3/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^3*a^3*b^6-1
/2/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^3*a*b^8-27/4/d/(a^2+b^2)^6/(
1+tan(d*x+c)^2)^3*tan(d*x+c)^2*a^8*b-3/16/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*
tan(d*x+c)^5*a*b^8-9/2/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^4*a^8*b+
3/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^3*tan(d*x+c)^4*a^6*b^3+27/4/d/(a^2+b^2)^6/
(1+tan(d*x+c)^2)^3*tan(d*x+c)^5*a^7*b^2
```

maxima [B] time = 0.56, size = 1088, normalized size = 2.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(5*a^9 - 180*a^7*b^2 + 390*a^5*b^4 - 68*a^3*b^6 - 3*a*b^8)*(d*x + c
)/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
b^12) + 48*(3*a^8*b - 22*a^6*b^3 + 15*a^4*b^5)*log(b*tan(d*x + c) + a)/(a^1
2 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)
- 24*(3*a^8*b - 22*a^6*b^3 + 15*a^4*b^5)*log(tan(d*x + c)^2 + 1)/(a^12 + 6*
a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12) - (252
*a^8*b - 644*a^6*b^3 + 68*a^4*b^5 + 4*a^2*b^7 + 3*(43*a^7*b^2 - 215*a^5*b^4
 + 65*a^3*b^6 + 3*a*b^8)*tan(d*x + c)^7 + 6*(31*a^8*b - 127*a^6*b^3 + 5*a^4
*b^5 + 3*a^2*b^7)*tan(d*x + c)^6 + (33*a^9 + 403*a^7*b^2 - 2005*a^5*b^4 + 5
29*a^3*b^6 + 24*a*b^8)*tan(d*x + c)^5 + 4*(164*a^8*b - 515*a^6*b^3 + 65*a^4
*b^5 + 27*a^2*b^7 + 3*b^9)*tan(d*x + c)^4 + (40*a^9 + 335*a^7*b^2 - 2171*a^
5*b^4 + 429*a^3*b^6 + 15*a*b^8)*tan(d*x + c)^3 + 2*(357*a^8*b - 987*a^6*b^3
 + 125*a^4*b^5 + 31*a^2*b^7 + 2*b^9)*tan(d*x + c)^2 + (15*a^9 + 93*a^7*b^2
- 763*a^5*b^4 + 127*a^3*b^6 + 8*a*b^8)*tan(d*x + c))/(a^12 + 5*a^10*b^2 + 1
0*a^8*b^4 + 10*a^6*b^6 + 5*a^4*b^8 + a^2*b^10 + (a^10*b^2 + 5*a^8*b^4 + 10*
a^6*b^6 + 10*a^4*b^8 + 5*a^2*b^10 + b^12)*tan(d*x + c)^8 + 2*(a^11*b + 5*a^
9*b^3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9 + a*b^11)*tan(d*x + c)^7 + (a^1
2 + 8*a^10*b^2 + 25*a^8*b^4 + 40*a^6*b^6 + 35*a^4*b^8 + 16*a^2*b^10 + 3*b^1
2)*tan(d*x + c)^6 + 6*(a^11*b + 5*a^9*b^3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3
*b^9 + a*b^11)*tan(d*x + c)^5 + 3*(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*
b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*tan(d*x + c)^4 + 6*(a^11*b + 5*a^9*b^
3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9 + a*b^11)*tan(d*x + c)^3 + (3*a^12
+ 16*a^10*b^2 + 35*a^8*b^4 + 40*a^6*b^6 + 25*a^4*b^8 + 8*a^2*b^10 + b^12)*t
an(d*x + c)^2 + 2*(a^11*b + 5*a^9*b^3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9
 + a*b^11)*tan(d*x + c)))/d
```

mupad [B] time = 5.84, size = 1068, normalized size = 2.80

$$\frac{\ln\left(a+b\tan\left(c+d\,x\right)\right)\left(\frac{3\,b}{{\left(a^2+b^2\right)}^2}-\frac{34\,b^3}{{\left(a^2+b^2\right)}^3}+\frac{99\,b^5}{{\left(a^2+b^2\right)}^4}-\frac{108\,b^7}{{\left(a^2+b^2\right)}^5}+\frac{40\,b^9}{{\left(a^2+b^2\right)}^6}\right)}{d}-\frac{{\mathrm{tan}\left(c+d\,x\right)}^6\,\left(31\,a^8\,b-127\,a^6\,b^3+5\,a^4\,b^5+3\,a^2\,b^7}{8\,\left(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+10\,a^4\,b^6+5\,a^2\,b^8+b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + b*tan(c + d*x))^3,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((3*b)/(a^2 + b^2)^2 - (34*b^3)/(a^2 + b^2)^3 + (9
9*b^5)/(a^2 + b^2)^4 - (108*b^7)/(a^2 + b^2)^5 + (40*b^9)/(a^2 + b^2)^6))/d
 - ((tan(c + d*x)^6*(31*a^8*b + 3*a^2*b^7 + 5*a^4*b^5 - 127*a^6*b^3))/(8*(a
^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (tan(c + d
*x)^7*(3*a*b^8 + 65*a^3*b^6 - 215*a^5*b^4 + 43*a^7*b^2))/(16*(a^10 + b^10 +
 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (tan(c + d*x)^5*(24*a*
b^8 + 33*a^9 + 529*a^3*b^6 - 2005*a^5*b^4 + 403*a^7*b^2))/(48*(a^10 + b^10
```

```
+ 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (tan(c + d*x)^4*(164*
a^8*b + 3*b^9 + 27*a^2*b^7 + 65*a^4*b^5 - 515*a^6*b^3))/(12*(a^10 + b^10 +
5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (a^2*(63*a^6*b + b^7 +
17*a^2*b^5 - 161*a^4*b^3))/(12*(a^2 + b^2)*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b
^4 + 4*a^6*b^2)) + (tan(c + d*x)^3*(15*a*b^8 + 40*a^9 + 429*a^3*b^6 - 2171*
a^5*b^4 + 335*a^7*b^2))/(48*(a^2 + b^2)*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4
+ 4*a^6*b^2)) + (tan(c + d*x)^2*(357*a^8*b + 2*b^9 + 31*a^2*b^7 + 125*a^4*b
^5 - 987*a^6*b^3))/(24*(a^2 + b^2)*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a
^6*b^2)) + (a*tan(c + d*x)*(15*a^8 + 8*b^8 + 127*a^2*b^6 - 763*a^4*b^4 + 93
*a^6*b^2))/(48*(a^2 + b^2)*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2))
)/(d*(tan(c + d*x)^2*(3*a^2 + b^2) + tan(c + d*x)^6*(a^2 + 3*b^2) + a^2 + t
an(c + d*x)^4*(3*a^2 + 3*b^2) + b^2*tan(c + d*x)^8 + 2*a*b*tan(c + d*x) + 6
*a*b*tan(c + d*x)^3 + 6*a*b*tan(c + d*x)^5 + 2*a*b*tan(c + d*x)^7)) - (log(
tan(c + d*x) + 1i)*(a*b^2*3i - 18*a^2*b + a^3*5i))/(32*d*(a*b^5*6i + a^5*b*
6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)) - (log(tan(c + d*x
) - 1i)*(3*a*b^2 - a^2*b*18i + 5*a^3))/(32*d*(6*a*b^5 + 6*a^5*b - a^6*1i +
b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i))
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*tan(d*x+c))**3,x)

[Out] Exception raised: AttributeError
```

3.68 $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=285

$$\frac{\cos^4(c+dx)\left(a\left(a^2-3b^2\right)\tan(c+dx)+b\left(3a^2-b^2\right)\right)}{4d\left(a^2+b^2\right)^3}-\frac{a^4b}{2d\left(a^2+b^2\right)^3(a+b\tan(c+dx))^2}-\frac{a\cos^2(c+dx)\left(24a\right.}{}$$

```
[Out] 3/8*a*(a^6-25*a^4*b^2+35*a^2*b^4-3*b^6)*x/(a^2+b^2)^5+3*a^2*b*(a^4-5*a^2*b^
2+2*b^4)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^5/d-1/2*a^4*b/(a^2+b^2)^3/
d/(a+b*tan(d*x+c))^2-2*a^3*b*(a^2-2*b^2)/(a^2+b^2)^4/d/(a+b*tan(d*x+c))+1/4
*cos(d*x+c)^4*(b*(3*a^2-b^2)+a*(a^2-3*b^2)*tan(d*x+c))/(a^2+b^2)^3/d-1/8*a*
cos(d*x+c)^2*(24*a*b*(a^2-b^2)+(5*a^4-34*a^2*b^2+9*b^4)*tan(d*x+c))/(a^2+b^
2)^4/d
```

Rubi [A] time = 0.85, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$-\frac{a^4b}{2d\left(a^2+b^2\right)^3(a+b\tan(c+dx))^2}-\frac{2a^3b\left(a^2-2b^2\right)}{d\left(a^2+b^2\right)^4(a+b\tan(c+dx))}-\frac{a\cos^2(c+dx)\left(\left(-34a^2b^2+5a^4+9b^4\right)\mathrm{ta}\right.}{8d\left(a^2+b^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*(a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*x)/(8*(a^2 + b^2)^5) + (3*a^2*
b*(a^4 - 5*a^2*b^2 + 2*b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b
^2)^5*d) - (a^4*b)/(2*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])^2) - (2*a^3*b*(a
^2 - 2*b^2))/((a^2 + b^2)^4*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^4*(b*(3
*a^2 - b^2) + a*(a^2 - 3*b^2)*Tan[c + d*x]))/(4*(a^2 + b^2)^3*d) - (a*Cos[c
 + d*x]^2*(24*a*b*(a^2 - b^2) + (5*a^4 - 34*a^2*b^2 + 9*b^4)*Tan[c + d*x]))
/(8*(a^2 + b^2)^4*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^4}{(a+x)^3\left(b^2+x^2\right)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^4(c+dx)\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^3 d} - \frac{\text{Subst}\left(\int \frac{\frac{a^4b^4\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^3}-\frac{a^3b^4\left(9a^2+\cdots\right.}{\left(a^2+b^{\cdots}\right.}}{\cdots}\right.}{\cdots}$$

$$= \frac{\cos^4(c+dx)\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^3 d} - \frac{a\cos^2(c+dx)\left(24ab\left(a^2-\cdots\right.\right.}{8\cdots}$$

$$= \frac{\cos^4(c+dx)\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^3 d} - \frac{a\cos^2(c+dx)\left(24ab\left(a^2-\cdots\right.\right.}{8\cdots}$$

$$= \frac{3a^2b\left(a^4-5a^2b^2+2b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} - \frac{a^4b}{2\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))^2} \cdots$$

$$= \frac{3a^2b\left(a^4-5a^2b^2+2b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} - \frac{a^4b}{2\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))^2} \cdots$$

$$= \frac{3a\left(a^6-25a^4b^2+35a^2b^4-3b^6\right)x}{8\left(a^2+b^2\right)^5} + \frac{3a^2b\left(a^4-5a^2b^2+2b^4\right)\log(\cos(c+dx))}{\left(a^2+b^2\right)^5 d} + \frac{3a\cdots}{\cdots}$$

Mathematica [A] time = 6.49, size = 501, normalized size = 1.76

$$\frac{b\left(\frac{\left(3a^2-b^2\right)\cos^4(c+dx)}{4\left(a^2+b^2\right)^3} - \frac{3a^2(a-b)(a+b)\cos^2(c+dx)}{\left(a^2+b^2\right)^4} + \frac{a\left(a^2-3b^2\right)\sin(c+dx)\cos^3(c+dx)}{4b\left(a^2+b^2\right)^3} + \frac{3a\left(a^2-3b^2\right)\left(\frac{\tan^{-1}(\tan(c+dx))}{b}+\frac{\sin(c+dx)\cos(c+dx)}{b}\right)}{8\left(a^2+b^2\right)^3} - \frac{\cdots}{2(\cdots}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b*(-((a^3*(a^2 - 5*b^2)*ArcTan[Tan[c + d*x]])/(b*(a^2 + b^2)^4)) - (3*a^2*
(a - b)*(a + b)*Cos[c + d*x]^2)/(a^2 + b^2)^4 + ((3*a^2 - b^2)*Cos[c + d*x]
^4)/(4*(a^2 + b^2)^3) - (a^2*(3*a^4 - 15*a^2*b^2 + 6*b^4 - (a^5 - 13*a^3*b^
2 + 10*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(2*(a^2 + b^2)^
5) + (3*a^2*(a^4 - 5*a^2*b^2 + 2*b^4)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^
5 - (a^2*(3*a^4 - 15*a^2*b^2 + 6*b^4 + (a^5 - 13*a^3*b^2 + 10*a*b^4)/Sqrt[-
b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^5) - (a^3*(a^2 - 5*b
^2)*Cos[c + d*x]*Sin[c + d*x])/(b*(a^2 + b^2)^4) + (a*(a^2 - 3*b^2)*Cos[c +
 d*x]^3*Sin[c + d*x])/(4*b*(a^2 + b^2)^3) + (3*a*(a^2 - 3*b^2)*(ArcTan[Tan[
c + d*x]]/b + (Cos[c + d*x]*Sin[c + d*x])/b))/(8*(a^2 + b^2)^3) - a^4/(2*(a
^2 + b^2)^3*(a + b*Tan[c + d*x])^2) - (2*a^3*(a^2 - 2*b^2))/((a^2 + b^2)^4*
(a + b*Tan[c + d*x]))))/d
```

fricas [B] time = 0.57, size = 705, normalized size = 2.47

$$\frac{119\,a^6b^3 - 159\,a^4b^5 - 51\,a^2b^7 + 3\,b^9 + 8\left(a^8b + 4\,a^6b^3 + 6\,a^4b^5 + 4\,a^2b^7 + b^9\right)\cos\left(dx+c\right)^6 - 8\left(5\,a^8b + 16\,a^6b^3 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(119*a^6*b^3 - 159*a^4*b^5 - 51*a^2*b^7 + 3*b^9 + 8*(a^8*b + 4*a^6*b^3
 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^6 - 8*(5*a^8*b + 16*a^6*b^3 +
18*a^4*b^5 + 8*a^2*b^7 + b^9)*cos(d*x + c)^4 + 12*(a^7*b^2 - 25*a^5*b^4 + 3
5*a^3*b^6 - 3*a*b^8)*d*x - (a^8*b + 110*a^6*b^3 - 420*a^4*b^5 - 78*a^2*b^7
+ 3*b^9 - 12*(a^9 - 26*a^7*b^2 + 60*a^5*b^4 - 38*a^3*b^6 + 3*a*b^8)*d*x)*co
s(d*x + c)^2 + 48*(a^6*b^3 - 5*a^4*b^5 + 2*a^2*b^7 + (a^8*b - 6*a^6*b^3 + 7
*a^4*b^5 - 2*a^2*b^7)*cos(d*x + c)^2 + 2*(a^7*b^2 - 5*a^5*b^4 + 2*a^3*b^6)*
cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2
)*cos(d*x + c)^2 + b^2) + 2*(4*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a
*b^8)*cos(d*x + c)^5 - 2*(5*a^9 + 12*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 - 3*a*
b^8)*cos(d*x + c)^3 + (77*a^7*b^2 - 69*a^5*b^4 + 63*a^3*b^6 - 15*a*b^8 + 12
*(a^8*b - 25*a^6*b^3 + 35*a^4*b^5 - 3*a^2*b^7)*d*x)*cos(d*x + c))*sin(d*x +
 c))/((a^12 + 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^8 - 4*a^2*b^10 - b^12)*d*cos
(d*x + c)^2 + 2*(a^11*b + 5*a^9*b^3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9 +
 a*b^11)*d*cos(d*x + c)*sin(d*x + c) + (a^10*b^2 + 5*a^8*b^4 + 10*a^6*b^6 +
 10*a^4*b^8 + 5*a^2*b^10 + b^12)*d)
```

giac [B] time = 3.87, size = 588, normalized size = 2.06

$$\frac{3\left(a^7-25\,a^5b^2+35\,a^3b^4-3\,ab^6\right)(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} - \frac{12\left(a^6b-5\,a^4b^3+2\,a^2b^5\right)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} + \frac{24\left(a^6b^2-5\,a^4b^4+2\,a^2b^6\right)\log(|b\tan(dx+c)+a|)}{a^{10}b+5\,a^8b^3+10\,a^6b^5+10\,a^4b^7+5\,a^2b^9+b^{11}} - \frac{21\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^7 - 25*a^5*b^2 + 35*a^3*b^4 - 3*a*b^6)*(d*x + c)/(a^10 + 5*a^8*b^
2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - 12*(a^6*b - 5*a^4*b^3 + 2
*a^2*b^5)*log(tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b
^6 + 5*a^2*b^8 + b^10) + 24*(a^6*b^2 - 5*a^4*b^4 + 2*a^2*b^6)*log(abs(b*tan
(d*x + c) + a))/(a^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 +
 b^11) - (21*a^5*b^2*tan(d*x + c)^5 - 66*a^3*b^4*tan(d*x + c)^5 + 9*a*b^6*t
an(d*x + c)^5 + 30*a^6*b*tan(d*x + c)^4 - 72*a^4*b^3*tan(d*x + c)^4 - 6*a^2
*b^5*tan(d*x + c)^4 + 5*a^7*tan(d*x + c)^3 + 49*a^5*b^2*tan(d*x + c)^3 - 13
3*a^3*b^4*tan(d*x + c)^3 + 15*a*b^6*tan(d*x + c)^3 + 70*a^6*b*tan(d*x + c)^
2 - 122*a^4*b^3*tan(d*x + c)^2 + 2*a^2*b^5*tan(d*x + c)^2 + 2*b^7*tan(d*x +
```

```
 c)^2 + 3*a^7*tan(d*x + c) + 22*a^5*b^2*tan(d*x + c) - 73*a^3*b^4*tan(d*x +
 c) + 4*a*b^6*tan(d*x + c) + 38*a^6*b - 56*a^4*b^3 + 2*a^2*b^5)/((a^8 + 4*a
^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*x + c)^3 + a*tan(d*x + c)^2
+ b*tan(d*x + c) + a)^2))/d
```

maple [B] time = 0.49, size = 882, normalized size = 3.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x)
```

```
[Out] 3/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*tan(d*x+c)^2*a^2*b^5+29/8/d/(a^2+b^2)^5/
(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*a^5*b^2-3/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*
tan(d*x+c)^2*a^6*b+27/8/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a^5*b^2
-9/8/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*a*b^6+25/8/d/(a^2+b^2)^5
/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*a^3*b^4-15/8/d/(a^2+b^2)^5/(1+tan(d*x+c)^2
)^2*tan(d*x+c)*a*b^6-1/4/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*b^7+3/8/d/(a^2+b^
2)^5*arctan(tan(d*x+c))*a^7+15/8/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*tan(d*x+c
)*a^3*b^4-2/d*b*a^5/(a^2+b^2)^4/(a+b*tan(d*x+c))+4/d*b^3*a^3/(a^2+b^2)^4/(a
+b*tan(d*x+c))-3/d/(a^2+b^2)^5*ln(1+tan(d*x+c)^2)*a^2*b^5+3/d*a^6*b/(a^2+b^
2)^5*ln(a+b*tan(d*x+c))-9/8/d/(a^2+b^2)^5*arctan(tan(d*x+c))*a*b^6-75/8/d/(
a^2+b^2)^5*arctan(tan(d*x+c))*a^5*b^2+105/8/d/(a^2+b^2)^5*arctan(tan(d*x+c)
)*a^3*b^4+5/4/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*a^4*b^3+13/4/d/(a^2+b^2)^5/(
1+tan(d*x+c)^2)^2*a^2*b^5-3/2/d/(a^2+b^2)^5*ln(1+tan(d*x+c)^2)*a^6*b+15/2/d
/(a^2+b^2)^5*ln(1+tan(d*x+c)^2)*a^4*b^3-1/2*a^4*b/(a^2+b^2)^3/d/(a+b*tan(d*
x+c))^2+6/d*a^2*b^5/(a^2+b^2)^5*ln(a+b*tan(d*x+c))-15/d*a^4*b^3/(a^2+b^2)^5
*ln(a+b*tan(d*x+c))-5/8/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*a^7-3
/8/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a^7-9/4/d/(a^2+b^2)^5/(1+tan
(d*x+c)^2)^2*a^6*b
```

maxima [B] time = 0.70, size = 744, normalized size = 2.61

$$\frac{3\left(a^7-25\,a^5b^2+35\,a^3b^4-3\,ab^6\right)(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}}+\frac{24\left(a^6b-5\,a^4b^3+2\,a^2b^5\right)\log(b\tan(dx+c)+a)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}}-\frac{12\left(a^6b-5\,a^4b^3+2\,a^2b^5\right)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}}-\frac{}{a^{10}+4\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(a^7 - 25*a^5*b^2 + 35*a^3*b^4 - 3*a*b^6)*(d*x + c)/(a^10 + 5*a^8*b^
2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + 24*(a^6*b - 5*a^4*b^3 + 2
*a^2*b^5)*log(b*tan(d*x + c) + a)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b
^6 + 5*a^2*b^8 + b^10) - 12*(a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*log(tan(d*x + c
)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) -
(38*a^6*b - 56*a^4*b^3 + 2*a^2*b^5 + 3*(7*a^5*b^2 - 22*a^3*b^4 + 3*a*b^6)*t
an(d*x + c)^5 + 6*(5*a^6*b - 12*a^4*b^3 - a^2*b^5)*tan(d*x + c)^4 + (5*a^7
+ 49*a^5*b^2 - 133*a^3*b^4 + 15*a*b^6)*tan(d*x + c)^3 + 2*(35*a^6*b - 61*a^
4*b^3 + a^2*b^5 + b^7)*tan(d*x + c)^2 + (3*a^7 + 22*a^5*b^2 - 73*a^3*b^4 +
4*a*b^6)*tan(d*x + c))/(a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^8
+ (a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*tan(d*x + c)^6 + 2*(
a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)^5 + (a^10 +
 6*a^8*b^2 + 14*a^6*b^4 + 16*a^4*b^6 + 9*a^2*b^8 + 2*b^10)*tan(d*x + c)^4 +
 4*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)^3 + (2*
a^10 + 9*a^8*b^2 + 16*a^6*b^4 + 14*a^4*b^6 + 6*a^2*b^8 + b^10)*tan(d*x + c)
^2 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)))/d
```

mupad [B] time = 5.34, size = 717, normalized size = 2.52

$$\frac{\ln\left(a+b\tan\left(c+d\,x\right)\right)\left(\frac{3\,b}{\left(a^2+b^2\right)^2}-\frac{24\,b^3}{\left(a^2+b^2\right)^3}+\frac{45\,b^5}{\left(a^2+b^2\right)^4}-\frac{24\,b^7}{\left(a^2+b^2\right)^5}\right)}{d}-\frac{\frac{19\,a^6\,b-28\,a^4\,b^3+a^2\,b^5}{4\left(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8\right)}+\frac{\tan(c+d\,x)^2\left(35\,a^6\,b-\right.}{4\left(a^8+4\,a^6\,b^2+6\,a^4\right.}}{d\left(\tan\left(c+d\,x\right)^2\,\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b*tan(c + d*x))^3,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((3*b)/(a^2 + b^2)^2 - (24*b^3)/(a^2 + b^2)^3 + (4
5*b^5)/(a^2 + b^2)^4 - (24*b^7)/(a^2 + b^2)^5))/d - ((19*a^6*b + a^2*b^5 -
28*a^4*b^3)/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (tan(c +
d*x)^2*(35*a^6*b + b^7 + a^2*b^5 - 61*a^4*b^3))/(4*(a^8 + b^8 + 4*a^2*b^6 +
 6*a^4*b^4 + 4*a^6*b^2)) - (3*tan(c + d*x)^4*(a^2*b^5 - 5*a^6*b + 12*a^4*b^
3))/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (3*tan(c + d*x)^5
*(3*a*b^6 - 22*a^3*b^4 + 7*a^5*b^2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4
+ 4*a^6*b^2)) + (tan(c + d*x)^3*(15*a*b^6 + 5*a^7 - 133*a^3*b^4 + 49*a^5*b^
2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (a*tan(c + d*x)*(
3*a^6 + 4*b^6 - 73*a^2*b^4 + 22*a^4*b^2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4
*b^4 + 4*a^6*b^2)))/(d*(tan(c + d*x)^2*(2*a^2 + b^2) + tan(c + d*x)^4*(a^2
+ 2*b^2) + a^2 + b^2*tan(c + d*x)^6 + 2*a*b*tan(c + d*x) + 4*a*b*tan(c + d*
x)^3 + 2*a*b*tan(c + d*x)^5)) - (3*log(tan(c + d*x) - 1i)*(3*a*b + a^2*1i))
/(16*d*(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)) - (3
*log(tan(c + d*x) + 1i)*(3*a*b - a^2*1i))/(16*d*(5*a*b^4 - a^4*b*5i + a^5 -
 b^5*1i + a^2*b^3*10i - 10*a^3*b^2))
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

3.69 $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=206

$$-\frac{a^2b}{2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}-\frac{2ab\left(a^2-b^2\right)}{d\left(a^2+b^2\right)^3(a+b\tan(c+dx))}-\frac{\cos^2(c+dx)\left(a\left(a^2-3b^2\right)\tan(c+dx)+b\left(\right.\right.}{2d\left(a^2+b^2\right)^3}$$

```
[Out] 1/2*a*(a^4-14*a^2*b^2+9*b^4)*x/(a^2+b^2)^4+b*(3*a^4-8*a^2*b^2+b^4)*ln(a*cos
(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^4/d-1/2*a^2*b/(a^2+b^2)^2/d/(a+b*tan(d*x+c)
)^2-2*a*b*(a^2-b^2)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))-1/2*cos(d*x+c)^2*(b*(3*a
^2-b^2)+a*(a^2-3*b^2)*tan(d*x+c))/(a^2+b^2)^3/d
```

Rubi [A] time = 0.40, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$-\frac{a^2b}{2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}-\frac{2ab\left(a^2-b^2\right)}{d\left(a^2+b^2\right)^3(a+b\tan(c+dx))}-\frac{\cos^2(c+dx)\left(a\left(a^2-3b^2\right)\tan(c+dx)+b\left(\right.\right.}{2d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(a^4 - 14*a^2*b^2 + 9*b^4)*x)/(2*(a^2 + b^2)^4) + (b*(3*a^4 - 8*a^2*b^2
+ b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) - (a^2*b)/(2
*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) - (2*a*b*(a^2 - b^2))/((a^2 + b^2)
^3*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^2*(b*(3*a^2 - b^2) + a*(a^2 - 3*
b^2)*Tan[c + d*x]))/(2*(a^2 + b^2)^3*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
```

```
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^2}{(a+x)^3\left(b^2+x^2\right)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^2(c+dx)\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^3 d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^4b^2\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^3}+\ldots}{\ldots}\right)}{\ldots}$$

$$= -\frac{\cos^2(c+dx)\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^3 d} - \frac{\text{Subst}\left(\int \left(-\frac{2a^2b^2}{\left(a^2+b^2\right)^2(a+x)\ldots}\right)\right)}{\ldots}$$

$$= \frac{b\left(3a^4-8a^2b^2+b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^4 d} - \frac{a^2b}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} - \ldots$$

$$= \frac{b\left(3a^4-8a^2b^2+b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^4 d} - \frac{a^2b}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} - \ldots$$

$$= \frac{a\left(a^4-14a^2b^2+9b^4\right)x}{2\left(a^2+b^2\right)^4} + \frac{b\left(3a^4-8a^2b^2+b^4\right)\log(\cos(c+dx))}{\left(a^2+b^2\right)^4 d} + \frac{b\left(3a^4-8a^2b^2\ldots\right.}{\ldots}$$

Mathematica [A] time = 4.00, size = 316, normalized size = 1.53

$$-\frac{b\left(\frac{4\left(a^5-ab^4\right)}{a+b\tan(c+dx)} + \frac{a\left(a^2-3b^2\right)\left(a^2+b^2\right)\sin(2(c+dx))}{2b} + \left(3a^2-b^2\right)\left(a^2+b^2\right)\cos^2(c+dx) + \frac{a\left(a^2-3b^2\right)\left(a^2+b^2\right)\tan^{-1}(\tan(c+dx))}{b} + \ldots\right)}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*(b*((a*(a^2 - 3*b^2)*(a^2 + b^2)*ArcTan[Tan[c + d*x]])/b + (3*a^2 - b^
2)*(a^2 + b^2)*Cos[c + d*x]^2 + (3*a^4 - 8*a^2*b^2 + b^4 - (a^5 - 8*a^3*b^2
 + 3*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*(3*a^4 - 8*a^2
*b^2 + b^4)*Log[a + b*Tan[c + d*x]] + (3*a^4 - 8*a^2*b^2 + b^4 + (a^5 - 8*a
^3*b^2 + 3*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (a*(a^2 -
3*b^2)*(a^2 + b^2)*Sin[2*(c + d*x)])/(2*b) + (a^2*(a^2 + b^2)^2)/(a + b*Tan
[c + d*x])^2 + (4*(a^5 - a*b^4))/(a + b*Tan[c + d*x])))/((a^2 + b^2)^4*d)
```

fricas [B] time = 0.52, size = 526, normalized size = 2.55

$$\frac{13\,a^4b^3 - 8\,a^2b^5 - b^7 - 2\left(a^6b + 3\,a^4b^3 + 3\,a^2b^5 + b^7\right)\cos\left(dx+c\right)^4 + 2\left(a^5b^2 - 14\,a^3b^4 + 9\,ab^6\right)dx - \left(a^6b + 23\,a^4b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(13*a^4*b^3 - 8*a^2*b^5 - b^7 - 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)
*cos(d*x + c)^4 + 2*(a^5*b^2 - 14*a^3*b^4 + 9*a*b^6)*d*x - (a^6*b + 23*a^4*
b^3 - 21*a^2*b^5 - 3*b^7 - 2*(a^7 - 15*a^5*b^2 + 23*a^3*b^4 - 9*a*b^6)*d*x)
*cos(d*x + c)^2 + 2*(3*a^4*b^3 - 8*a^2*b^5 + b^7 + (3*a^6*b - 11*a^4*b^3 +
9*a^2*b^5 - b^7)*cos(d*x + c)^2 + 2*(3*a^5*b^2 - 8*a^3*b^4 + a*b^6)*cos(d*x
 + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d
*x + c)^2 + b^2) - 2*((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^3
- 2*(4*a^5*b^2 - 3*a^3*b^4 + 3*a*b^6 + (a^6*b - 14*a^4*b^3 + 9*a^2*b^5)*d*x
)*cos(d*x + c))*sin(d*x + c))/((a^10 + 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 -
3*a^2*b^8 - b^10)*d*cos(d*x + c)^2 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a
^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) + (a^8*b^2 + 4*a^6*b^4 + 6*a^4*
b^6 + 4*a^2*b^8 + b^10)*d)
```

giac [B] time = 1.23, size = 482, normalized size = 2.34

$$\frac{\frac{\left(a^5-14\,a^3b^2+9\,ab^4\right)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} - \frac{\left(3\,a^4b-8\,a^2b^3+b^5\right)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{2\left(3\,a^4b^2-8\,a^2b^4+b^6\right)\log(|b\tan(dx+c)+a|)}{a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9} + \frac{3\,a^4b\tan(dx+c)^2-8\,a^2b^3\tan}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*((a^5 - 14*a^3*b^2 + 9*a*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
4*a^2*b^6 + b^8) - (3*a^4*b - 8*a^2*b^3 + b^5)*log(tan(d*x + c)^2 + 1)/(a^8
 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 2*(3*a^4*b^2 - 8*a^2*b^4 + b^
6)*log(abs(b*tan(d*x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7
+ b^9) + (3*a^4*b*tan(d*x + c)^2 - 8*a^2*b^3*tan(d*x + c)^2 + b^5*tan(d*x +
 c)^2 - a^5*tan(d*x + c) + 2*a^3*b^2*tan(d*x + c) + 3*a*b^4*tan(d*x + c) -
10*a^2*b^3 + 2*b^5)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(tan(d
*x + c)^2 + 1)) - (9*a^4*b^3*tan(d*x + c)^2 - 24*a^2*b^5*tan(d*x + c)^2 + 3
*b^7*tan(d*x + c)^2 + 22*a^5*b^2*tan(d*x + c) - 48*a^3*b^4*tan(d*x + c) + 2
*a*b^6*tan(d*x + c) + 14*a^6*b - 22*a^4*b^3)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8)*(b*tan(d*x + c) + a)^2))/d
```

maple [B] time = 0.49, size = 542, normalized size = 2.63

$$-\frac{a^2b}{2\left(a^2+b^2\right)^2 d\left(a+b\tan\left(dx+c\right)\right)^2} + \frac{3b\ln\left(a+b\tan\left(dx+c\right)\right)a^4}{d\left(a^2+b^2\right)^4} - \frac{8b^3\ln\left(a+b\tan\left(dx+c\right)\right)a^2}{d\left(a^2+b^2\right)^4} + \frac{b^5\ln\left(a+b\tan\right.}{d\left(a^2+b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x)
```

```
[Out] -1/2*a^2*b/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2+3/d*b/(a^2+b^2)^4*ln(a+b*tan(d*
x+c))*a^4-8/d*b^3/(a^2+b^2)^4*ln(a+b*tan(d*x+c))*a^2+1/d*b^5/(a^2+b^2)^4*ln
(a+b*tan(d*x+c))-2/d*a^3*b/(a^2+b^2)^3/(a+b*tan(d*x+c))+2/d*a*b^3/(a^2+b^2)
^3/(a+b*tan(d*x+c))-1/2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)*tan(d*x+c)*a^5+1/d/(
a^2+b^2)^4/(1+tan(d*x+c)^2)*tan(d*x+c)*b^2*a^3+3/2/d/(a^2+b^2)^4/(1+tan(d*x
+c)^2)*tan(d*x+c)*a*b^4-3/2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)*b*a^4-1/d/(a^2+b
^2)^4/(1+tan(d*x+c)^2)*a^2*b^3+1/2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)*b^5-3/2/d
```

```
/(a^2+b^2)^4*ln(1+tan(d*x+c)^2)*b*a^4+4/d/(a^2+b^2)^4*ln(1+tan(d*x+c)^2)*a^
2*b^3-1/2/d/(a^2+b^2)^4*ln(1+tan(d*x+c)^2)*b^5-7/d/(a^2+b^2)^4*arctan(tan(d
*x+c))*b^2*a^3+9/2/d/(a^2+b^2)^4*arctan(tan(d*x+c))*a*b^4+1/2/d/(a^2+b^2)^4
*arctan(tan(d*x+c))*a^5
```

maxima [B] time = 0.91, size = 463, normalized size = 2.25

$$\frac{\frac{(a^5-14a^3b^2+9ab^4)(dx+c)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} + \frac{2(3a^4b-8a^2b^3+b^5)\log(b\tan(dx+c)+a)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} - \frac{(3a^4b-8a^2b^3+b^5)\log(\tan(dx+c)^2+1)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} - \frac{}{a^8+3a^6b^2+3a^4b^4+a^2b^6+(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((a^5 - 14*a^3*b^2 + 9*a*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
4*a^2*b^6 + b^8) + 2*(3*a^4*b - 8*a^2*b^3 + b^5)*log(b*tan(d*x + c) + a)/(a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - (3*a^4*b - 8*a^2*b^3 + b^5)
*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) -
(8*a^4*b - 4*a^2*b^3 + (5*a^3*b^2 - 7*a*b^4)*tan(d*x + c)^3 + (7*a^4*b - 6*
a^2*b^3 - b^5)*tan(d*x + c)^2 + (a^5 + 7*a^3*b^2 - 6*a*b^4)*tan(d*x + c))/(
a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 +
b^8)*tan(d*x + c)^4 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*tan(d*x + c
)^3 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*tan(d*x + c)^2 + 2*(a
^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*tan(d*x + c)))/d
```

mupad [B] time = 4.62, size = 433, normalized size = 2.10

$$\frac{\frac{\tan(c+d\,x)^2\,\left(-7\,a^4\,b+6\,a^2\,b^3+b^5\right)}{2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)} + \frac{\tan(c+d\,x)^3\,\left(7\,a\,b^4-5\,a^3\,b^2\right)}{2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)} - \frac{2\,a^2\,\left(2\,a^2\,b-b^3\right)}{\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)} - \frac{a\,\tan(c+d\,x)\,\left(a^4+7\,a^2\,b^2-6\,b^4\right)}{2\,\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(\tan\left(c+d\,x\right)^2\,\left(a^2+b^2\right)+a^2+b^2\,\tan\left(c+d\,x\right)^4+2\,a\,b\,\tan\left(c+d\,x\right)+2\,a\,b\,\tan\left(c+d\,x\right)^3\right)} + \frac{\ln\left(a+b\,\mathrm{tan}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b*tan(c + d*x))^3,x)
```

```
[Out] ((tan(c + d*x)^2*(b^5 - 7*a^4*b + 6*a^2*b^3))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3
*a^4*b^2)) + (tan(c + d*x)^3*(7*a*b^4 - 5*a^3*b^2))/(2*(a^6 + b^6 + 3*a^2*b
^4 + 3*a^4*b^2)) - (2*a^2*(2*a^2*b - b^3))/((a^2 + b^2)*(a^4 + b^4 + 2*a^2*
b^2)) - (a*tan(c + d*x)*(a^4 - 6*b^4 + 7*a^2*b^2))/(2*(a^2 + b^2)*(a^4 + b^
4 + 2*a^2*b^2)))/(d*(tan(c + d*x)^2*(a^2 + b^2) + a^2 + b^2*tan(c + d*x)^4
+ 2*a*b*tan(c + d*x) + 2*a*b*tan(c + d*x)^3)) + (log(a + b*tan(c + d*x))*((
3*b)/(a^2 + b^2)^2 - (14*b^3)/(a^2 + b^2)^3 + (12*b^5)/(a^2 + b^2)^4))/d +
(log(tan(c + d*x) + 1i)*(a*1i - 2*b))/(4*d*(a*b^3*4i - a^3*b*4i + a^4 + b^4
 - 6*a^2*b^2)) + (log(tan(c + d*x) - 1i)*(a - b*2i))/(4*d*(4*a*b^3 - 4*a^3*
b + a^4*1i + b^4*1i - a^2*b^2*6i))
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

3.70 $\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=95

$$-\frac{3b\log(\tan(c+dx))}{a^4d}+\frac{3b\log(a+b\tan(c+dx))}{a^4d}-\frac{2b}{a^3d(a+b\tan(c+dx))}-\frac{\cot(c+dx)}{a^3d}-\frac{b}{2a^2d(a+b\tan(c+dx))^2}$$

```
[Out] -cot(d*x+c)/a^3/d-3*b*ln(tan(d*x+c))/a^4/d+3*b*ln(a+b*tan(d*x+c))/a^4/d-1/2
*b/a^2/d/(a+b*tan(d*x+c))^2-2*b/a^3/d/(a+b*tan(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 44}

$$-\frac{2b}{a^3d(a+b\tan(c+dx))}-\frac{b}{2a^2d(a+b\tan(c+dx))^2}-\frac{3b\log(\tan(c+dx))}{a^4d}+\frac{3b\log(a+b\tan(c+dx))}{a^4d}-\frac{\cot(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(Cot[c + d*x]/(a^3*d)) - (3*b*Log[Tan[c + d*x]])/(a^4*d) + (3*b*Log[a + b*
Tan[c + d*x]])/(a^4*d) - b/(2*a^2*d*(a + b*Tan[c + d*x])^2) - (2*b)/(a^3*d*
(a + b*Tan[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= \frac{b\,\text{Subst}\left(\int \frac{1}{x^2(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{d}\\ &= \frac{b\,\text{Subst}\left(\int \left(\frac{1}{a^3x^2}-\frac{3}{a^4x}+\frac{1}{a^2(a+x)^3}+\frac{2}{a^3(a+x)^2}+\frac{3}{a^4(a+x)}\right)dx, x, b\tan(c+dx)\right)}{d}\\ &= -\frac{\cot(c+dx)}{a^3d}-\frac{3b\log(\tan(c+dx))}{a^4d}+\frac{3b\log(a+b\tan(c+dx))}{a^4d}-\frac{b}{2a^2d(a+b\tan(c\end{aligned}$$

Mathematica [B] time = 2.69, size = 241, normalized size = 2.54

$$b\left(a^2\left(-b^2\right)\sec^2(c+dx)-2a^2\left(a^2+b^2\right)(-3\log(a\cos(c+dx)+b\sin(c+dx))+3\log(\sin(c+dx))+2)-2b^2\tan$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-2*a^3*(a^2 + b^2)*Cot[c + d*x] + b*(-2*a^2*(a^2 + b^2)*(2 + 3*Log[Sin[c +
 d*x]] - 3*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]) - a^2*b^2*Sec[c + d*x]^2 +
 2*a*b*(2*a^2 + b^2 - 6*(a^2 + b^2)*Log[Sin[c + d*x]] + 6*(a^2 + b^2)*Log[a
*Cos[c + d*x] + b*Sin[c + d*x]])*Tan[c + d*x] - 2*b^2*(-3*a^2 - 2*b^2 + 3*(
a^2 + b^2)*Log[Sin[c + d*x]] - 3*(a^2 + b^2)*Log[a*Cos[c + d*x] + b*Sin[c +
 d*x]])*Tan[c + d*x]^2))/(2*a^4*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
```

fricas [B] time = 0.54, size = 565, normalized size = 5.95

$$\frac{2\left(a^7 + 4\,a^5b^2 - 2\,a^3b^4 - 3\,ab^6\right)\cos\left(dx+c\right)^3 - 2\left(2\,a^5b^2 - 3\,a^3b^4 - 3\,ab^6\right)\cos\left(dx+c\right) + 3\left(2\left(a^5b^2 + 2\,a^3b^4 + \ldots\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a^7 + 4*a^5*b^2 - 2*a^3*b^4 - 3*a*b^6)*cos(d*x + c)^3 - 2*(2*a^5*b^
2 - 3*a^3*b^4 - 3*a*b^6)*cos(d*x + c) + 3*(2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*
cos(d*x + c)^3 - 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c) - (a^4*b^3 +
2*a^2*b^5 + b^7 + (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*cos(d*x + c)^2)*sin(d*x
 + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b
^2) - 3*(2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 2*(a^5*b^2 + 2*a^
3*b^4 + a*b^6)*cos(d*x + c) - (a^4*b^3 + 2*a^2*b^5 + b^7 + (a^6*b + a^4*b^3
 - a^2*b^5 - b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/4*cos(d*x + c)^2 + 1
/4) - (5*a^4*b^3 + 3*a^2*b^5 - 4*(a^6*b + 5*a^4*b^3 + 3*a^2*b^5)*cos(d*x +
c)^2)*sin(d*x + c))/(2*(a^9*b + 2*a^7*b^3 + a^5*b^5)*d*cos(d*x + c)^3 - 2*(
a^9*b + 2*a^7*b^3 + a^5*b^5)*d*cos(d*x + c) - ((a^10 + a^8*b^2 - a^6*b^4 -
a^4*b^6)*d*cos(d*x + c)^2 + (a^8*b^2 + 2*a^6*b^4 + a^4*b^6)*d)*sin(d*x + c)
)
```

giac [A] time = 1.04, size = 113, normalized size = 1.19

$$\frac{\frac{6\,b\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^4} - \frac{6\,b\log\left(\left|\tan\left(dx+c\right)\right|\right)}{a^4} + \frac{2\left(3\,b\tan\left(dx+c\right)-a\right)}{a^4\tan\left(dx+c\right)} - \frac{9\,b^3\tan\left(dx+c\right)^2+22\,ab^2\tan\left(dx+c\right)+14\,a^2b}{\left(b\tan\left(dx+c\right)+a\right)^2a^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*b*log(abs(b*tan(d*x + c) + a))/a^4 - 6*b*log(abs(tan(d*x + c)))/a^4
+ 2*(3*b*tan(d*x + c) - a)/(a^4*tan(d*x + c)) - (9*b^3*tan(d*x + c)^2 + 22*
a*b^2*tan(d*x + c) + 14*a^2*b)/((b*tan(d*x + c) + a)^2*a^4))/d
```

maple [A] time = 0.52, size = 96, normalized size = 1.01

$$-\frac{b}{2a^2d\left(a+b\tan\left(dx+c\right)\right)^2}+\frac{3b\ln\left(a+b\tan\left(dx+c\right)\right)}{a^4d}-\frac{2b}{a^3d\left(a+b\tan\left(dx+c\right)\right)}-\frac{1}{d\,a^3\tan\left(dx+c\right)}-\frac{3b\ln\left(\tan\left(\ldots\right.\right.}{a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x)
```

```
[Out] -1/2*b/a^2/d/(a+b*tan(d*x+c))^2+3*b*ln(a+b*tan(d*x+c))/a^4/d-2*b/a^3/d/(a+b
*tan(d*x+c))-1/d/a^3/tan(d*x+c)-3*b*ln(tan(d*x+c))/a^4/d
```

maxima [A] time = 0.48, size = 108, normalized size = 1.14

$$-\frac{\frac{6\,b^2\tan\left(dx+c\right)^2+9\,ab\tan\left(dx+c\right)+2\,a^2}{a^3b^2\tan\left(dx+c\right)^3+2\,a^4b\tan\left(dx+c\right)^2+a^5\tan\left(dx+c\right)} - \frac{6\,b\log\left(b\tan\left(dx+c\right)+a\right)}{a^4} + \frac{6\,b\log\left(\tan\left(dx+c\right)\right)}{a^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((6*b^2*tan(d*x + c)^2 + 9*a*b*tan(d*x + c) + 2*a^2)/(a^3*b^2*tan(d*x
+ c)^3 + 2*a^4*b*tan(d*x + c)^2 + a^5*tan(d*x + c)) - 6*b*log(b*tan(d*x + c
) + a)/a^4 + 6*b*log(tan(d*x + c))/a^4)/d
```

mupad [B] time = 3.87, size = 99, normalized size = 1.04

$$\frac{6\,b\,\mathrm{atanh}\left(\frac{2\,b\,\tan(c+d\,x)}{a}+1\right)}{a^4\,d}-\frac{\frac{1}{a}+\frac{3\,b^2\,\tan(c+d\,x)^2}{a^3}+\frac{9\,b\,\tan(c+d\,x)}{2\,a^2}}{d\,\left(a^2\,\tan\left(c+d\,x\right)+2\,a\,b\,{\tan\left(c+d\,x\right)}^2+b^2\,{\tan\left(c+d\,x\right)}^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b*tan(c + d*x))^3),x)
```

```
[Out] (6*b*atanh((2*b*tan(c + d*x))/a + 1))/(a^4*d) - (1/a + (3*b^2*tan(c + d*x)^
2)/a^3 + (9*b*tan(c + d*x))/(2*a^2))/(d*(a^2*tan(c + d*x) + b^2*tan(c + d*x
)^3 + 2*a*b*tan(c + d*x)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*tan(c + d*x))**3, x)
```

3.71 $\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=178

$$\frac{3b\cot^2(c+dx)}{2a^4d}-\frac{\cot^3(c+dx)}{3a^3d}-\frac{b\left(3a^2+10b^2\right)\log(\tan(c+dx))}{a^6d}+\frac{b\left(3a^2+10b^2\right)\log(a+b\tan(c+dx))}{a^6d}-\frac{2\dots}{a^5d(a\dots}$$

```
[Out] -(a^2+6*b^2)*cot(d*x+c)/a^5/d+3/2*b*cot(d*x+c)^2/a^4/d-1/3*cot(d*x+c)^3/a^3
/d-b*(3*a^2+10*b^2)*ln(tan(d*x+c))/a^6/d+b*(3*a^2+10*b^2)*ln(a+b*tan(d*x+c)
)/a^6/d-1/2*b*(a^2+b^2)/a^4/d/(a+b*tan(d*x+c))^2-2*b*(a^2+2*b^2)/a^5/d/(a+b
*tan(d*x+c))
```

Rubi [A] time = 0.15, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 894}

$$-\frac{2b\left(a^2+2b^2\right)}{a^5d(a+b\tan(c+dx))}-\frac{b\left(a^2+b^2\right)}{2a^4d(a+b\tan(c+dx))^2}-\frac{\left(a^2+6b^2\right)\cot(c+dx)}{a^5d}-\frac{b\left(3a^2+10b^2\right)\log(\tan(c+dx))}{a^6d}+\dots$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(((a^2 + 6*b^2)*Cot[c + d*x])/(a^5*d)) + (3*b*Cot[c + d*x]^2)/(2*a^4*d) -
Cot[c + d*x]^3/(3*a^3*d) - (b*(3*a^2 + 10*b^2)*Log[Tan[c + d*x]])/(a^6*d) +
 (b*(3*a^2 + 10*b^2)*Log[a + b*Tan[c + d*x]])/(a^6*d) - (b*(a^2 + b^2))/(2*
a^4*d*(a + b*Tan[c + d*x])^2) - (2*b*(a^2 + 2*b^2))/(a^5*d*(a + b*Tan[c + d
*x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= \frac{b\,\text{Subst}\left(\int \frac{b^2+x^2}{x^4(a+x)^3}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\,\text{Subst}\left(\int \left(\frac{b^2}{a^3x^4}-\frac{3b^2}{a^4x^3}+\frac{a^2+6b^2}{a^5x^2}+\frac{-3a^2-10b^2}{a^6x}+\frac{a^2+b^2}{a^4(a+x)^3}+\frac{2\left(a^2+2b^2\right)}{a^5(a+x)^2}+\frac{3a^2+10b^2}{a^6(a+x)}\right)dx,\dots\right)}{d}\\ &= -\frac{\left(a^2+6b^2\right)\cot(c+dx)}{a^5d}+\frac{3b\cot^2(c+dx)}{2a^4d}-\frac{\cot^3(c+dx)}{3a^3d}-\frac{b\left(3a^2+10b^2\right)\log(\dots}{a^6d}\end{aligned}$$

Mathematica [B] time = 3.41, size = 456, normalized size = 2.56

$$-\frac{b^3\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))}{2a^4d(a+b\tan(c+dx))^3}+\frac{3b\csc^2(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3}{2a^4d(a+b\tan(c+dx))^3}-\dots$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*(b^3*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(a^4*d*(a + b*T
an[c + d*x])^3) - (Csc[c + d*x]^3*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^3)/(3*a^3*d*(a + b*Tan[c + d*x])^3) - (2*(a^2*Cos[c + d*x] + 9*b^2*
Cos[c + d*x])*Csc[c + d*x]*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])
^3)/(3*a^5*d*(a + b*Tan[c + d*x])^3) + (3*b*Csc[c + d*x]^2*Sec[c + d*x]^3*(
a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(2*a^4*d*(a + b*Tan[c + d*x])^3) + ((-3
*a^2*b - 10*b^3)*Log[Sin[c + d*x]]*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^3)/(a^6*d*(a + b*Tan[c + d*x])^3) + ((3*a^2*b + 10*b^3)*Log[a*Cos[
c + d*x] + b*Sin[c + d*x]]*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])
^3)/(a^6*d*(a + b*Tan[c + d*x])^3) + (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Si
n[c + d*x])^2*(3*a^2*b^2*Sin[c + d*x] + 4*b^4*Sin[c + d*x]))/(a^6*d*(a + b*
Tan[c + d*x])^3)
```

fricas [B] time = 0.56, size = 811, normalized size = 4.56

$$\frac{2\left(2a^{7}+27a^{5}b^{2}+a^{3}b^{4}-30ab^{6}\right)\cos\left(dx+c\right)^{5}-2\left(3a^{7}+43a^{5}b^{2}-8a^{3}b^{4}-60ab^{6}\right)\cos\left(dx+c\right)^{3}+6\left(5a^{5}b^{2}-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(2*a^7 + 27*a^5*b^2 + a^3*b^4 - 30*a*b^6)*cos(d*x + c)^5 - 2*(3*a^7
+ 43*a^5*b^2 - 8*a^3*b^4 - 60*a*b^6)*cos(d*x + c)^3 + 6*(5*a^5*b^2 - 3*a^3*
b^4 - 10*a*b^6)*cos(d*x + c) + 3*(2*(3*a^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos
(d*x + c)^5 - 4*(3*a^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos(d*x + c)^3 + 2*(3*a
^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos(d*x + c) + (3*a^4*b^3 + 13*a^2*b^5 + 10
*b^7 - (3*a^6*b + 10*a^4*b^3 - 3*a^2*b^5 - 10*b^7)*cos(d*x + c)^4 + (3*a^6*
b + 7*a^4*b^3 - 16*a^2*b^5 - 20*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*
b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 3*(2*(3*a
^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos(d*x + c)^5 - 4*(3*a^5*b^2 + 13*a^3*b^4
+ 10*a*b^6)*cos(d*x + c)^3 + 2*(3*a^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos(d*x
+ c) + (3*a^4*b^3 + 13*a^2*b^5 + 10*b^7 - (3*a^6*b + 10*a^4*b^3 - 3*a^2*b^5
 - 10*b^7)*cos(d*x + c)^4 + (3*a^6*b + 7*a^4*b^3 - 16*a^2*b^5 - 20*b^7)*cos
(d*x + c)^2)*sin(d*x + c))*log(-1/4*cos(d*x + c)^2 + 1/4) + (24*a^4*b^3 + 3
0*a^2*b^5 + 4*(2*a^6*b + 29*a^4*b^3 + 30*a^2*b^5)*cos(d*x + c)^4 - 3*(a^6*b
 + 45*a^4*b^3 + 50*a^2*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(2*(a^9*b + a^7*b
^3)*d*cos(d*x + c)^5 - 4*(a^9*b + a^7*b^3)*d*cos(d*x + c)^3 + 2*(a^9*b + a^
7*b^3)*d*cos(d*x + c) - ((a^10 - a^6*b^4)*d*cos(d*x + c)^4 - (a^10 - a^8*b^
2 - 2*a^6*b^4)*d*cos(d*x + c)^2 - (a^8*b^2 + a^6*b^4)*d)*sin(d*x + c))
```

giac [A] time = 2.07, size = 237, normalized size = 1.33

$$-\frac{\frac{6\left(3a^{2}b+10b^{3}\right)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{a^{6}}-\frac{6\left(3a^{2}b^{2}+10b^{4}\right)\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^{6}b}+\frac{3\left(9a^{2}b^{3}\tan\left(dx+c\right)^{2}+30b^{5}\tan\left(dx+c\right)^{2}+22a^{3}b^{2}\tan\left(dx+c\right)+68ab^{4}\tan\left(\right.\right.}{\left(b\tan\left(dx+c\right)+a\right)^{2}a^{6}}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(6*(3*a^2*b + 10*b^3)*log(abs(tan(d*x + c)))/a^6 - 6*(3*a^2*b^2 + 10*b
^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b) + 3*(9*a^2*b^3*tan(d*x + c)^2 + 30
*b^5*tan(d*x + c)^2 + 22*a^3*b^2*tan(d*x + c) + 68*a*b^4*tan(d*x + c) + 14*
a^4*b + 39*a^2*b^3)/((b*tan(d*x + c) + a)^2*a^6) - (33*a^2*b*tan(d*x + c)^3
 + 110*b^3*tan(d*x + c)^3 - 6*a^3*tan(d*x + c)^2 - 36*a*b^2*tan(d*x + c)^2
+ 9*a^2*b*tan(d*x + c) - 2*a^3)/(a^6*tan(d*x + c)^3))/d
```

maple [A] time = 0.55, size = 234, normalized size = 1.31

$$\frac{3b\ln\left(a+b\tan\left(dx+c\right)\right)}{a^4d}+\frac{10b^3\ln\left(a+b\tan\left(dx+c\right)\right)}{d\,a^6}-\frac{b}{2a^2d\left(a+b\tan\left(dx+c\right)\right)^2}-\frac{b^3}{2d\,a^4\left(a+b\tan\left(dx+c\right)\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x)
```

```
[Out] 3*b*ln(a+b*tan(d*x+c))/a^4/d+10/d*b^3/a^6*ln(a+b*tan(d*x+c))-1/2*b/a^2/d/(a
+b*tan(d*x+c))^2-1/2/d*b^3/a^4/(a+b*tan(d*x+c))^2-2*b/a^3/d/(a+b*tan(d*x+c)
)-4/d*b^3/a^5/(a+b*tan(d*x+c))-1/3/d/a^3/tan(d*x+c)^3-1/d/a^3/tan(d*x+c)-6/
d/a^5/tan(d*x+c)*b^2+3/2/d/a^4*b/tan(d*x+c)^2-3*b*ln(tan(d*x+c))/a^4/d-10/d
*b^3/a^6*ln(tan(d*x+c))
```

maxima [A] time = 0.40, size = 192, normalized size = 1.08

$$\frac{\frac{5\,a^3b\tan(dx+c)-6\left(3\,a^2b^2+10\,b^4\right)\tan(dx+c)^4-2\,a^4-9\left(3\,a^3b+10\,ab^3\right)\tan(dx+c)^3-2\left(3\,a^4+10\,a^2b^2\right)\tan(dx+c)^2}{a^5b^2\tan(dx+c)^5+2\,a^6b\tan(dx+c)^4+a^7\tan(dx+c)^3}+\frac{6\left(3\,a^2b+10\,b^3\right)\log(b\tan(dx+c)+}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*((5*a^3*b*tan(d*x + c) - 6*(3*a^2*b^2 + 10*b^4)*tan(d*x + c)^4 - 2*a^4
- 9*(3*a^3*b + 10*a*b^3)*tan(d*x + c)^3 - 2*(3*a^4 + 10*a^2*b^2)*tan(d*x +
c)^2)/(a^5*b^2*tan(d*x + c)^5 + 2*a^6*b*tan(d*x + c)^4 + a^7*tan(d*x + c)^3
) + 6*(3*a^2*b + 10*b^3)*log(b*tan(d*x + c) + a)/a^6 - 6*(3*a^2*b + 10*b^3)
*log(tan(d*x + c))/a^6)/d
```

mupad [B] time = 4.37, size = 200, normalized size = 1.12

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{b\left(3\,a^2+10\,b^2\right)\left(a+2\,b\tan(c+d\,x)\right)}{a\left(3\,a^2\,b+10\,b^3\right)}\right)\left(3\,a^2+10\,b^2\right)}{a^6\,d}-\frac{\frac{1}{3\,a}+\frac{\tan(c+d\,x)^2\left(3\,a^2+10\,b^2\right)}{3\,a^3}-\frac{5\,b\tan(c+d\,x)}{6\,a^2}+\frac{b^2\tan(c+d\,x)^4\left(3\,a^2+\right.}{a^5}}{d\left(a^2\tan\left(c+d\,x\right)^3+2\,a\,b\tan\left(c+d\,x\right)^4+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + b*tan(c + d*x))^3),x)
```

```
[Out] (2*b*atanh((b*(3*a^2 + 10*b^2)*(a + 2*b*tan(c + d*x)))/(a*(3*a^2*b + 10*b^3
)))*(3*a^2 + 10*b^2))/(a^6*d) - (1/(3*a) + (tan(c + d*x)^2*(3*a^2 + 10*b^2)
)/(3*a^3) - (5*b*tan(c + d*x))/(6*a^2) + (b^2*tan(c + d*x)^4*(3*a^2 + 10*b^
2))/a^5 + (3*b*tan(c + d*x)^3*(3*a^2 + 10*b^2))/(2*a^4))/(d*(a^2*tan(c + d*
x)^3 + b^2*tan(c + d*x)^5 + 2*a*b*tan(c + d*x)^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*tan(c + d*x))**3, x)
```

3.72 $\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=265

$$\frac{3b\cot^4(c+dx)}{4a^4d}-\frac{\cot^5(c+dx)}{5a^3d}-\frac{2b\left(a^2+b^2\right)\left(a^2+3b^2\right)}{a^7d(a+b\tan(c+dx))}-\frac{b\left(a^2+b^2\right)^2}{2a^6d(a+b\tan(c+dx))^2}+\frac{b\left(3a^2+5b^2\right)\cot^2(c+dx)}{a^6d}-\frac{2(}{}$$

```
[Out] -(a^4+12*a^2*b^2+15*b^4)*cot(d*x+c)/a^7/d+b*(3*a^2+5*b^2)*cot(d*x+c)^2/a^6/
d-2/3*(a^2+3*b^2)*cot(d*x+c)^3/a^5/d+3/4*b*cot(d*x+c)^4/a^4/d-1/5*cot(d*x+c
)^5/a^3/d-b*(3*a^4+20*a^2*b^2+21*b^4)*ln(tan(d*x+c))/a^8/d+b*(3*a^4+20*a^2*
b^2+21*b^4)*ln(a+b*tan(d*x+c))/a^8/d-1/2*b*(a^2+b^2)^2/a^6/d/(a+b*tan(d*x+c
))^2-2*b*(a^2+b^2)*(a^2+3*b^2)/a^7/d/(a+b*tan(d*x+c))
```

Rubi [A] time = 0.24, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 894}

$$-\frac{2b\left(a^2+b^2\right)\left(a^2+3b^2\right)}{a^7d(a+b\tan(c+dx))}-\frac{b\left(a^2+b^2\right)^2}{2a^6d(a+b\tan(c+dx))^2}-\frac{2\left(a^2+3b^2\right)\cot^3(c+dx)}{3a^5d}+\frac{b\left(3a^2+5b^2\right)\cot^2(c+dx)}{a^6d}-\frac{(12a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(((a^4 + 12*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(a^7*d)) + (b*(3*a^2 + 5*b^2)*
Cot[c + d*x]^2)/(a^6*d) - (2*(a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*a^5*d) + (3*b
*Cot[c + d*x]^4)/(4*a^4*d) - Cot[c + d*x]^5/(5*a^3*d) - (b*(3*a^4 + 20*a^2*
b^2 + 21*b^4)*Log[Tan[c + d*x]])/(a^8*d) + (b*(3*a^4 + 20*a^2*b^2 + 21*b^4)
*Log[a + b*Tan[c + d*x]])/(a^8*d) - (b*(a^2 + b^2)^2)/(2*a^6*d*(a + b*Tan[c
 + d*x])^2) - (2*b*(a^2 + b^2)*(a^2 + 3*b^2))/(a^7*d*(a + b*Tan[c + d*x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= \frac{b\,\text{Subst}\left(\int \frac{\left(b^2+x^2\right)^2}{x^6(a+x)^3}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\,\text{Subst}\left(\int \left(\frac{b^4}{a^3x^6}-\frac{3b^4}{a^4x^5}+\frac{2b^2\left(a^2+3b^2\right)}{a^5x^4}-\frac{2\left(3a^2b^2+5b^4\right)}{a^6x^3}+\frac{a^4+12a^2b^2+15b^4}{a^7x^2}+\frac{-3a^4-20a^2b^2-21b^4}{a^8x}\right.\right.}{d}\\ &= -\frac{\left(a^4+12a^2b^2+15b^4\right)\cot(c+dx)}{a^7d}+\frac{b\left(3a^2+5b^2\right)\cot^2(c+dx)}{a^6d}-\frac{2\left(a^2+3b^2\right)\cot^3(}{3a^5d}\end{aligned}$$

Mathematica [A] time = 4.76, size = 494, normalized size = 1.86

$$-\frac{\csc^5(c+dx)\left(960b\left(3a^4+20a^2b^2+21b^4\right)\sin^5(c+dx)(a+b\tan(c+dx))^2(\log(\sin(c+dx))-\log(a\cos(c+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/960*(Csc[c + d*x]^5*(Sec[c + d*x]^2*((8*a^7 + 567*a^5*b^2 + 630*a^3*b^4
- 1215*a*b^6)*Cos[3*(c + d*x)] - (24*a^7 + 619*a^5*b^2 + 630*a^3*b^4 - 675*
a*b^6)*Cos[5*(c + d*x)] + 8*a^7*Cos[7*(c + d*x)] + 187*a^5*b^2*Cos[7*(c + d
*x)] + 210*a^3*b^4*Cos[7*(c + d*x)] - 135*a*b^6*Cos[7*(c + d*x)] - 126*a^6*
b*Sin[3*(c + d*x)] + 1665*a^4*b^3*Sin[3*(c + d*x)] + 4635*a^2*b^5*Sin[3*(c
+ d*x)] + 1890*b^7*Sin[3*(c + d*x)] + 10*a^6*b*Sin[5*(c + d*x)] - 1215*a^4*
b^3*Sin[5*(c + d*x)] - 2565*a^2*b^5*Sin[5*(c + d*x)] - 630*b^7*Sin[5*(c + d
*x)] + 16*a^6*b*Sin[7*(c + d*x)] + 345*a^4*b^3*Sin[7*(c + d*x)] + 585*a^2*b
^5*Sin[7*(c + d*x)] + 90*b^7*Sin[7*(c + d*x)]) + 960*b*(3*a^4 + 20*a^2*b^2
+ 21*b^4)*(Log[Sin[c + d*x]] - Log[a*Cos[c + d*x] + b*Sin[c + d*x]])*Sin[c
+ d*x]^5*(a + b*Tan[c + d*x])^2 + 5*Sec[c + d*x]*(40*a^7 - 27*a^5*b^2 - 42*
a^3*b^4 + 135*a*b^6 - 3*b*(8*a^6 + 89*a^4*b^2 + 345*a^2*b^4 + 210*b^6)*Tan[
c + d*x])))/(a^8*d*(a + b*Tan[c + d*x])^2)
```

fricas [B] time = 0.56, size = 1018, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(4*(8*a^7 + 187*a^5*b^2 + 120*a^3*b^4 - 315*a*b^6)*cos(d*x + c)^7 - 4*
(20*a^7 + 482*a^5*b^2 + 255*a^3*b^4 - 945*a*b^6)*cos(d*x + c)^5 + 10*(6*a^7
 + 157*a^5*b^2 + 60*a^3*b^4 - 378*a*b^6)*cos(d*x + c)^3 - 30*(13*a^5*b^2 +
2*a^3*b^4 - 42*a*b^6)*cos(d*x + c) + 30*(2*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b
^6)*cos(d*x + c)^7 - 6*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c)^5 +
 6*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c)^3 - 2*(3*a^5*b^2 + 20*a
^3*b^4 + 21*a*b^6)*cos(d*x + c) - (3*a^4*b^3 + 20*a^2*b^5 + 21*b^7 + (3*a^6
*b + 17*a^4*b^3 + a^2*b^5 - 21*b^7)*cos(d*x + c)^6 - (6*a^6*b + 31*a^4*b^3
- 18*a^2*b^5 - 63*b^7)*cos(d*x + c)^4 + (3*a^6*b + 11*a^4*b^3 - 39*a^2*b^5
- 63*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c)
 + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 30*(2*(3*a^5*b^2 + 20*a^3*b^4 + 21*a
*b^6)*cos(d*x + c)^7 - 6*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c)^5
 + 6*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c)^3 - 2*(3*a^5*b^2 + 20
*a^3*b^4 + 21*a*b^6)*cos(d*x + c) - (3*a^4*b^3 + 20*a^2*b^5 + 21*b^7 + (3*a
^6*b + 17*a^4*b^3 + a^2*b^5 - 21*b^7)*cos(d*x + c)^6 - (6*a^6*b + 31*a^4*b^
3 - 18*a^2*b^5 - 63*b^7)*cos(d*x + c)^4 + (3*a^6*b + 11*a^4*b^3 - 39*a^2*b^
5 - 63*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/4*cos(d*x + c)^2 + 1/4) -
(285*a^4*b^3 + 630*a^2*b^5 - 8*(8*a^6*b + 195*a^4*b^3 + 315*a^2*b^5)*cos(d*
x + c)^6 + 10*(7*a^6*b + 330*a^4*b^3 + 567*a^2*b^5)*cos(d*x + c)^4 + 15*(a^
6*b - 135*a^4*b^3 - 252*a^2*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(2*a^9*b*d*c
os(d*x + c)^7 - 6*a^9*b*d*cos(d*x + c)^5 + 6*a^9*b*d*cos(d*x + c)^3 - 2*a^9
*b*d*cos(d*x + c) - (a^8*b^2*d + (a^10 - a^8*b^2)*d*cos(d*x + c)^6 - (2*a^1
0 - 3*a^8*b^2)*d*cos(d*x + c)^4 + (a^10 - 3*a^8*b^2)*d*cos(d*x + c)^2)*sin(
d*x + c))
```

giac [A] time = 1.36, size = 382, normalized size = 1.44

$$-\frac{\frac{60\left(3a^4b+20a^2b^3+21b^5\right)\log(|\tan(dx+c)|)}{a^8}-\frac{60\left(3a^4b^2+20a^2b^4+21b^6\right)\log(|b\tan(dx+c)+a|)}{a^8b}+\frac{30\left(9a^4b^3\tan(dx+c)^2+60a^2b^5\tan(dx+c)^2+63\ldots}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*(3*a^4*b + 20*a^2*b^3 + 21*b^5)*log(abs(tan(d*x + c)))/a^8 - 60*(
3*a^4*b^2 + 20*a^2*b^4 + 21*b^6)*log(abs(b*tan(d*x + c) + a))/(a^8*b) + 30*
(9*a^4*b^3*tan(d*x + c)^2 + 60*a^2*b^5*tan(d*x + c)^2 + 63*b^7*tan(d*x + c)
^2 + 22*a^5*b^2*tan(d*x + c) + 136*a^3*b^4*tan(d*x + c) + 138*a*b^6*tan(d*x
 + c) + 14*a^6*b + 78*a^4*b^3 + 76*a^2*b^5)/((b*tan(d*x + c) + a)^2*a^8) -
(411*a^4*b*tan(d*x + c)^5 + 2740*a^2*b^3*tan(d*x + c)^5 + 2877*b^5*tan(d*x
+ c)^5 - 60*a^5*tan(d*x + c)^4 - 720*a^3*b^2*tan(d*x + c)^4 - 900*a*b^4*tan
(d*x + c)^4 + 180*a^4*b*tan(d*x + c)^3 + 300*a^2*b^3*tan(d*x + c)^3 - 40*a^
5*tan(d*x + c)^2 - 120*a^3*b^2*tan(d*x + c)^2 + 45*a^4*b*tan(d*x + c) - 12*
a^5)/(a^8*tan(d*x + c)^5))/d
```

maple [A] time = 0.57, size = 410, normalized size = 1.55

$$\frac{3b\ln\left(a+b\tan\left(dx+c\right)\right)}{a^4d}+\frac{20b^3\ln\left(a+b\tan\left(dx+c\right)\right)}{d\,a^6}+\frac{21b^5\ln\left(a+b\tan\left(dx+c\right)\right)}{d\,a^8}-\frac{b}{2a^2d\left(a+b\tan\left(dx+c\right)\right)^2}-\frac{\ }{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x)
```

```
[Out] 3*b*ln(a+b*tan(d*x+c))/a^4/d+20/d*b^3/a^6*ln(a+b*tan(d*x+c))+21/d*b^5/a^8*l
n(a+b*tan(d*x+c))-1/2*b/a^2/d/(a+b*tan(d*x+c))^2-1/d*b^3/a^4/(a+b*tan(d*x+c
))^2-1/2/d*b^5/a^6/(a+b*tan(d*x+c))^2-2*b/a^3/d/(a+b*tan(d*x+c))-8/d*b^3/a^
5/(a+b*tan(d*x+c))-6/d*b^5/a^7/(a+b*tan(d*x+c))-1/5/d/a^3/tan(d*x+c)^5-2/3/
d/a^3/tan(d*x+c)^3-2/d/a^5/tan(d*x+c)^3*b^2-1/d/a^3/tan(d*x+c)-12/d/a^5/tan
(d*x+c)*b^2-15/d/a^7/tan(d*x+c)*b^4+3/4/d/a^4*b/tan(d*x+c)^4+3/d/a^4*b/tan(
d*x+c)^2+5/d*b^3/a^6/tan(d*x+c)^2-3*b*ln(tan(d*x+c))/a^4/d-20/d*b^3/a^6*ln(
tan(d*x+c))-21/d*b^5/a^8*ln(tan(d*x+c))
```

maxima [A] time = 0.78, size = 281, normalized size = 1.06

$$\frac{\frac{21\,a^5b\tan(dx+c)-60\left(3\,a^4b^2+20\,a^2b^4+21\,b^6\right)\tan(dx+c)^6-12\,a^6-90\left(3\,a^5b+20\,a^3b^3+21\,ab^5\right)\tan(dx+c)^5-20\left(3\,a^6+20\,a^4b^2+21\,a^2b^4\right)\tan(dx+c)^4+5\left(20\,a\right.}{a^7b^2\tan(dx+c)^7+2\,a^8b\tan(dx+c)^6+a^9\tan(dx+c)^5}}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*((21*a^5*b*tan(d*x + c) - 60*(3*a^4*b^2 + 20*a^2*b^4 + 21*b^6)*tan(d*x
 + c)^6 - 12*a^6 - 90*(3*a^5*b + 20*a^3*b^3 + 21*a*b^5)*tan(d*x + c)^5 - 20
*(3*a^6 + 20*a^4*b^2 + 21*a^2*b^4)*tan(d*x + c)^4 + 5*(20*a^5*b + 21*a^3*b^
3)*tan(d*x + c)^3 - 2*(20*a^6 + 21*a^4*b^2)*tan(d*x + c)^2)/(a^7*b^2*tan(d*
x + c)^7 + 2*a^8*b*tan(d*x + c)^6 + a^9*tan(d*x + c)^5) + 60*(3*a^4*b + 20*
a^2*b^3 + 21*b^5)*log(b*tan(d*x + c) + a)/a^8 - 60*(3*a^4*b + 20*a^2*b^3 +
21*b^5)*log(tan(d*x + c))/a^8)/d
```

mupad [B] time = 5.32, size = 297, normalized size = 1.12

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{b\,(a+2\,b\tan(c+d\,x))\left(3\,a^4+20\,a^2\,b^2+21\,b^4\right)}{a\left(3\,a^4\,b+20\,a^2\,b^3+21\,b^5\right)}\right)\left(3\,a^4+20\,a^2\,b^2+21\,b^4\right)}{a^8\,d}-\frac{\frac{1}{5\,a}+\frac{\tan(c+d\,x)^4\left(3\,a^4+20\,a^2\,b^2+21\,b^4\right)}{3\,a^5}+\frac{\tan(c+d\,x)}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a + b*tan(c + d*x))^3),x)
```

```
[Out] (2*b*atanh((b*(a + 2*b*tan(c + d*x))*(3*a^4 + 21*b^4 + 20*a^2*b^2))/(a*(3*a
^4*b + 21*b^5 + 20*a^2*b^3)))*(3*a^4 + 21*b^4 + 20*a^2*b^2))/(a^8*d) - (1/(
```

```
5*a) + (tan(c + d*x)^4*(3*a^4 + 21*b^4 + 20*a^2*b^2))/(3*a^5) + (tan(c + d*
x)^2*(20*a^2 + 21*b^2))/(30*a^3) - (7*b*tan(c + d*x))/(20*a^2) + (b^2*tan(c
 + d*x)^6*(3*a^4 + 21*b^4 + 20*a^2*b^2))/a^7 + (3*b*tan(c + d*x)^5*(3*a^4 +
 21*b^4 + 20*a^2*b^2))/(2*a^6) - (b*tan(c + d*x)^3*(20*a^2 + 21*b^2))/(12*a
^4))/(d*(a^2*tan(c + d*x)^5 + b^2*tan(c + d*x)^7 + 2*a*b*tan(c + d*x)^6))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**6/(a + b*tan(c + d*x))**3, x)
```

3.73 $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4} \, dx$

Optimal. Leaf size=366

$$-\frac{a^4b}{3d\left(a^2+b^2\right)^3(a+b\tan(c+dx))^3}-\frac{3a^2b\left(a^4-5a^2b^2+2b^4\right)}{d\left(a^2+b^2\right)^5(a+b\tan(c+dx))}+\frac{\cos^4(c+dx)\left(4ab\left(a^2-b^2\right)+\left(a^4-6a^2b^2+\right.\right.}{4d\left(a^2+b^2\right)^4}$$

```
[Out] 1/8*(3*a^8-132*a^6*b^2+370*a^4*b^4-132*a^2*b^6+3*b^8)*x/(a^2+b^2)^6+4*a*b*(
a^2-b^2)*(a^4-8*a^2*b^2+b^4)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^6/d-1/
3*a^4*b/(a^2+b^2)^3/d/(a+b*tan(d*x+c))^3-a^3*b*(a^2-2*b^2)/(a^2+b^2)^4/d/(a
+b*tan(d*x+c))^2-3*a^2*b*(a^4-5*a^2*b^2+2*b^4)/(a^2+b^2)^5/d/(a+b*tan(d*x+c
))+1/4*cos(d*x+c)^4*(4*a*b*(a^2-b^2)+(a^4-6*a^2*b^2+b^4)*tan(d*x+c))/(a^2+b
^2)^4/d-1/8*cos(d*x+c)^2*(16*a*b*(2*a^4-5*a^2*b^2+b^4)+(5*a^6-65*a^4*b^2+55
*a^2*b^4-3*b^6)*tan(d*x+c))/(a^2+b^2)^5/d
```

Rubi [A] time = 1.37, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$-\frac{a^4b}{3d\left(a^2+b^2\right)^3(a+b\tan(c+dx))^3}-\frac{a^3b\left(a^2-2b^2\right)}{d\left(a^2+b^2\right)^4(a+b\tan(c+dx))^2}-\frac{3a^2b\left(-5a^2b^2+a^4+2b^4\right)}{d\left(a^2+b^2\right)^5(a+b\tan(c+dx))}+\frac{\cos^4(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] ((3*a^8 - 132*a^6*b^2 + 370*a^4*b^4 - 132*a^2*b^6 + 3*b^8)*x)/(8*(a^2 + b^2
)^6) + (4*a*b*(a^2 - b^2)*(a^4 - 8*a^2*b^2 + b^4)*Log[a*Cos[c + d*x] + b*Si
n[c + d*x]])/((a^2 + b^2)^6*d) - (a^4*b)/(3*(a^2 + b^2)^3*d*(a + b*Tan[c +
d*x])^3) - (a^3*b*(a^2 - 2*b^2))/((a^2 + b^2)^4*d*(a + b*Tan[c + d*x])^2) -
 (3*a^2*b*(a^4 - 5*a^2*b^2 + 2*b^4))/((a^2 + b^2)^5*d*(a + b*Tan[c + d*x]))
 + (Cos[c + d*x]^4*(4*a*b*(a^2 - b^2) + (a^4 - 6*a^2*b^2 + b^4)*Tan[c + d*x
]))/(4*(a^2 + b^2)^4*d) - (Cos[c + d*x]^2*(16*a*b*(2*a^4 - 5*a^2*b^2 + b^4)
 + (5*a^6 - 65*a^4*b^2 + 55*a^2*b^4 - 3*b^6)*Tan[c + d*x]))/(8*(a^2 + b^2)^
5*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
```

```
d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^4}{(a+x)^4\left(b^2+x^2\right)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^4(c+dx)\left(4ab\left(a^2-b^2\right)+\left(a^4-6a^2b^2+b^4\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^4 d} - \frac{\text{Subst}\left(\int \frac{a^4b^4\left(a^4-6a\right.}{\left(a^2+b\right.}\right.}{}$$

$$= \frac{\cos^4(c+dx)\left(4ab\left(a^2-b^2\right)+\left(a^4-6a^2b^2+b^4\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^4 d} - \frac{\cos^2(c+dx)\left(16a\right.}{}$$

$$= \frac{\cos^4(c+dx)\left(4ab\left(a^2-b^2\right)+\left(a^4-6a^2b^2+b^4\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^4 d} - \frac{\cos^2(c+dx)\left(16a\right.}{}$$

$$= \frac{4ab\left(a^2-b^2\right)\left(a^4-8a^2b^2+b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^6 d} - \frac{a^4b}{3\left(a^2+b^2\right)^3 d(a+b\tan(}$$

$$= \frac{4ab\left(a^2-b^2\right)\left(a^4-8a^2b^2+b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^6 d} - \frac{a^4b}{3\left(a^2+b^2\right)^3 d(a+b\tan(}$$

$$= \frac{\left(3a^8-132a^6b^2+370a^4b^4-132a^2b^6+3b^8\right)x}{8\left(a^2+b^2\right)^6} + \frac{4ab\left(a^2-b^2\right)\left(a^4-8a^2b^2+b^4\right)\mathrm{l}}{\left(a^2+b^2\right)^6 d}$$

Mathematica [A] time = 5.66, size = 564, normalized size = 1.54

$$-\frac{b\left(-24a(a-b)(a+b)\left(a^2+b^2\right)^2\cos^4(c+dx)+\frac{8a^4\left(a^2+b^2\right)^3}{(a+b\tan(c+dx))^3}+\frac{12a^2\left(a^2+b^2\right)\left(a^4-10a^2b^2+5b^4\right)\sin(2(c+dx))}{b}+48a\left(a^2+b^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/24*(b*((24*a^2*(a^2 + b^2)*(a^4 - 10*a^2*b^2 + 5*b^4)*ArcTan[Tan[c + d*x
]])/b + 48*a*(a^2 + b^2)*(2*a^4 - 5*a^2*b^2 + b^4)*Cos[c + d*x]^2 - 24*a*(a
 - b)*(a + b)*(a^2 + b^2)^2*Cos[c + d*x]^4 + 12*a*(4*a^6 - 36*a^4*b^2 + 36*
a^2*b^4 - 4*b^6 + (-a^7 + 24*a^5*b^2 - 45*a^3*b^4 + 10*a*b^6)/Sqrt[-b^2])*L
og[Sqrt[-b^2] - b*Tan[c + d*x]] - 96*a*(a - b)*(a + b)*(a^4 - 8*a^2*b^2 + b
^4)*Log[a + b*Tan[c + d*x]] + 12*a*(4*a^6 - 36*a^4*b^2 + 36*a^2*b^4 - 4*b^6
 + (a^7 - 24*a^5*b^2 + 45*a^3*b^4 - 10*a*b^6)/Sqrt[-b^2])*Log[Sqrt[-b^2] +
b*Tan[c + d*x]] - (6*(a^2 + b^2)^2*(a^4 - 6*a^2*b^2 + b^4)*Cos[c + d*x]^3*S
in[c + d*x])/b + (12*a^2*(a^2 + b^2)*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[2*(c +
d*x)])/b - (9*(a^2 + b^2)^2*(a^4 - 6*a^2*b^2 + b^4)*(2*ArcTan[Tan[c + d*x]]
 + Sin[2*(c + d*x)]))/(2*b) + (8*a^4*(a^2 + b^2)^3)/(a + b*Tan[c + d*x])^3
+ (24*a^3*(a^2 - 2*b^2)*(a^2 + b^2)^2)/(a + b*Tan[c + d*x])^2 + (72*a^2*(a^
2 + b^2)*(a^4 - 5*a^2*b^2 + 2*b^4))/(a + b*Tan[c + d*x])))/((a^2 + b^2)^6*d
)
```

fricas [B] time = 0.67, size = 1053, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(6*(a^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11)*c
os(d*x + c)^7 - 3*(11*a^10*b + 45*a^8*b^3 + 70*a^6*b^5 + 50*a^4*b^7 + 15*a^
2*b^9 + b^11)*cos(d*x + c)^5 - (6*a^10*b + 342*a^8*b^3 - 1830*a^6*b^5 + 614
*a^4*b^7 - 216*a^2*b^9 + 12*b^11 - 3*(3*a^11 - 141*a^9*b^2 + 766*a^7*b^4 -
1242*a^5*b^6 + 399*a^3*b^8 - 9*a*b^10)*d*x)*cos(d*x + c)^3 + 3*(114*a^8*b^3
 - 381*a^6*b^5 + 187*a^4*b^7 - 67*a^2*b^9 + 3*b^11 + 3*(3*a^9*b^2 - 132*a^7
*b^4 + 370*a^5*b^6 - 132*a^3*b^8 + 3*a*b^10)*d*x)*cos(d*x + c) + 48*((a^10*
b - 12*a^8*b^3 + 36*a^6*b^5 - 28*a^4*b^7 + 3*a^2*b^9)*cos(d*x + c)^3 + 3*(a
^8*b^3 - 9*a^6*b^5 + 9*a^4*b^7 - a^2*b^9)*cos(d*x + c) + (a^7*b^4 - 9*a^5*b
^6 + 9*a^3*b^8 - a*b^10 + (3*a^9*b^2 - 28*a^7*b^4 + 36*a^5*b^6 - 12*a^3*b^8
 + a*b^10)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c
) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) + (143*a^7*b^4 - 537*a^5*b^6 + 105*a^
3*b^8 + 33*a*b^10 + 6*(a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^5*b^6 + 5*a^3*b
^8 + a*b^10)*cos(d*x + c)^6 - 15*(a^11 + 3*a^9*b^2 + 2*a^7*b^4 - 2*a^5*b^6
- 3*a^3*b^8 - a*b^10)*cos(d*x + c)^4 + 3*(3*a^8*b^3 - 132*a^6*b^5 + 370*a^4
*b^7 - 132*a^2*b^9 + 3*b^11)*d*x + (216*a^9*b^2 - 734*a^7*b^4 + 1590*a^5*b^
6 - 522*a^3*b^8 - 54*a*b^10 + 3*(9*a^10*b - 399*a^8*b^3 + 1242*a^6*b^5 - 76
6*a^4*b^7 + 141*a^2*b^9 - 3*b^11)*d*x)*cos(d*x + c)^2)*sin(d*x + c))/((a^15
 + 3*a^13*b^2 - 3*a^11*b^4 - 25*a^9*b^6 - 45*a^7*b^8 - 39*a^5*b^10 - 17*a^3
*b^12 - 3*a*b^14)*d*cos(d*x + c)^3 + 3*(a^13*b^2 + 6*a^11*b^4 + 15*a^9*b^6
+ 20*a^7*b^8 + 15*a^5*b^10 + 6*a^3*b^12 + a*b^14)*d*cos(d*x + c) + ((3*a^14
*b + 17*a^12*b^3 + 39*a^10*b^5 + 45*a^8*b^7 + 25*a^6*b^9 + 3*a^4*b^11 - 3*a
^2*b^13 - b^15)*d*cos(d*x + c)^2 + (a^12*b^3 + 6*a^10*b^5 + 15*a^8*b^7 + 20
*a^6*b^9 + 15*a^4*b^11 + 6*a^2*b^13 + b^15)*d)*sin(d*x + c))
```

giac [B] time = 6.92, size = 902, normalized size = 2.46

$$\frac{3\left(3a^{8}-132a^{6}b^{2}+370a^{4}b^{4}-132a^{2}b^{6}+3b^{8}\right)(dx+c)}{a^{12}+6a^{10}b^{2}+15a^{8}b^{4}+20a^{6}b^{6}+15a^{4}b^{8}+6a^{2}b^{10}+b^{12}}-\frac{48\left(a^{7}b-9a^{5}b^{3}+9a^{3}b^{5}-ab^{7}\right)\log\left(\tan(dx+c)^{2}+1\right)}{a^{12}+6a^{10}b^{2}+15a^{8}b^{4}+20a^{6}b^{6}+15a^{4}b^{8}+6a^{2}b^{10}+b^{12}}+\frac{96\left(a^{7}b^{2}-9a^{5}b^{4}+9a^{3}b^{6}-ab^{8}\right)\log}{a^{12}b+6a^{10}b^{3}+15a^{8}b^{5}+20a^{6}b^{7}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^8 - 132*a^6*b^2 + 370*a^4*b^4 - 132*a^2*b^6 + 3*b^8)*(d*x + c)
/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b
^12) - 48*(a^7*b - 9*a^5*b^3 + 9*a^3*b^5 - a*b^7)*log(tan(d*x + c)^2 + 1)/(
a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^1
2) + 96*(a^7*b^2 - 9*a^5*b^4 + 9*a^3*b^6 - a*b^8)*log(abs(b*tan(d*x + c) +
a))/(a^12*b + 6*a^10*b^3 + 15*a^8*b^5 + 20*a^6*b^7 + 15*a^4*b^9 + 6*a^2*b^1
1 + b^13) + 3*(24*a^7*b*tan(d*x + c)^4 - 216*a^5*b^3*tan(d*x + c)^4 + 216*a
^3*b^5*tan(d*x + c)^4 - 24*a*b^7*tan(d*x + c)^4 - 5*a^8*tan(d*x + c)^3 + 60
*a^6*b^2*tan(d*x + c)^3 + 10*a^4*b^4*tan(d*x + c)^3 - 52*a^2*b^6*tan(d*x +
c)^3 + 3*b^8*tan(d*x + c)^3 + 16*a^7*b*tan(d*x + c)^2 - 384*a^5*b^3*tan(d*x
 + c)^2 + 496*a^3*b^5*tan(d*x + c)^2 - 64*a*b^7*tan(d*x + c)^2 - 3*a^8*tan(
d*x + c) + 52*a^6*b^2*tan(d*x + c) - 10*a^4*b^4*tan(d*x + c) - 60*a^2*b^6*t
an(d*x + c) + 5*b^8*tan(d*x + c) - 160*a^5*b^3 + 272*a^3*b^5 - 48*a*b^7)/((
a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^1
2)*(tan(d*x + c)^2 + 1)^2) - 8*(22*a^7*b^4*tan(d*x + c)^3 - 198*a^5*b^6*tan
(d*x + c)^3 + 198*a^3*b^8*tan(d*x + c)^3 - 22*a*b^10*tan(d*x + c)^3 + 75*a^
8*b^3*tan(d*x + c)^2 - 630*a^6*b^5*tan(d*x + c)^2 + 567*a^4*b^7*tan(d*x + c
)^2 - 48*a^2*b^9*tan(d*x + c)^2 + 87*a^9*b^2*tan(d*x + c) - 666*a^7*b^4*tan
(d*x + c) + 531*a^5*b^6*tan(d*x + c) - 36*a^3*b^8*tan(d*x + c) + 35*a^10*b
- 231*a^8*b^3 + 165*a^6*b^5 - 9*a^4*b^7)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 +
 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*(b*tan(d*x + c) + a)^3))/d
```

maple [B] time = 0.54, size = 1215, normalized size = 3.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x)
```

```
[Out] -4/d*b^7*a/(a^2+b^2)^6*ln(a+b*tan(d*x+c))-1/d*b*a^5/(a^2+b^2)^4/(a+b*tan(d*
x+c))^2-6/d*a^2*b^5/(a^2+b^2)^5/(a+b*tan(d*x+c))+4/d*b*a^7/(a^2+b^2)^6*ln(a
+b*tan(d*x+c))+2/d*b^3*a^3/(a^2+b^2)^4/(a+b*tan(d*x+c))^2-33/2/d/(a^2+b^2)^
6*arctan(tan(d*x+c))*a^2*b^6-5/8/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*tan(d*x+c
)^3*a^8+3/8/d/(a^2+b^2)^6*arctan(tan(d*x+c))*b^8+3/8/d/(a^2+b^2)^6*arctan(t
an(d*x+c))*a^8-2/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*tan(d*x+c)^2*a*b^7+13/2/d
/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a^6*b^2-15/2/d/(a^2+b^2)^6/(1+ta
n(d*x+c)^2)^2*tan(d*x+c)*a^2*b^6-5/4/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*tan(d
*x+c)*a^4*b^4+15/2/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*a^6*b^2+5/
4/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*a^4*b^4-13/2/d/(a^2+b^2)^6/
(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*a^2*b^6-4/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*
tan(d*x+c)^2*a^7*b+6/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*tan(d*x+c)^2*a^5*b^3+
8/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*tan(d*x+c)^2*a^3*b^5-36/d*b^3*a^5/(a^2+b
^2)^6*ln(a+b*tan(d*x+c))+7/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*a^3*b^5-3/d/(a^
2+b^2)^6/(1+tan(d*x+c)^2)^2*a*b^7-2/d/(a^2+b^2)^6*ln(1+tan(d*x+c)^2)*a^7*b+
18/d/(a^2+b^2)^6*ln(1+tan(d*x+c)^2)*a^5*b^3-18/d/(a^2+b^2)^6*ln(1+tan(d*x+c
)^2)*a^3*b^5+36/d*b^5*a^3/(a^2+b^2)^6*ln(a+b*tan(d*x+c))-1/3*a^4*b/(a^2+b^2
)^3/d/(a+b*tan(d*x+c))^3+2/d/(a^2+b^2)^6*ln(1+tan(d*x+c)^2)*a*b^7-33/2/d/(a
^2+b^2)^6*arctan(tan(d*x+c))*a^6*b^2+185/4/d/(a^2+b^2)^6*arctan(tan(d*x+c))
*a^4*b^4+3/8/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*b^8-3/8/d/(a^2+b
^2)^6/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a^8+5/8/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^
```

```
2*tan(d*x+c)*b^8-3/d/(a^2+b^2)^6/(1+tan(d*x+c)^2)^2*a^7*b+7/d/(a^2+b^2)^6/(
1+tan(d*x+c)^2)^2*b^3*a^5-3/d*a^6*b/(a^2+b^2)^5/(a+b*tan(d*x+c))+15/d*a^4*b
^3/(a^2+b^2)^5/(a+b*tan(d*x+c))
```

maxima [B] time = 0.94, size = 997, normalized size = 2.72

$$\frac{3\left(3a^8-132a^6b^2+370a^4b^4-132a^2b^6+3b^8\right)(dx+c)}{a^{12}+6a^{10}b^2+15a^8b^4+20a^6b^6+15a^4b^8+6a^2b^{10}+b^{12}}+\frac{96\left(a^7b-9a^5b^3+9a^3b^5-ab^7\right)\log(b\tan(dx+c)+a)}{a^{12}+6a^{10}b^2+15a^8b^4+20a^6b^6+15a^4b^8+6a^2b^{10}+b^{12}}-\frac{48\left(a^7b-9a^5b^3+9a^3b^5-ab^7\right)\log(\text{ta}}{a^{12}+6a^{10}b^2+15a^8b^4+20a^6b^6+15a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/24*(3*(3*a^8 - 132*a^6*b^2 + 370*a^4*b^4 - 132*a^2*b^6 + 3*b^8)*(d*x + c)
/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b
^12) + 96*(a^7*b - 9*a^5*b^3 + 9*a^3*b^5 - a*b^7)*log(b*tan(d*x + c) + a)/(
a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^1
2) - 48*(a^7*b - 9*a^5*b^3 + 9*a^3*b^5 - a*b^7)*log(tan(d*x + c)^2 + 1)/(a^
12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)
 - (176*a^8*b - 608*a^6*b^3 + 176*a^4*b^5 + 3*(29*a^6*b^3 - 185*a^4*b^5 + 1
03*a^2*b^7 - 3*b^9)*tan(d*x + c)^6 + 3*(71*a^7*b^2 - 411*a^5*b^4 + 165*a^3*
b^6 + 7*a*b^8)*tan(d*x + c)^5 + (149*a^8*b - 512*a^6*b^3 - 1006*a^4*b^5 + 6
00*a^2*b^7 - 15*b^9)*tan(d*x + c)^4 + 3*(5*a^9 + 152*a^7*b^2 - 822*a^5*b^4
+ 320*a^3*b^6 + 9*a*b^8)*tan(d*x + c)^3 + (331*a^8*b - 1183*a^6*b^3 - 239*a
^4*b^5 + 315*a^2*b^7)*tan(d*x + c)^2 + 3*(3*a^9 + 73*a^7*b^2 - 423*a^5*b^4
+ 147*a^3*b^6)*tan(d*x + c))/(a^13 + 5*a^11*b^2 + 10*a^9*b^4 + 10*a^7*b^6 +
 5*a^5*b^8 + a^3*b^10 + (a^10*b^3 + 5*a^8*b^5 + 10*a^6*b^7 + 10*a^4*b^9 + 5
*a^2*b^11 + b^13)*tan(d*x + c)^7 + 3*(a^11*b^2 + 5*a^9*b^4 + 10*a^7*b^6 + 1
0*a^5*b^8 + 5*a^3*b^10 + a*b^12)*tan(d*x + c)^6 + (3*a^12*b + 17*a^10*b^3 +
 40*a^8*b^5 + 50*a^6*b^7 + 35*a^4*b^9 + 13*a^2*b^11 + 2*b^13)*tan(d*x + c)^
5 + (a^13 + 11*a^11*b^2 + 40*a^9*b^4 + 70*a^7*b^6 + 65*a^5*b^8 + 31*a^3*b^1
0 + 6*a*b^12)*tan(d*x + c)^4 + (6*a^12*b + 31*a^10*b^3 + 65*a^8*b^5 + 70*a^
6*b^7 + 40*a^4*b^9 + 11*a^2*b^11 + b^13)*tan(d*x + c)^3 + (2*a^13 + 13*a^11
*b^2 + 35*a^9*b^4 + 50*a^7*b^6 + 40*a^5*b^8 + 17*a^3*b^10 + 3*a*b^12)*tan(d
*x + c)^2 + 3*(a^12*b + 5*a^10*b^3 + 10*a^8*b^5 + 10*a^6*b^7 + 5*a^4*b^9 +
a^2*b^11)*tan(d*x + c)))/d
```

mupad [B] time = 5.66, size = 962, normalized size = 2.63

$$\frac{\ln(a+b\tan(c+d\,x))\left(\frac{4\,a\,b}{\left(a^2+b^2\right)^3}-\frac{48\,a\,b^3}{\left(a^2+b^2\right)^4}+\frac{120\,a\,b^5}{\left(a^2+b^2\right)^5}-\frac{80\,a\,b^7}{\left(a^2+b^2\right)^6}\right)}{d}-\frac{\frac{2\left(11\,a^8\,b-38\,a^6\,b^3+11\,a^4\,b^5\right)}{3\left(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+10\,a^4\,b^6+5\,a^2\,b^8+b^{10}\right)}-\frac{\tan(c+d\,x)^6\,(}{8\left(a^{10}+5\,a^8\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b*tan(c + d*x))^4,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((4*a*b)/(a^2 + b^2)^3 - (48*a*b^3)/(a^2 + b^2)^4
+ (120*a*b^5)/(a^2 + b^2)^5 - (80*a*b^7)/(a^2 + b^2)^6))/d - ((2*(11*a^8*b
+ 11*a^4*b^5 - 38*a^6*b^3))/(3*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a
^6*b^4 + 5*a^8*b^2)) - (tan(c + d*x)^6*(3*b^9 - 103*a^2*b^7 + 185*a^4*b^5 -
 29*a^6*b^3))/(8*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8
*b^2)) + (tan(c + d*x)^5*(7*a*b^8 + 165*a^3*b^6 - 411*a^5*b^4 + 71*a^7*b^2)
)/(8*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (ta
n(c + d*x)^2*(331*a^8*b + 315*a^2*b^7 - 239*a^4*b^5 - 1183*a^6*b^3))/(24*(a
^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (tan(c + d
*x)^3*(9*a*b^8 + 5*a^9 + 320*a^3*b^6 - 822*a^5*b^4 + 152*a^7*b^2))/(8*(a^10
 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) - (tan(c + d*x)
^4*(15*b^9 - 149*a^8*b - 600*a^2*b^7 + 1006*a^4*b^5 + 512*a^6*b^3))/(24*(a^
10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (a*tan(c +
```

```
d*x)*(3*a^8 + 147*a^2*b^6 - 423*a^4*b^4 + 73*a^6*b^2))/(8*(a^10 + b^10 + 5*
a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)))/(d*(tan(c + d*x)^2*(3*a*b^
2 + 2*a^3) + tan(c + d*x)^5*(3*a^2*b + 2*b^3) + a^3 + tan(c + d*x)^4*(6*a*b
^2 + a^3) + tan(c + d*x)^3*(6*a^2*b + b^3) + b^3*tan(c + d*x)^7 + 3*a*b^2*t
an(c + d*x)^6 + 3*a^2*b*tan(c + d*x))) + (log(tan(c + d*x) - 1i)*(a*b*14i -
 3*a^2 + 3*b^2))/(16*d*(6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i -
 20*a^3*b^3 + a^4*b^2*15i)) - (log(tan(c + d*x) + 1i)*(a*b*14i + 3*a^2 - 3*
b^2))/(16*d*(6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3
 - a^4*b^2*15i))
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

3.74 $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4} dx$

Optimal. Leaf size=264

$$-\frac{a^2b}{3d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^3}-\frac{ab\left(a^2-b^2\right)}{d\left(a^2+b^2\right)^3(a+b\tan(c+dx))^2}-\frac{b\left(3a^4-8a^2b^2+b^4\right)}{d\left(a^2+b^2\right)^4(a+b\tan(c+dx))}-\frac{\cos^2(c+$$

```
[Out] 1/2*(a^6-25*a^4*b^2+35*a^2*b^4-3*b^6)*x/(a^2+b^2)^5+4*a*b*(a^4-5*a^2*b^2+2*
b^4)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^5/d-1/3*a^2*b/(a^2+b^2)^2/d/(a
+b*tan(d*x+c))^3-a*b*(a^2-b^2)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))^2-b*(3*a^4-8*
a^2*b^2+b^4)/(a^2+b^2)^4/d/(a+b*tan(d*x+c))-1/2*cos(d*x+c)^2*(4*a*b*(a^2-b^
2)+(a^4-6*a^2*b^2+b^4)*tan(d*x+c))/(a^2+b^2)^4/d
```

Rubi [A] time = 0.57, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$-\frac{a^2b}{3d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^3}-\frac{ab\left(a^2-b^2\right)}{d\left(a^2+b^2\right)^3(a+b\tan(c+dx))^2}-\frac{b\left(-8a^2b^2+3a^4+b^4\right)}{d\left(a^2+b^2\right)^4(a+b\tan(c+dx))}-\frac{\cos^2(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] ((a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*x)/(2*(a^2 + b^2)^5) + (4*a*b*(a^4
 - 5*a^2*b^2 + 2*b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^5*
d) - (a^2*b)/(3*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^3) - (a*b*(a^2 - b^2))
/((a^2 + b^2)^3*d*(a + b*Tan[c + d*x])^2) - (b*(3*a^4 - 8*a^2*b^2 + b^4))/(
(a^2 + b^2)^4*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^2*(4*a*b*(a^2 - b^2)
+ (a^4 - 6*a^2*b^2 + b^4)*Tan[c + d*x]))/(2*(a^2 + b^2)^4*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
```

```
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{x^2}{(a+x)^4\left(b^2+x^2\right)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^2(c+dx)\left(4ab\left(a^2-b^2\right)+\left(a^4-6a^2b^2+b^4\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^4 d} - \frac{\mathrm{Subst}\left(\int \frac{-\frac{a^4b^2(a^4}{(a}}{\ldots}\right.}{\ldots}$$

$$= -\frac{\cos^2(c+dx)\left(4ab\left(a^2-b^2\right)+\left(a^4-6a^2b^2+b^4\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^4 d} - \frac{\mathrm{Subst}\left(\int\left(-\frac{\ldots}{(a^2+\ldots}\right.\right.}{\ldots}$$

$$= \frac{4ab\left(a^4-5a^2b^2+2b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} - \frac{a^2b}{3\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))}$$

$$= \frac{4ab\left(a^4-5a^2b^2+2b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} - \frac{a^2b}{3\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))}$$

$$= \frac{\left(a^6-25a^4b^2+35a^2b^4-3b^6\right)x}{2\left(a^2+b^2\right)^5} + \frac{4ab\left(a^4-5a^2b^2+2b^4\right)\log(\cos(c+dx))}{\left(a^2+b^2\right)^5 d} + \frac{4ab}{\ldots}$$

Mathematica [A] time = 3.77, size = 395, normalized size = 1.50

$$-\frac{b\left(12a(a-b)(a+b)\left(a^2+b^2\right)\cos^2(c+dx)+\frac{2a^2\left(a^2+b^2\right)^3}{(a+b\tan(c+dx))^3}+\frac{6a(a-b)(a+b)\left(a^2+b^2\right)^2}{(a+b\tan(c+dx))^2}+\frac{3\left(a^4-6a^2b^2+b^4\right)\left(a^2+b^2\right)\sin(2(c+dx}{2b}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/6*(b*((3*(a^2 + b^2)*(a^4 - 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]])/b + 1
2*a*(a - b)*(a + b)*(a^2 + b^2)*Cos[c + d*x]^2 + 3*(4*a^5 - 20*a^3*b^2 + 8*
a*b^4 + (-a^6 + 15*a^4*b^2 - 15*a^2*b^4 + b^6)/Sqrt[-b^2])*Log[Sqrt[-b^2] -
 b*Tan[c + d*x]] - 24*a*(a^4 - 5*a^2*b^2 + 2*b^4)*Log[a + b*Tan[c + d*x]] +
 3*(4*a^5 - 20*a^3*b^2 + 8*a*b^4 + (a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)/Sq
rt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (3*(a^2 + b^2)*(a^4 - 6*a^2*b^
```

```
2 + b^4)*Sin[2*(c + d*x)])/(2*b) + (2*a^2*(a^2 + b^2)^3)/(a + b*Tan[c + d*x
])^3 + (6*a*(a - b)*(a + b)*(a^2 + b^2)^2)/(a + b*Tan[c + d*x])^2 + (6*(a^2
 + b^2)*(3*a^4 - 8*a^2*b^2 + b^4))/(a + b*Tan[c + d*x]))))/((a^2 + b^2)^5*d)
```

fricas [B] time = 0.56, size = 802, normalized size = 3.04

$$-\frac{3\left(a^8b+4a^6b^3+6a^4b^5+4a^2b^7+b^9\right)\cos(dx+c)^5+\left(3a^8b+111a^6b^3-231a^4b^5+65a^2b^7-12b^9-3\left(a^9-2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^5 +
(3*a^8*b + 111*a^6*b^3 - 231*a^4*b^5 + 65*a^2*b^7 - 12*b^9 - 3*(a^9 - 28*a^
7*b^2 + 110*a^5*b^4 - 108*a^3*b^6 + 9*a*b^8)*d*x)*cos(d*x + c)^3 - 3*(25*a^
6*b^3 - 51*a^4*b^5 + 25*a^2*b^7 - 3*b^9 + 3*(a^7*b^2 - 25*a^5*b^4 + 35*a^3*
b^6 - 3*a*b^8)*d*x)*cos(d*x + c) - 12*((a^8*b - 8*a^6*b^3 + 17*a^4*b^5 - 6*
a^2*b^7)*cos(d*x + c)^3 + 3*(a^6*b^3 - 5*a^4*b^5 + 2*a^2*b^7)*cos(d*x + c)
+ (a^5*b^4 - 5*a^3*b^6 + 2*a*b^8 + (3*a^7*b^2 - 16*a^5*b^4 + 11*a^3*b^6 - 2
*a*b^8)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) +
 (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (32*a^5*b^4 - 66*a^3*b^6 + 6*a*b^8 - 3
*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cos(d*x + c)^4 + 3*(a^6*
b^3 - 25*a^4*b^5 + 35*a^2*b^7 - 3*b^9)*d*x + (45*a^7*b^2 - 143*a^5*b^4 + 21
9*a^3*b^6 - 9*a*b^8 + 3*(3*a^8*b - 76*a^6*b^3 + 130*a^4*b^5 - 44*a^2*b^7 +
3*b^9)*d*x)*cos(d*x + c)^2)*sin(d*x + c))/((a^13 + 2*a^11*b^2 - 5*a^9*b^4 -
 20*a^7*b^6 - 25*a^5*b^8 - 14*a^3*b^10 - 3*a*b^12)*d*cos(d*x + c)^3 + 3*(a^
11*b^2 + 5*a^9*b^4 + 10*a^7*b^6 + 10*a^5*b^8 + 5*a^3*b^10 + a*b^12)*d*cos(d
*x + c) + ((3*a^12*b + 14*a^10*b^3 + 25*a^8*b^5 + 20*a^6*b^7 + 5*a^4*b^9 -
2*a^2*b^11 - b^13)*d*cos(d*x + c)^2 + (a^10*b^3 + 5*a^8*b^5 + 10*a^6*b^7 +
10*a^4*b^9 + 5*a^2*b^11 + b^13)*d)*sin(d*x + c))
```

giac [B] time = 3.89, size = 642, normalized size = 2.43

$$\frac{\frac{3\left(a^6-25a^4b^2+35a^2b^4-3b^6\right)(dx+c)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}}-\frac{12\left(a^5b-5a^3b^3+2ab^5\right)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}}+\frac{24\left(a^5b^2-5a^3b^4+2ab^6\right)\log(|b\tan(dx+c)+a|)}{a^{10}b+5a^8b^3+10a^6b^5+10a^4b^7+5a^2b^9+b^{11}}+\frac{3\left(4a^5b\tan\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*(a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*(d*x + c)/(a^10 + 5*a^8*b^2
+ 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - 12*(a^5*b - 5*a^3*b^3 + 2*a
*b^5)*log(tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 +
 5*a^2*b^8 + b^10) + 24*(a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*log(abs(b*tan(d*x +
 c) + a))/(a^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11)
 + 3*(4*a^5*b*tan(d*x + c)^2 - 20*a^3*b^3*tan(d*x + c)^2 + 8*a*b^5*tan(d*x
+ c)^2 - a^6*tan(d*x + c) + 5*a^4*b^2*tan(d*x + c) + 5*a^2*b^4*tan(d*x + c)
 - b^6*tan(d*x + c) - 20*a^3*b^3 + 12*a*b^5)/((a^10 + 5*a^8*b^2 + 10*a^6*b^
4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*(tan(d*x + c)^2 + 1)) - 2*(22*a^5*b^4*ta
n(d*x + c)^3 - 110*a^3*b^6*tan(d*x + c)^3 + 44*a*b^8*tan(d*x + c)^3 + 75*a^
6*b^3*tan(d*x + c)^2 - 345*a^4*b^5*tan(d*x + c)^2 + 111*a^2*b^7*tan(d*x + c
)^2 + 3*b^9*tan(d*x + c)^2 + 87*a^7*b^2*tan(d*x + c) - 357*a^5*b^4*tan(d*x
+ c) + 87*a^3*b^6*tan(d*x + c) + 3*a*b^8*tan(d*x + c) + 35*a^8*b - 119*a^6*
b^3 + 23*a^4*b^5 + a^2*b^7)/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 +
5*a^2*b^8 + b^10)*(b*tan(d*x + c) + a)^3))/d
```

maple [B] time = 0.55, size = 668, normalized size = 2.53

$$-\frac{a^2b}{3\left(a^2+b^2\right)^2d\left(a+b\tan(dx+c)\right)^3}-\frac{3b\,a^4}{d\left(a^2+b^2\right)^4\left(a+b\tan(dx+c)\right)}+\frac{8b^3a^2}{d\left(a^2+b^2\right)^4\left(a+b\tan(dx+c)\right)}-\frac{}{d\left(a^2+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x)
```

```
[Out] -1/3*a^2*b/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^3-3/d*b/(a^2+b^2)^4/(a+b*tan(d*x+
c))*a^4+8/d*b^3/(a^2+b^2)^4/(a+b*tan(d*x+c))*a^2-1/d*b^5/(a^2+b^2)^4/(a+b*t
an(d*x+c))-1/d*a^3*b/(a^2+b^2)^3/(a+b*tan(d*x+c))^2+1/d*a*b^3/(a^2+b^2)^3/(
a+b*tan(d*x+c))^2+4/d*a^5*b/(a^2+b^2)^5*ln(a+b*tan(d*x+c))-20/d*a^3*b^3/(a^
2+b^2)^5*ln(a+b*tan(d*x+c))+8/d*a*b^5/(a^2+b^2)^5*ln(a+b*tan(d*x+c))-1/2/d/
(a^2+b^2)^5/(1+tan(d*x+c)^2)*tan(d*x+c)*a^6+5/2/d/(a^2+b^2)^5/(1+tan(d*x+c)
^2)*tan(d*x+c)*a^4*b^2+5/2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)*tan(d*x+c)*b^4*a^
2-1/2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)*tan(d*x+c)*b^6-2/d/(a^2+b^2)^5/(1+tan(
d*x+c)^2)*a^5*b+2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)*a*b^5-2/d/(a^2+b^2)^5*ln(1
+tan(d*x+c)^2)*a^5*b+10/d/(a^2+b^2)^5*ln(1+tan(d*x+c)^2)*a^3*b^3-4/d/(a^2+b
^2)^5*ln(1+tan(d*x+c)^2)*a*b^5-25/2/d/(a^2+b^2)^5*arctan(tan(d*x+c))*a^4*b^
2+35/2/d/(a^2+b^2)^5*arctan(tan(d*x+c))*b^4*a^2-3/2/d/(a^2+b^2)^5*arctan(ta
n(d*x+c))*b^6+1/2/d/(a^2+b^2)^5*arctan(tan(d*x+c))*a^6
```

maxima [B] time = 0.60, size = 662, normalized size = 2.51

$$\frac{\frac{3\left(a^6-25a^4b^2+35a^2b^4-3b^6\right)(dx+c)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}} + \frac{24\left(a^5b-5a^3b^3+2ab^5\right)\log(b\tan(dx+c)+a)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}} - \frac{12\left(a^5b-5a^3b^3+2ab^5\right)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}} - \frac{\cdots}{a^{11}+4a\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*(d*x + c)/(a^10 + 5*a^8*b^2
+ 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + 24*(a^5*b - 5*a^3*b^3 + 2*a
*b^5)*log(b*tan(d*x + c) + a)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 +
 5*a^2*b^8 + b^10) - 12*(a^5*b - 5*a^3*b^3 + 2*a*b^5)*log(tan(d*x + c)^2 +
1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - (38*a^
6*b - 56*a^4*b^3 + 2*a^2*b^5 + 3*(7*a^4*b^3 - 22*a^2*b^5 + 3*b^7)*tan(d*x +
 c)^4 + 3*(17*a^5*b^2 - 46*a^3*b^4 + a*b^6)*tan(d*x + c)^3 + (35*a^6*b - 44
*a^4*b^3 - 73*a^2*b^5 + 6*b^7)*tan(d*x + c)^2 + 3*(a^7 + 20*a^5*b^2 - 43*a^
3*b^4 + 2*a*b^6)*tan(d*x + c))/(a^11 + 4*a^9*b^2 + 6*a^7*b^4 + 4*a^5*b^6 +
a^3*b^8 + (a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*tan(d*x + c)
^5 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*tan(d*x + c)^
4 + (3*a^10*b + 13*a^8*b^3 + 22*a^6*b^5 + 18*a^4*b^7 + 7*a^2*b^9 + b^11)*ta
n(d*x + c)^3 + (a^11 + 7*a^9*b^2 + 18*a^7*b^4 + 22*a^5*b^6 + 13*a^3*b^8 + 3
*a*b^10)*tan(d*x + c)^2 + 3*(a^10*b + 4*a^8*b^3 + 6*a^6*b^5 + 4*a^4*b^7 + a
^2*b^9)*tan(d*x + c)))/d
```

mupad [B] time = 5.09, size = 597, normalized size = 2.26

$$\frac{\ln\left(a+b\tan\left(c+d\,x\right)\right)\left(\frac{4\,a\,b}{{\left(a^2+b^2\right)}^3}-\frac{28\,a\,b^3}{{\left(a^2+b^2\right)}^4}+\frac{32\,a\,b^5}{{\left(a^2+b^2\right)}^5}\right)}{d}-\frac{\frac{{\mathrm{tan}\left(c+d\,x\right)}^2\,\left(35\,a^4\,b-79\,a^2\,b^3+6\,b^5\right)}{6\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{{\mathrm{tan}\left(c+d\,x\right)}^4\,\left(7\,a^4\,b^3-22\,a^2\,b^5+3\,b^7\cdots\right.}{2\,\left(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8\cdots\right.}\cdots}{d\,\left(a^3+{\mathrm{tan}\left(c+d\,x\right)}^2\,\left(a^3+3\,a\,b^2\right)+\mathrm{tan}\left(c\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b*tan(c + d*x))^4,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((4*a*b)/(a^2 + b^2)^3 - (28*a*b^3)/(a^2 + b^2)^4
+ (32*a*b^5)/(a^2 + b^2)^5))/d - ((tan(c + d*x)^2*(35*a^4*b + 6*b^5 - 79*a^
2*b^3))/(6*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (tan(c + d*x)^4*(3*b^7 -
22*a^2*b^5 + 7*a^4*b^3))/(2*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)
) + (tan(c + d*x)^3*(a*b^6 - 46*a^3*b^4 + 17*a^5*b^2))/(2*(a^8 + b^8 + 4*a^
2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (a^2*(19*a^4*b + b^5 - 28*a^2*b^3))/(3*(a
^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a*tan(c + d*x)*(a^6 + 2*b
```

```
^6 - 43*a^2*b^4 + 20*a^4*b^2))/(2*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)))/(d*(a^3 + tan(c + d*x)^2*(3*a*b^2 + a^3) + tan(c + d*x)^3*(3*a^2*b
 + b^3) + b^3*tan(c + d*x)^5 + 3*a*b^2*tan(c + d*x)^4 + 3*a^2*b*tan(c + d*x
))) - (log(tan(c + d*x) - 1i)*(a*1i + 3*b))/(4*d*(5*a*b^4 + a^4*b*5i + a^5
+ b^5*1i - a^2*b^3*10i - 10*a^3*b^2)) + (log(tan(c + d*x) + 1i)*(a*1i - 3*b
))/(4*d*(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2))
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

3.75 $\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal. Leaf size=116

$$-\frac{4b\log(\tan(c+dx))}{a^5d}+\frac{4b\log(a+b\tan(c+dx))}{a^5d}-\frac{3b}{a^4d(a+b\tan(c+dx))}-\frac{\cot(c+dx)}{a^4d}-\frac{b}{a^3d(a+b\tan(c+dx))}$$

```
[Out] -cot(d*x+c)/a^4/d-4*b*ln(tan(d*x+c))/a^5/d+4*b*ln(a+b*tan(d*x+c))/a^5/d-1/3
*b/a^2/d/(a+b*tan(d*x+c))^3-b/a^3/d/(a+b*tan(d*x+c))^2-3*b/a^4/d/(a+b*tan(d
*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 44}

$$-\frac{3b}{a^4d(a+b\tan(c+dx))}-\frac{b}{a^3d(a+b\tan(c+dx))^2}-\frac{b}{3a^2d(a+b\tan(c+dx))^3}-\frac{4b\log(\tan(c+dx))}{a^5d}+\frac{4b\log(a+$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(Cot[c + d*x]/(a^4*d)) - (4*b*Log[Tan[c + d*x]])/(a^5*d) + (4*b*Log[a + b*
Tan[c + d*x]])/(a^5*d) - b/(3*a^2*d*(a + b*Tan[c + d*x])^3) - b/(a^3*d*(a +
 b*Tan[c + d*x])^2) - (3*b)/(a^4*d*(a + b*Tan[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx &= \frac{b\,\text{Subst}\left(\int \frac{1}{x^2(a+x)^4}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\,\text{Subst}\left(\int \left(\frac{1}{a^4x^2}-\frac{4}{a^5x}+\frac{1}{a^2(a+x)^4}+\frac{2}{a^3(a+x)^3}+\frac{3}{a^4(a+x)^2}+\frac{4}{a^5(a+x)}\right)dx,x,b\tan(c+dx\right.}{d}\\ &= -\frac{\cot(c+dx)}{a^4d}-\frac{4b\log(\tan(c+dx))}{a^5d}+\frac{4b\log(a+b\tan(c+dx))}{a^5d}-\frac{b}{3a^2d(a+b\tan}\end{aligned}$$

Mathematica [B] time = 2.22, size = 259, normalized size = 2.23

$$\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{a^2b^4\tan(c+dx)}{a^2+b^2}-\frac{2a^2b^3(3a^2+2b^2)(a+b\tan(c+dx))}{(a^2+b^2)^2}+\frac{b^2(18a^4+23a^2b^2+9b^4)\tan(c+d}{(a^2+b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*(-3*a*(b + a*Cot[c + d*x]
)^3*Sin[c + d*x]^2 + (a^2*b^4*Tan[c + d*x])/(a^2 + b^2) + (b^2*(18*a^4 + 23
*a^2*b^2 + 9*b^4)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2*Tan[c + d*x])/(a^2 +
b^2)^2 - (2*a^2*b^3*(3*a^2 + 2*b^2)*(a + b*Tan[c + d*x]))/(a^2 + b^2)^2 - 1
2*b*Cos[c + d*x]^2*Log[Sin[c + d*x]]*(a + b*Tan[c + d*x])^3 + 12*b*Cos[c +
d*x]^2*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]*(a + b*Tan[c + d*x])^3))/(3*a^5
*d*(a + b*Tan[c + d*x])^4)
```

fricas [B] time = 0.54, size = 874, normalized size = 7.53

$$-\frac{13\,a^6b^4+15\,a^4b^6+6\,a^2b^8-\left(3\,a^{10}+18\,a^8b^2-49\,a^6b^4-84\,a^4b^6-36\,a^2b^8\right)\cos\left(dx+c\right)^4+\left(9\,a^8b^2-71\,a^6b^4-1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(13*a^6*b^4 + 15*a^4*b^6 + 6*a^2*b^8 - (3*a^10 + 18*a^8*b^2 - 49*a^6*b
^4 - 84*a^4*b^6 - 36*a^2*b^8)*cos(d*x + c)^4 + (9*a^8*b^2 - 71*a^6*b^4 - 10
2*a^4*b^6 - 42*a^2*b^8)*cos(d*x + c)^2 + 6*(a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8
 + b^10 - (3*a^8*b^2 + 8*a^6*b^4 + 6*a^4*b^6 - b^10)*cos(d*x + c)^4 + (3*a^
8*b^2 + 7*a^6*b^4 + 3*a^4*b^6 - 3*a^2*b^8 - 2*b^10)*cos(d*x + c)^2 + ((a^9*
b - 6*a^5*b^5 - 8*a^3*b^7 - 3*a*b^9)*cos(d*x + c)^3 + 3*(a^7*b^3 + 3*a^5*b^
5 + 3*a^3*b^7 + a*b^9)*cos(d*x + c))*sin(d*x + c))*log(2*a*b*cos(d*x + c)*s
in(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 6*(a^6*b^4 + 3*a^4*b^6 +
3*a^2*b^8 + b^10 - (3*a^8*b^2 + 8*a^6*b^4 + 6*a^4*b^6 - b^10)*cos(d*x + c)^
4 + (3*a^8*b^2 + 7*a^6*b^4 + 3*a^4*b^6 - 3*a^2*b^8 - 2*b^10)*cos(d*x + c)^2
 + ((a^9*b - 6*a^5*b^5 - 8*a^3*b^7 - 3*a*b^9)*cos(d*x + c)^3 + 3*(a^7*b^3 +
 3*a^5*b^5 + 3*a^3*b^7 + a*b^9)*cos(d*x + c))*sin(d*x + c))*log(-1/4*cos(d*
x + c)^2 + 1/4) - ((9*a^9*b + 78*a^7*b^3 + 69*a^5*b^5 + 4*a^3*b^7 - 12*a*b^
9)*cos(d*x + c)^3 - 3*(9*a^7*b^3 + 3*a^5*b^5 - 6*a^3*b^7 - 4*a*b^9)*cos(d*x
 + c))*sin(d*x + c))/((3*a^13*b + 8*a^11*b^3 + 6*a^9*b^5 - a^5*b^9)*d*cos(d
*x + c)^4 - (3*a^13*b + 7*a^11*b^3 + 3*a^9*b^5 - 3*a^7*b^7 - 2*a^5*b^9)*d*c
os(d*x + c)^2 - (a^11*b^3 + 3*a^9*b^5 + 3*a^7*b^7 + a^5*b^9)*d - ((a^14 - 6
*a^10*b^4 - 8*a^8*b^6 - 3*a^6*b^8)*d*cos(d*x + c)^3 + 3*(a^12*b^2 + 3*a^10*
b^4 + 3*a^8*b^6 + a^6*b^8)*d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 2.57, size = 129, normalized size = 1.11

$$\frac{\frac{12\,b\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^5}-\frac{12\,b\log\left(\left|\tan\left(dx+c\right)\right|\right)}{a^5}+\frac{3\,\left(4\,b\tan\left(dx+c\right)-a\right)}{a^5\tan\left(dx+c\right)}-\frac{22\,b^4\tan\left(dx+c\right)^3+75\,ab^3\tan\left(dx+c\right)^2+87\,a^2b^2\tan\left(dx+c\right)+35\,a^3b}{\left(b\tan\left(dx+c\right)+a\right)^3a^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(12*b*log(abs(b*tan(d*x + c) + a))/a^5 - 12*b*log(abs(tan(d*x + c)))/a^
5 + 3*(4*b*tan(d*x + c) - a)/(a^5*tan(d*x + c)) - (22*b^4*tan(d*x + c)^3 +
75*a*b^3*tan(d*x + c)^2 + 87*a^2*b^2*tan(d*x + c) + 35*a^3*b)/((b*tan(d*x +
 c) + a)^3*a^5))/d
```

maple [A] time = 0.53, size = 117, normalized size = 1.01

$$-\frac{b}{3a^2d\left(a+b\tan\left(dx+c\right)\right)^3}+\frac{4b\ln\left(a+b\tan\left(dx+c\right)\right)}{a^5d}-\frac{3b}{a^4d\left(a+b\tan\left(dx+c\right)\right)}-\frac{b}{a^3d\left(a+b\tan\left(dx+c\right)\right)^2}-\frac{}{d\,a^4\,\mathrm{ta}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x)
```

```
[Out] -1/3*b/a^2/d/(a+b*tan(d*x+c))^3+4*b*ln(a+b*tan(d*x+c))/a^5/d-3*b/a^4/d/(a+b
*tan(d*x+c))-b/a^3/d/(a+b*tan(d*x+c))^2-1/d/a^4/tan(d*x+c)-4*b*ln(tan(d*x+c
))/a^5/d
```

maxima [A] time = 0.38, size = 140, normalized size = 1.21

$$-\frac{\frac{12\,b^3\tan(dx+c)^3+30\,ab^2\tan(dx+c)^2+22\,a^2b\tan(dx+c)+3\,a^3}{a^4b^3\tan(dx+c)^4+3\,a^5b^2\tan(dx+c)^3+3\,a^6b\tan(dx+c)^2+a^7\tan(dx+c)}-\frac{12\,b\log(b\tan(dx+c)+a)}{a^5}+\frac{12\,b\log(\tan(dx+c))}{a^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*((12*b^3*tan(d*x + c)^3 + 30*a*b^2*tan(d*x + c)^2 + 22*a^2*b*tan(d*x +
 c) + 3*a^3)/(a^4*b^3*tan(d*x + c)^4 + 3*a^5*b^2*tan(d*x + c)^3 + 3*a^6*b*t
an(d*x + c)^2 + a^7*tan(d*x + c)) - 12*b*log(b*tan(d*x + c) + a)/a^5 + 12*b
*log(tan(d*x + c))/a^5)/d
```

mupad [B] time = 3.98, size = 131, normalized size = 1.13

$$\frac{8\,b\,\mathrm{atanh}\left(\frac{2\,b\tan(c+d\,x)}{a}+1\right)}{a^5\,d}-\frac{\frac{1}{a}+\frac{10\,b^2\tan(c+d\,x)^2}{a^3}+\frac{4\,b^3\tan(c+d\,x)^3}{a^4}+\frac{22\,b\tan(c+d\,x)}{3\,a^2}}{d\,\left(a^3\tan\left(c+d\,x\right)+3\,a^2\,b\tan\left(c+d\,x\right)^2+3\,a\,b^2\tan\left(c+d\,x\right)^3+b^3\tan\left(c+d\,x\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b*tan(c + d*x))^4),x)
```

```
[Out] (8*b*atanh((2*b*tan(c + d*x))/a + 1))/(a^5*d) - (1/a + (10*b^2*tan(c + d*x)
^2)/a^3 + (4*b^3*tan(c + d*x)^3)/a^4 + (22*b*tan(c + d*x))/(3*a^2))/(d*(a^3
*tan(c + d*x) + b^3*tan(c + d*x)^4 + 3*a^2*b*tan(c + d*x)^2 + 3*a*b^2*tan(c
 + d*x)^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*tan(c + d*x))**4, x)
```

3.76 $\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal. Leaf size=205

$$\frac{2b\cot^2(c+dx)}{a^5d}-\frac{\cot^3(c+dx)}{3a^4d}-\frac{4b\left(a^2+5b^2\right)\log(\tan(c+dx))}{a^7d}+\frac{4b\left(a^2+5b^2\right)\log(a+b\tan(c+dx))}{a^7d}-\frac{b\left(3a^2\cdots\right.}{a^6d(a+b\,\mathrm{ta}\cdots}$$

```
[Out] -(a^2+10*b^2)*cot(d*x+c)/a^6/d+2*b*cot(d*x+c)^2/a^5/d-1/3*cot(d*x+c)^3/a^4/
d-4*b*(a^2+5*b^2)*ln(tan(d*x+c))/a^7/d+4*b*(a^2+5*b^2)*ln(a+b*tan(d*x+c))/a
^7/d-1/3*b*(a^2+b^2)/a^4/d/(a+b*tan(d*x+c))^3-b*(a^2+2*b^2)/a^5/d/(a+b*tan(
d*x+c))^2-b*(3*a^2+10*b^2)/a^6/d/(a+b*tan(d*x+c))
```

Rubi [A] time = 0.17, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 894}

$$-\frac{b\left(3a^2+10b^2\right)}{a^6d(a+b\tan(c+dx))}-\frac{b\left(a^2+2b^2\right)}{a^5d(a+b\tan(c+dx))^2}-\frac{b\left(a^2+b^2\right)}{3a^4d(a+b\tan(c+dx))^3}-\frac{\left(a^2+10b^2\right)\cot(c+dx)}{a^6d}-\frac{4b\left(a^2+5\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(((a^2 + 10*b^2)*Cot[c + d*x])/(a^6*d)) + (2*b*Cot[c + d*x]^2)/(a^5*d) - C
ot[c + d*x]^3/(3*a^4*d) - (4*b*(a^2 + 5*b^2)*Log[Tan[c + d*x]])/(a^7*d) + (
4*b*(a^2 + 5*b^2)*Log[a + b*Tan[c + d*x]])/(a^7*d) - (b*(a^2 + b^2))/(3*a^4
*d*(a + b*Tan[c + d*x])^3) - (b*(a^2 + 2*b^2))/(a^5*d*(a + b*Tan[c + d*x])^
2) - (b*(3*a^2 + 10*b^2))/(a^6*d*(a + b*Tan[c + d*x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx &= \frac{b\,\mathrm{Subst}\left(\int \frac{b^2+x^2}{x^4(a+x)^4}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\,\mathrm{Subst}\left(\int \left(\frac{b^2}{a^4x^4}-\frac{4b^2}{a^5x^3}+\frac{a^2+10b^2}{a^6x^2}-\frac{4\left(a^2+5b^2\right)}{a^7x}+\frac{a^2+b^2}{a^4(a+x)^4}+\frac{2\left(a^2+2b^2\right)}{a^5(a+x)^3}+\frac{3a^2+10b^2}{a^6(a+x)^2}+\frac{4\left(a^2+5\cdots\right.}{a^7(a+\cdots}\right.\right.}{d}\\ &= -\frac{\left(a^2+10b^2\right)\cot(c+dx)}{a^6d}+\frac{2b\cot^2(c+dx)}{a^5d}-\frac{\cot^3(c+dx)}{3a^4d}-\frac{4b\left(a^2+5b^2\right)\log(\tan(\cdots}{a^7d}\end{aligned}$$

Mathematica [B] time = 2.08, size = 528, normalized size = 2.58

$$\frac{\sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(-192b\left(a^2+5b^2\right)\log(\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])*(-192*b*(a^2 + 5*b^2)*Log
[Sin[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3 + 192*b*(a^2 + 5*b^2)*Lo
g[a*Cos[c + d*x] + b*Sin[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3 - (C
sc[c + d*x]^3*(8*a^8 - 4*a^6*b^2 - 50*a^4*b^4 - 190*a^2*b^6 - 150*b^8 + 3*(
3*a^8 + 10*a^6*b^2 + 45*a^4*b^4 + 115*a^2*b^6 + 75*b^8)*Cos[2*(c + d*x)] +
6*(2*a^6*b^2 - 17*a^4*b^4 - 35*a^2*b^6 - 15*b^8)*Cos[4*(c + d*x)] - a^8*Cos
[6*(c + d*x)] - 22*a^6*b^2*Cos[6*(c + d*x)] + 17*a^4*b^4*Cos[6*(c + d*x)] +
 55*a^2*b^6*Cos[6*(c + d*x)] + 15*b^8*Cos[6*(c + d*x)] - 3*a^7*b*Sin[2*(c +
 d*x)] + 3*a^5*b^3*Sin[2*(c + d*x)] - 75*a^3*b^5*Sin[2*(c + d*x)] - 75*a*b^
7*Sin[2*(c + d*x)] - 6*a^7*b*Sin[4*(c + d*x)] + 84*a^5*b^3*Sin[4*(c + d*x)]
 + 156*a^3*b^5*Sin[4*(c + d*x)] + 60*a*b^7*Sin[4*(c + d*x)] - 3*a^7*b*Sin[6
*(c + d*x)] - 65*a^5*b^3*Sin[6*(c + d*x)] - 79*a^3*b^5*Sin[6*(c + d*x)] - 1
5*a*b^7*Sin[6*(c + d*x)]))/(a^2 + b^2)))/(48*a^7*d*(a + b*Tan[c + d*x])^4)
```

fricas [B] time = 0.60, size = 1235, normalized size = 6.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(19*a^6*b^4 + 51*a^4*b^6 + 30*a^2*b^8 + 2*(a^10 + 23*a^8*b^2 - 22*a^6*b
^4 - 138*a^4*b^6 - 90*a^2*b^8)*cos(d*x + c)^6 - 3*(a^10 + 25*a^8*b^2 - 46*a
^6*b^4 - 206*a^4*b^6 - 130*a^2*b^8)*cos(d*x + c)^4 + 3*(9*a^8*b^2 - 38*a^6*
b^4 - 131*a^4*b^6 - 80*a^2*b^8)*cos(d*x + c)^2 + 6*(a^6*b^4 + 7*a^4*b^6 + 1
1*a^2*b^8 + 5*b^10 + (3*a^8*b^2 + 20*a^6*b^4 + 26*a^4*b^6 + 4*a^2*b^8 - 5*b
^10)*cos(d*x + c)^6 - 3*(2*a^8*b^2 + 13*a^6*b^4 + 15*a^4*b^6 - a^2*b^8 - 5*
b^10)*cos(d*x + c)^4 + 3*(a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 - 6*a^2*b^8 - 5*b
^10)*cos(d*x + c)^2 - ((a^9*b + 4*a^7*b^3 - 10*a^5*b^5 - 28*a^3*b^7 - 15*a*
b^9)*cos(d*x + c)^5 - (a^9*b + a^7*b^3 - 31*a^5*b^5 - 61*a^3*b^7 - 30*a*b^9
)*cos(d*x + c)^3 - 3*(a^7*b^3 + 7*a^5*b^5 + 11*a^3*b^7 + 5*a*b^9)*cos(d*x +
 c))*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*
x + c)^2 + b^2) - 6*(a^6*b^4 + 7*a^4*b^6 + 11*a^2*b^8 + 5*b^10 + (3*a^8*b^2
 + 20*a^6*b^4 + 26*a^4*b^6 + 4*a^2*b^8 - 5*b^10)*cos(d*x + c)^6 - 3*(2*a^8*
b^2 + 13*a^6*b^4 + 15*a^4*b^6 - a^2*b^8 - 5*b^10)*cos(d*x + c)^4 + 3*(a^8*b
^2 + 6*a^6*b^4 + 4*a^4*b^6 - 6*a^2*b^8 - 5*b^10)*cos(d*x + c)^2 - ((a^9*b +
 4*a^7*b^3 - 10*a^5*b^5 - 28*a^3*b^7 - 15*a*b^9)*cos(d*x + c)^5 - (a^9*b +
a^7*b^3 - 31*a^5*b^5 - 61*a^3*b^7 - 30*a*b^9)*cos(d*x + c)^3 - 3*(a^7*b^3 +
 7*a^5*b^5 + 11*a^3*b^7 + 5*a*b^9)*cos(d*x + c))*sin(d*x + c))*log(-1/4*cos
(d*x + c)^2 + 1/4) + (2*(3*a^9*b + 77*a^7*b^3 + 142*a^5*b^5 + 34*a^3*b^7 -
30*a*b^9)*cos(d*x + c)^5 - (3*a^9*b + 193*a^7*b^3 + 350*a^5*b^5 + 26*a^3*b^
7 - 120*a*b^9)*cos(d*x + c)^3 + 3*(15*a^7*b^3 + 23*a^5*b^5 - 14*a^3*b^7 - 2
0*a*b^9)*cos(d*x + c))*sin(d*x + c))/((3*a^13*b + 5*a^11*b^3 + a^9*b^5 - a^
7*b^7)*d*cos(d*x + c)^6 - 3*(2*a^13*b + 3*a^11*b^3 - a^7*b^7)*d*cos(d*x + c
)^4 + 3*(a^13*b + a^11*b^3 - a^9*b^5 - a^7*b^7)*d*cos(d*x + c)^2 + (a^11*b^
3 + 2*a^9*b^5 + a^7*b^7)*d - ((a^14 - a^12*b^2 - 5*a^10*b^4 - 3*a^8*b^6)*d*
cos(d*x + c)^5 - (a^14 - 4*a^12*b^2 - 11*a^10*b^4 - 6*a^8*b^6)*d*cos(d*x +
c)^3 - 3*(a^12*b^2 + 2*a^10*b^4 + a^8*b^6)*d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 5.00, size = 222, normalized size = 1.08

$$-\frac{\frac{12\left(a^2b+5\,b^3\right)\log(|\tan(dx+c)|)}{a^7}-\frac{12\left(a^2b^2+5\,b^4\right)\log(|b\tan(dx+c)+a|)}{a^7b}+\frac{12\,a^2b^3\tan(dx+c)^5+60\,b^5\tan(dx+c)^5+30\,a^3b^2\tan(dx+c)^4+150\,ab^4\tan(dx}{}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(12*(a^2*b + 5*b^3)*log(abs(tan(d*x + c)))/a^7 - 12*(a^2*b^2 + 5*b^4)*
log(abs(b*tan(d*x + c) + a))/(a^7*b) + (12*a^2*b^3*tan(d*x + c)^5 + 60*b^5*
tan(d*x + c)^5 + 30*a^3*b^2*tan(d*x + c)^4 + 150*a*b^4*tan(d*x + c)^4 + 22*
a^4*b*tan(d*x + c)^3 + 110*a^2*b^3*tan(d*x + c)^3 + 3*a^5*tan(d*x + c)^2 +
15*a^3*b^2*tan(d*x + c)^2 - 3*a^4*b*tan(d*x + c) + a^5)/((b*tan(d*x + c)^2
+ a*tan(d*x + c))^3*a^6))/d
```

maple [A] time = 0.59, size = 278, normalized size = 1.36

$$-\frac{3b}{a^4d\,(a+b\tan(dx+c))}-\frac{10b^3}{d\,a^6\,(a+b\tan(dx+c))}-\frac{b}{3a^2d\,(a+b\tan(dx+c))^3}-\frac{b^3}{3d\,a^4\,(a+b\tan(dx+c))^3}-\frac{}{a^3d\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x)
```

```
[Out] -3*b/a^4/d/(a+b*tan(d*x+c))-10/d*b^3/a^6/(a+b*tan(d*x+c))-1/3*b/a^2/d/(a+b*
tan(d*x+c))^3-1/3/d*b^3/a^4/(a+b*tan(d*x+c))^3-b/a^3/d/(a+b*tan(d*x+c))^2-2
/d*b^3/a^5/(a+b*tan(d*x+c))^2+4*b*ln(a+b*tan(d*x+c))/a^5/d+20/d*b^3/a^7*ln(
a+b*tan(d*x+c))-1/3/d/a^4/tan(d*x+c)^3-1/d/a^4/tan(d*x+c)-10/d/a^6/tan(d*x+
c)*b^2+2/d/a^5*b/tan(d*x+c)^2-4*b*ln(tan(d*x+c))/a^5/d-20/d*b^3/a^7*ln(tan(
d*x+c))
```

maxima [A] time = 0.56, size = 228, normalized size = 1.11

$$\frac{\frac{3\,a^4b\tan(dx+c)-12\left(a^2b^3+5\,b^5\right)\tan(dx+c)^5-a^5-30\left(a^3b^2+5\,ab^4\right)\tan(dx+c)^4-22\left(a^4b+5\,a^2b^3\right)\tan(dx+c)^3-3\left(a^5+5\,a^3b^2\right)\tan(dx+c)^2}{a^6b^3\tan(dx+c)^6+3\,a^7b^2\tan(dx+c)^5+3\,a^8b\tan(dx+c)^4+a^9\tan(dx+c)^3}+\frac{12\left(a^2b+5\,b^3\right)}{}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*((3*a^4*b*tan(d*x + c) - 12*(a^2*b^3 + 5*b^5)*tan(d*x + c)^5 - a^5 - 30
*(a^3*b^2 + 5*a*b^4)*tan(d*x + c)^4 - 22*(a^4*b + 5*a^2*b^3)*tan(d*x + c)^3
 - 3*(a^5 + 5*a^3*b^2)*tan(d*x + c)^2)/(a^6*b^3*tan(d*x + c)^6 + 3*a^7*b^2*
tan(d*x + c)^5 + 3*a^8*b*tan(d*x + c)^4 + a^9*tan(d*x + c)^3) + 12*(a^2*b +
 5*b^3)*log(b*tan(d*x + c) + a)/a^7 - 12*(a^2*b + 5*b^3)*log(tan(d*x + c))/
a^7)/d
```

mupad [B] time = 5.06, size = 232, normalized size = 1.13

$$\frac{8\,b\,\mathrm{atanh}\left(\frac{4\,b\left(a^2+5\,b^2\right)(a+2\,b\tan(c+d\,x))}{a\left(4\,a^2\,b+20\,b^3\right)}\right)\left(a^2+5\,b^2\right)}{a^7\,d}-\frac{\frac{1}{3\,a}+\frac{\tan(c+d\,x)^2\left(a^2+5\,b^2\right)}{a^3}-\frac{b\tan(c+d\,x)}{a^2}+\frac{22\,b\tan(c+d\,x)^3\left(a^2+5\,b^2\right)}{3\,a^4}+\frac{10\,b}{}}{d\left(a^3\tan(c+d\,x)^3+3\,a^2\,b\tan(c+d\,x)^4+3\,a\,b^2\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + b*tan(c + d*x))^4),x)
```

```
[Out] (8*b*atanh((4*b*(a^2 + 5*b^2)*(a + 2*b*tan(c + d*x)))/(a*(4*a^2*b + 20*b^3)
))*(a^2 + 5*b^2))/(a^7*d) - (1/(3*a) + (tan(c + d*x)^2*(a^2 + 5*b^2))/a^3 -
```

```
 (b*tan(c + d*x))/a^2 + (22*b*tan(c + d*x)^3*(a^2 + 5*b^2))/(3*a^4) + (10*b
^2*tan(c + d*x)^4*(a^2 + 5*b^2))/a^5 + (4*b^3*tan(c + d*x)^5*(a^2 + 5*b^2))
/a^6)/(d*(a^3*tan(c + d*x)^3 + b^3*tan(c + d*x)^6 + 3*a^2*b*tan(c + d*x)^4
+ 3*a*b^2*tan(c + d*x)^5))
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*tan(c + d*x))**4, x)
```

3.77 $\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4} \, dx$

Optimal. Leaf size=300

$$\frac{b\cot^4(c+dx)}{a^5d}-\frac{\cot^5(c+dx)}{5a^4d}-\frac{b\left(a^2+b^2\right)\left(a^2+3b^2\right)}{a^7d(a+b\tan(c+dx))^2}+\frac{2b\left(2a^2+5b^2\right)\cot^2(c+dx)}{a^7d}-\frac{b\left(a^2+b^2\right)^2}{3a^6d(a+b\tan(c+dx))^3}-\frac{2}{}$$

```
[Out] -(a^4+20*a^2*b^2+35*b^4)*cot(d*x+c)/a^8/d+2*b*(2*a^2+5*b^2)*cot(d*x+c)^2/a^
7/d-2/3*(a^2+5*b^2)*cot(d*x+c)^3/a^6/d+b*cot(d*x+c)^4/a^5/d-1/5*cot(d*x+c)^
5/a^4/d-4*b*(a^4+10*a^2*b^2+14*b^4)*ln(tan(d*x+c))/a^9/d+4*b*(a^4+10*a^2*b^
2+14*b^4)*ln(a+b*tan(d*x+c))/a^9/d-1/3*b*(a^2+b^2)^2/a^6/d/(a+b*tan(d*x+c))
^3-b*(a^2+b^2)*(a^2+3*b^2)/a^7/d/(a+b*tan(d*x+c))^2-b*(3*a^4+20*a^2*b^2+21*
b^4)/a^8/d/(a+b*tan(d*x+c))
```

Rubi [A] time = 0.27, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 894}

$$-\frac{b\left(20a^2b^2+3a^4+21b^4\right)}{a^8d(a+b\tan(c+dx))}-\frac{b\left(a^2+b^2\right)\left(a^2+3b^2\right)}{a^7d(a+b\tan(c+dx))^2}-\frac{b\left(a^2+b^2\right)^2}{3a^6d(a+b\tan(c+dx))^3}-\frac{2\left(a^2+5b^2\right)\cot^3(c+dx)}{3a^6d}+\frac{2b\left(2a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(((a^4 + 20*a^2*b^2 + 35*b^4)*Cot[c + d*x])/(a^8*d)) + (2*b*(2*a^2 + 5*b^2
)*Cot[c + d*x]^2)/(a^7*d) - (2*(a^2 + 5*b^2)*Cot[c + d*x]^3)/(3*a^6*d) + (b
*Cot[c + d*x]^4)/(a^5*d) - Cot[c + d*x]^5/(5*a^4*d) - (4*b*(a^4 + 10*a^2*b^
2 + 14*b^4)*Log[Tan[c + d*x]])/(a^9*d) + (4*b*(a^4 + 10*a^2*b^2 + 14*b^4)*L
og[a + b*Tan[c + d*x]])/(a^9*d) - (b*(a^2 + b^2)^2)/(3*a^6*d*(a + b*Tan[c +
 d*x])^3) - (b*(a^2 + b^2)*(a^2 + 3*b^2))/(a^7*d*(a + b*Tan[c + d*x])^2) -
(b*(3*a^4 + 20*a^2*b^2 + 21*b^4))/(a^8*d*(a + b*Tan[c + d*x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{\left(b^2+x^2\right)^2}{x^6(a+x)^4}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\,\mathrm{Subst}\left(\int \left(\frac{b^4}{a^4x^6} - \frac{4b^4}{a^5x^5} + \frac{2b^2\left(a^2+5b^2\right)}{a^6x^4} - \frac{4\left(2a^2b^2+5b^4\right)}{a^7x^3} + \frac{a^4+20a^2b^2+35b^4}{a^8x^2} - \frac{4\left(a^4+10a^2b^2+14\right.}{a^9x}\right.\right.}{}$$

$$= -\frac{\left(a^4+20a^2b^2+35b^4\right)\cot(c+dx)}{a^8d} + \frac{2b\left(2a^2+5b^2\right)\cot^2(c+dx)}{a^7d} - \frac{2\left(a^2+5b^2\right)}{3a}$$

Mathematica [B] time = 1.76, size = 673, normalized size = 2.24

$$\frac{\sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(-7680b\left(a^4+10a^2b^2+14b^4\right)\log(\sin(c+dx))(a\cos(c+dx)+b\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])*(-7680*b*(a^4 + 10*a^2*b^
2 + 14*b^4)*Log[Sin[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3 + 7680*b*
(a^4 + 10*a^2*b^2 + 14*b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]*(a*Cos[c +
 d*x] + b*Sin[c + d*x])^3 + Csc[c + d*x]^5*(-200*a^8 + 380*a^6*b^2 + 3070*a
^4*b^4 + 11375*a^2*b^6 + 11025*b^8 - 4*(52*a^8 + 194*a^6*b^2 + 1510*a^4*b^4
 + 5705*a^2*b^6 + 4410*b^8)*Cos[2*(c + d*x)] + 4*(4*a^8 - 16*a^6*b^2 + 1010
*a^4*b^4 + 4585*a^2*b^6 + 2205*b^8)*Cos[4*(c + d*x)] + 16*a^8*Cos[6*(c + d*
x)] + 776*a^6*b^2*Cos[6*(c + d*x)] - 1000*a^4*b^4*Cos[6*(c + d*x)] - 8540*a
^2*b^6*Cos[6*(c + d*x)] - 2520*b^8*Cos[6*(c + d*x)] - 8*a^8*Cos[8*(c + d*x)
] - 316*a^6*b^2*Cos[8*(c + d*x)] - 70*a^4*b^4*Cos[8*(c + d*x)] + 1645*a^2*b
^6*Cos[8*(c + d*x)] + 315*b^8*Cos[8*(c + d*x)] + 264*a^7*b*Sin[2*(c + d*x)]
 + 372*a^5*b^3*Sin[2*(c + d*x)] + 4830*a^3*b^5*Sin[2*(c + d*x)] + 1470*a*b^
7*Sin[2*(c + d*x)] + 144*a^7*b*Sin[4*(c + d*x)] - 2476*a^5*b^3*Sin[4*(c + d
*x)] - 9730*a^3*b^5*Sin[4*(c + d*x)] - 1470*a*b^7*Sin[4*(c + d*x)] - 24*a^7
*b*Sin[6*(c + d*x)] + 2756*a^5*b^3*Sin[6*(c + d*x)] + 7670*a^3*b^5*Sin[6*(c
 + d*x)] + 630*a*b^7*Sin[6*(c + d*x)] - 24*a^7*b*Sin[8*(c + d*x)] - 922*a^5
*b^3*Sin[8*(c + d*x)] - 2095*a^3*b^5*Sin[8*(c + d*x)] - 105*a*b^7*Sin[8*(c
+ d*x)])))/(1920*a^9*d*(a + b*Tan[c + d*x])^4)
```

fricas [B] time = 0.65, size = 1536, normalized size = 5.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/15*(110*a^6*b^4 + 510*a^4*b^6 + 420*a^2*b^8 - 4*(2*a^10 + 81*a^8*b^2 + 2
9*a^6*b^4 - 660*a^4*b^6 - 630*a^2*b^8)*cos(d*x + c)^8 + 2*(10*a^10 + 423*a^
8*b^2 - 47*a^6*b^4 - 4320*a^4*b^6 - 3990*a^2*b^8)*cos(d*x + c)^6 - 15*(a^10
 + 47*a^8*b^2 - 44*a^6*b^4 - 658*a^4*b^6 - 588*a^2*b^8)*cos(d*x + c)^4 + 20
*(9*a^8*b^2 - 28*a^6*b^4 - 219*a^4*b^6 - 189*a^2*b^8)*cos(d*x + c)^2 + 30*(
a^6*b^4 + 11*a^4*b^6 + 24*a^2*b^8 + 14*b^10 - (3*a^8*b^2 + 32*a^6*b^4 + 61*
a^4*b^6 + 18*a^2*b^8 - 14*b^10)*cos(d*x + c)^8 + (9*a^8*b^2 + 95*a^6*b^4 +
172*a^4*b^6 + 30*a^2*b^8 - 56*b^10)*cos(d*x + c)^6 - 3*(3*a^8*b^2 + 31*a^6*
b^4 + 50*a^4*b^6 - 6*a^2*b^8 - 28*b^10)*cos(d*x + c)^4 + (3*a^8*b^2 + 29*a^
6*b^4 + 28*a^4*b^6 - 54*a^2*b^8 - 56*b^10)*cos(d*x + c)^2 + ((a^9*b + 8*a^7
*b^3 - 9*a^5*b^5 - 58*a^3*b^7 - 42*a*b^9)*cos(d*x + c)^7 - (2*a^9*b + 13*a^
```

```
7*b^3 - 51*a^5*b^5 - 188*a^3*b^7 - 126*a*b^9)*cos(d*x + c)^5 + (a^9*b + 2*a
^7*b^3 - 75*a^5*b^5 - 202*a^3*b^7 - 126*a*b^9)*cos(d*x + c)^3 + 3*(a^7*b^3
+ 11*a^5*b^5 + 24*a^3*b^7 + 14*a*b^9)*cos(d*x + c))*sin(d*x + c))*log(2*a*b
*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 30*(a^6*b^
4 + 11*a^4*b^6 + 24*a^2*b^8 + 14*b^10 - (3*a^8*b^2 + 32*a^6*b^4 + 61*a^4*b^
6 + 18*a^2*b^8 - 14*b^10)*cos(d*x + c)^8 + (9*a^8*b^2 + 95*a^6*b^4 + 172*a^
4*b^6 + 30*a^2*b^8 - 56*b^10)*cos(d*x + c)^6 - 3*(3*a^8*b^2 + 31*a^6*b^4 +
50*a^4*b^6 - 6*a^2*b^8 - 28*b^10)*cos(d*x + c)^4 + (3*a^8*b^2 + 29*a^6*b^4
+ 28*a^4*b^6 - 54*a^2*b^8 - 56*b^10)*cos(d*x + c)^2 + ((a^9*b + 8*a^7*b^3 -
 9*a^5*b^5 - 58*a^3*b^7 - 42*a*b^9)*cos(d*x + c)^7 - (2*a^9*b + 13*a^7*b^3
- 51*a^5*b^5 - 188*a^3*b^7 - 126*a*b^9)*cos(d*x + c)^5 + (a^9*b + 2*a^7*b^3
 - 75*a^5*b^5 - 202*a^3*b^7 - 126*a*b^9)*cos(d*x + c)^3 + 3*(a^7*b^3 + 11*a
^5*b^5 + 24*a^3*b^7 + 14*a*b^9)*cos(d*x + c))*sin(d*x + c))*log(-1/4*cos(d*
x + c)^2 + 1/4) - 2*(2*(6*a^9*b + 259*a^7*b^3 + 783*a^5*b^5 + 340*a^3*b^7 -
 210*a*b^9)*cos(d*x + c)^7 - (15*a^9*b + 1141*a^7*b^3 + 3546*a^5*b^5 + 1270
*a^3*b^7 - 1260*a*b^9)*cos(d*x + c)^5 + 5*(151*a^7*b^3 + 483*a^5*b^5 + 100*
a^3*b^7 - 252*a*b^9)*cos(d*x + c)^3 - 15*(9*a^7*b^3 + 29*a^5*b^5 - 6*a^3*b^
7 - 28*a*b^9)*cos(d*x + c))*sin(d*x + c))/((3*a^13*b + 2*a^11*b^3 - a^9*b^5
)*d*cos(d*x + c)^8 - (9*a^13*b + 5*a^11*b^3 - 4*a^9*b^5)*d*cos(d*x + c)^6 +
 3*(3*a^13*b + a^11*b^3 - 2*a^9*b^5)*d*cos(d*x + c)^4 - (3*a^13*b - a^11*b^
3 - 4*a^9*b^5)*d*cos(d*x + c)^2 - (a^11*b^3 + a^9*b^5)*d - ((a^14 - 2*a^12*
b^2 - 3*a^10*b^4)*d*cos(d*x + c)^7 - (2*a^14 - 7*a^12*b^2 - 9*a^10*b^4)*d*c
os(d*x + c)^5 + (a^14 - 8*a^12*b^2 - 9*a^10*b^4)*d*cos(d*x + c)^3 + 3*(a^12
*b^2 + a^10*b^4)*d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 1.95, size = 428, normalized size = 1.43

$$-\frac{\frac{60\left(a^4b+10\,a^2b^3+14\,b^5\right)\log(|\tan(dx+c)|)}{a^9}-\frac{60\left(a^4b^2+10\,a^2b^4+14\,b^6\right)\log(|b\tan(dx+c)+a|)}{a^9b}+\frac{5\left(22\,a^4b^4\tan(dx+c)^3+220\,a^2b^6\tan(dx+c)^3+308\,b^8\,\mathrm{ta}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/15*(60*(a^4*b + 10*a^2*b^3 + 14*b^5)*log(abs(tan(d*x + c)))/a^9 - 60*(a^
4*b^2 + 10*a^2*b^4 + 14*b^6)*log(abs(b*tan(d*x + c) + a))/(a^9*b) + 5*(22*a
^4*b^4*tan(d*x + c)^3 + 220*a^2*b^6*tan(d*x + c)^3 + 308*b^8*tan(d*x + c)^3
 + 75*a^5*b^3*tan(d*x + c)^2 + 720*a^3*b^5*tan(d*x + c)^2 + 987*a*b^7*tan(d
*x + c)^2 + 87*a^6*b^2*tan(d*x + c) + 792*a^4*b^4*tan(d*x + c) + 1059*a^2*b
^6*tan(d*x + c) + 35*a^7*b + 294*a^5*b^3 + 381*a^3*b^5)/((b*tan(d*x + c) +
a)^3*a^9) - (137*a^4*b*tan(d*x + c)^5 + 1370*a^2*b^3*tan(d*x + c)^5 + 1918*
b^5*tan(d*x + c)^5 - 15*a^5*tan(d*x + c)^4 - 300*a^3*b^2*tan(d*x + c)^4 - 5
25*a*b^4*tan(d*x + c)^4 + 60*a^4*b*tan(d*x + c)^3 + 150*a^2*b^3*tan(d*x + c
)^3 - 10*a^5*tan(d*x + c)^2 - 50*a^3*b^2*tan(d*x + c)^2 + 15*a^4*b*tan(d*x
+ c) - 3*a^5)/(a^9*tan(d*x + c)^5))/d
```

maple [A] time = 0.60, size = 476, normalized size = 1.59

$$-\frac{3b}{a^4d\left(a+b\tan\left(dx+c\right)\right)}-\frac{20b^3}{d\,a^6\left(a+b\tan\left(dx+c\right)\right)}-\frac{21b^5}{d\,a^8\left(a+b\tan\left(dx+c\right)\right)}-\frac{b}{3a^2d\left(a+b\tan\left(dx+c\right)\right)^3}-\frac{}{3d\,a^4\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+b*tan(d*x+c))^4,x)
```

```
[Out] -3*b/a^4/d/(a+b*tan(d*x+c))-20/d*b^3/a^6/(a+b*tan(d*x+c))-21/d*b^5/a^8/(a+b
*tan(d*x+c))-1/3*b/a^2/d/(a+b*tan(d*x+c))^3-2/3/d*b^3/a^4/(a+b*tan(d*x+c))^
3-1/3/d*b^5/a^6/(a+b*tan(d*x+c))^3-b/a^3/d/(a+b*tan(d*x+c))^2-4/d*b^3/a^5/(
a+b*tan(d*x+c))^2-3/d*b^5/a^7/(a+b*tan(d*x+c))^2+4*b*ln(a+b*tan(d*x+c))/a^5
/d+40/d*b^3/a^7*ln(a+b*tan(d*x+c))+56/d*b^5/a^9*ln(a+b*tan(d*x+c))-1/5/d/a^
```

```
4/tan(d*x+c)^5-2/3/d/a^4/tan(d*x+c)^3-10/3/d/a^6/tan(d*x+c)^3*b^2-1/d/a^4/t
an(d*x+c)-20/d/a^6/tan(d*x+c)*b^2-35/d/a^8/tan(d*x+c)*b^4+1/d/a^5*b/tan(d*x
+c)^4+4/d/a^5*b/tan(d*x+c)^2+10/d*b^3/a^7/tan(d*x+c)^2-4*b*ln(tan(d*x+c))/a
^5/d-40/d*b^3/a^7*ln(tan(d*x+c))-56/d*b^5/a^9*ln(tan(d*x+c))
```

maxima [A] time = 0.61, size = 325, normalized size = 1.08

$$\frac{\frac{6\,a^6b\tan(dx+c)-60\left(a^4b^3+10\,a^2b^5+14\,b^7\right)\tan(dx+c)^7-3\,a^7-150\left(a^5b^2+10\,a^3b^4+14\,ab^6\right)\tan(dx+c)^6-110\left(a^6b+10\,a^4b^3+14\,a^2b^5\right)\tan(dx+c)^5-15\,(a}{a^8b^3\tan(dx+c)^8+3\,a^9b^2\tan(dx+c)^7+3\,a^{10}b\tan(dx+c)^6+a^{11}\tan(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/15*((6*a^6*b*tan(d*x + c) - 60*(a^4*b^3 + 10*a^2*b^5 + 14*b^7)*tan(d*x +
c)^7 - 3*a^7 - 150*(a^5*b^2 + 10*a^3*b^4 + 14*a*b^6)*tan(d*x + c)^6 - 110*(
a^6*b + 10*a^4*b^3 + 14*a^2*b^5)*tan(d*x + c)^5 - 15*(a^7 + 10*a^5*b^2 + 14
*a^3*b^4)*tan(d*x + c)^4 + 6*(5*a^6*b + 7*a^4*b^3)*tan(d*x + c)^3 - 2*(5*a^
7 + 7*a^5*b^2)*tan(d*x + c)^2)/(a^8*b^3*tan(d*x + c)^8 + 3*a^9*b^2*tan(d*x
+ c)^7 + 3*a^10*b*tan(d*x + c)^6 + a^11*tan(d*x + c)^5) + 60*(a^4*b + 10*a^
2*b^3 + 14*b^5)*log(b*tan(d*x + c) + a)/a^9 - 60*(a^4*b + 10*a^2*b^3 + 14*b
^5)*log(tan(d*x + c))/a^9)/d
```

mupad [B] time = 5.65, size = 337, normalized size = 1.12

$$\frac{8\,b\,\mathrm{atanh}\left(\frac{4\,b\,(a+2\,b\tan(c+d\,x))\left(a^4+10\,a^2\,b^2+14\,b^4\right)}{a\left(4\,a^4\,b+40\,a^2\,b^3+56\,b^5\right)}\right)\left(a^4+10\,a^2\,b^2+14\,b^4\right)}{a^9\,d}-\frac{\frac{1}{5\,a}+\frac{\tan(c+d\,x)^4\left(a^4+10\,a^2\,b^2+14\,b^4\right)}{a^5}+\frac{2\tan(c+d}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a + b*tan(c + d*x))^4),x)
```

```
[Out] (8*b*atanh((4*b*(a + 2*b*tan(c + d*x))*(a^4 + 14*b^4 + 10*a^2*b^2))/(a*(4*a
^4*b + 56*b^5 + 40*a^2*b^3)))*(a^4 + 14*b^4 + 10*a^2*b^2))/(a^9*d) - (1/(5*
a) + (tan(c + d*x)^4*(a^4 + 14*b^4 + 10*a^2*b^2))/a^5 + (2*tan(c + d*x)^2*(
5*a^2 + 7*b^2))/(15*a^3) - (2*b*tan(c + d*x))/(5*a^2) + (22*b*tan(c + d*x)^
5*(a^4 + 14*b^4 + 10*a^2*b^2))/(3*a^6) + (10*b^2*tan(c + d*x)^6*(a^4 + 14*b
^4 + 10*a^2*b^2))/a^7 + (4*b^3*tan(c + d*x)^7*(a^4 + 14*b^4 + 10*a^2*b^2))/
a^8 - (2*b*tan(c + d*x)^3*(5*a^2 + 7*b^2))/(5*a^4))/(d*(a^3*tan(c + d*x)^5
+ b^3*tan(c + d*x)^8 + 3*a^2*b*tan(c + d*x)^6 + 3*a*b^2*tan(c + d*x)^7))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral(csc(c + d*x)**6/(a + b*tan(c + d*x))**4, x)
```

3.78 $\int \frac{\csc(x)}{1+\tan(x)}\,dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}-\tanh^{-1}(\cos(x))$$

```
[Out] -arctanh(cos(x))+1/2*arctanh(1/2*(cos(x)-sin(x))*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3518, 3110, 3770, 3074, 206}

$$\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}-\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(1 + Tan[x]),x]
```

```
[Out] -ArcTanh[Cos[x]] + ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc(x)}{1+\tan(x)}\,dx = \int \frac{\cot(x)}{\cos(x)+\sin(x)}\,dx$$
$$= \int \left(\csc(x) + \frac{1}{-\cos(x)-\sin(x)}\right) dx$$
$$= \int \csc(x)\,dx + \int \frac{1}{-\cos(x)-\sin(x)}\,dx$$
$$= -\tanh^{-1}(\cos(x)) - \text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, -\cos(x)+\sin(x)\right)$$
$$= -\tanh^{-1}(\cos(x)) - \frac{\tanh^{-1}\left(\frac{-\cos(x)+\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Mathematica [C] time = 0.04, size = 41, normalized size = 1.58

$$\log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) + (1+i)(-1)^{3/4}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(1 + Tan[x]),x]
```

```
[Out] (1 + I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] - Log[Cos[x/2]] + Log[S
in[x/2]]
```

fricas [B] time = 0.43, size = 55, normalized size = 2.12

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}+\cos(x)\right)\sin(x)-2\sqrt{2}\cos(x)-3}{2\cos(x)\sin(x)+1}\right)-\frac{1}{2}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{2}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(1+tan(x)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((2*(sqrt(2) + cos(x))*sin(x) - 2*sqrt(2)*cos(x) - 3)/(2*cos
(x)*sin(x) + 1)) - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

giac [A] time = 2.97, size = 44, normalized size = 1.69

$$\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)-2\right|}{\left|2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)-2\right|}\right)+\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(1+tan(x)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(1/2*x) - 2)/abs(2*sqrt(2) + 2*tan(1/
2*x) - 2)) + log(abs(tan(1/2*x)))
```

maple [A] time = 0.12, size = 26, normalized size = 1.00

$$-\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(2\tan\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(1+tan(x)),x)
```

```
[Out] -2^(1/2)*arctanh(1/4*(2*tan(1/2*x)-2)*2^(1/2))+ln(tan(1/2*x))
```

maxima [B] time = 0.85, size = 50, normalized size = 1.92

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(x)}{\cos(x)+1}+1}{\sqrt{2}+\frac{\sin(x)}{\cos(x)+1}-1}\right)+\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(1+tan(x)),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(-(sqrt(2) - sin(x)/(cos(x) + 1) + 1)/(sqrt(2) + sin(x)/(cos
(x) + 1) - 1)) + log(sin(x)/(cos(x) + 1))
```

mupad [B] time = 3.88, size = 38, normalized size = 1.46

$$\ln\left(\tan\left(\frac{x}{2}\right)\right)-\sqrt{2}\,\mathrm{atanh}\left(\frac{5\sqrt{2}\tan\left(\frac{x}{2}\right)+2\sqrt{2}}{7\tan\left(\frac{x}{2}\right)+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(tan(x) + 1)),x)
```

```
[Out] log(tan(x/2)) - 2^(1/2)*atanh((5*2^(1/2)*tan(x/2) + 2*2^(1/2))/(7*tan(x/2)
+ 3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc(x)}{\tan(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(1+tan(x)),x)
```

```
[Out] Integral(csc(x)/(tan(x) + 1), x)
```

3.79 $\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=229

$$\frac{a^3\cos(c+dx)\sin^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{3a^2b\sin^{m+2}(c+dx)\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(c+dx)\right)}{d(m+2)}$$

```
[Out] 3*a^2*b*hypergeom([1, 1+1/2*m],[2+1/2*m],sin(d*x+c)^2)*sin(d*x+c)^(2+m)/d/(
2+m)+b^3*hypergeom([2, 2+1/2*m],[3+1/2*m],sin(d*x+c)^2)*sin(d*x+c)^(4+m)/d/
(4+m)+a^3*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*s
in(d*x+c)^(1+m)/d/(1+m)/(cos(d*x+c)^2)^(1/2)+3*a*b^2*hypergeom([3/2, 3/2+1/
2*m],[5/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*sin(d*x+c)^(3+m)*(cos(d*x+c)^2)^(
1/2)/d/(3+m)
```

Rubi [A] time = 0.45, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4401, 2643, 2564, 364, 2577}

$$\frac{3a^2b\sin^{m+2}(c+dx)\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(c+dx)\right)}{d(m+2)}+\frac{a^3\cos(c+dx)\sin^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (3*a^2*b*Hyper
geometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))
/(d*(2 + m)) + (3*a*b^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)
/2, (5 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(3 + m))/(d*(3 + m
)) + (b^3*Hypergeometric2F1[2, (4 + m)/2, (6 + m)/2, Sin[c + d*x]^2]*Sin[c
+ d*x]^(4 + m))/(d*(4 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a^3\sin^m(c+dx)+3a^2b\sec(c+dx)\sin^{1+m}(c+dx)+3ab^2\sec^2(c+d\right.$$
$$= a^3\int \sin^m(c+dx)\,dx+\left(3a^2b\right)\int \sec(c+dx)\sin^{1+m}(c+dx)\,dx+\left(3ab^2\right.$$
$$= \frac{a^3\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}}+\frac{3ab^2\sqrt{c}}{}$$
$$= \frac{a^3\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}}+\frac{3a^2b\,{}_2F}{}$$

Mathematica [A] time = 2.56, size = 205, normalized size = 0.90

$$\frac{\sin^{m+1}(c+dx)\left(\frac{a^3\sqrt{\cos^2(c+dx)}\sec(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{m+1}+b\sin(c+dx)\left(\frac{3a^2\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(c+dx)\right)}{m+2}+b\left(\frac{3a\sqrt{\cos^2}}{}\right.\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sin[c + d*x]^(1 + m)*((a^3*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1
+ m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x])/(1 + m) + b*Sin[c + d*x]*(
(3*a^2*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2])/(2 + m)
+ b*((b*Hypergeometric2F1[2, (4 + m)/2, (6 + m)/2, Sin[c + d*x]^2]*Sin[c +
d*x]^2)/(4 + m) + (3*a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)/
2, (5 + m)/2, Sin[c + d*x]^2]*Tan[c + d*x])/(3 + m)))))/d
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\tan(dx+c)^3+3\,ab^2\tan(dx+c)^2+3\,a^2b\tan(dx+c)+a^3\right)\sin(dx+c)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*tan(d*x + c)^3 + 3*a*b^2*tan(d*x + c)^2 + 3*a^2*b*tan(d*x + c
) + a^3)*sin(d*x + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(dx+c)+a)^3\sin(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^3*sin(d*x + c)^m, x)
```

maple [F] time = 1.20, size = 0, normalized size = 0.00

$$\int \left(\sin^{m}\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x)
[Out] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(dx+c\right)+a\right)^{3}\sin\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^3*sin(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(c+d\,x\right)^{m}\,\left(a+b\tan\left(c+d\,x\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^m*(a + b*tan(c + d*x))^3,x)
[Out] int(sin(c + d*x)^m*(a + b*tan(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(c+dx\right)\right)^{3}\sin^{m}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**m*(a+b*tan(d*x+c))**3,x)
[Out] Integral((a + b*tan(c + d*x))**3*sin(c + d*x)**m, x)
```

3.80 $\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=179

$$\frac{a^2\cos(c+dx)\sin^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{2ab\sin^{m+2}(c+dx)\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(c+dx)\right)}{d(m+2)}$$

```
[Out] 2*a*b*hypergeom([1, 1+1/2*m],[2+1/2*m],sin(d*x+c)^2)*sin(d*x+c)^(2+m)/d/(2+
m)+a^2*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sin(
d*x+c)^(1+m)/d/(1+m)/(cos(d*x+c)^2)^(1/2)+b^2*hypergeom([3/2, 3/2+1/2*m],[5
/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*sin(d*x+c)^(3+m)*(cos(d*x+c)^2)^(1/2)/d/
(3+m)
```

Rubi [A] time = 0.27, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4401, 2643, 2564, 364, 2577}

$$\frac{a^2\cos(c+dx)\sin^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{2ab\sin^{m+2}(c+dx)\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(c+dx)\right)}{d(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (2*a*b*Hyperge
ometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(
d*(2 + m)) + (b^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5
 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(3 + m))/(d*(3 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a^2\sin^m(c+dx)+2ab\sec(c+dx)\sin^{1+m}(c+dx)+b^2\sec^2(c+d\right.$$

$$= a^2\int \sin^m(c+dx)\,dx+(2ab)\int \sec(c+dx)\sin^{1+m}(c+dx)\,dx+b^2\int$$

$$= \frac{a^2\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}}+\frac{b^2\sqrt{\cdots}}{\cdots}$$

$$= \frac{a^2\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}}+\frac{2ab\,{}_2\cdots}{\cdots}$$

Mathematica [A] time = 1.21, size = 166, normalized size = 0.93

$$\frac{\sin^{m+1}(c+dx)\left(\frac{a^2\sqrt{\cos^2(c+dx)}\sec(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{m+1}+\frac{b\sin(c+dx)\left(2a(m+3)\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(c+dx)\right)+b(m+2)\sqrt{\cos\cdots}\right.}{(m+2)(m+3)}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]^(1 + m)*((a^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1
+ m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x])/(1 + m) + (b*Sin[c + d*x]*
(2*a*(3 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2] + b
*(2 + m)*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5 + m)/2,
Sin[c + d*x]^2]*Tan[c + d*x]))/((2 + m)*(3 + m))))/d
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2\tan\left(dx+c\right)^2+2\,ab\tan\left(dx+c\right)+a^2\right)\sin\left(dx+c\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)*sin(d*x + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(dx+c\right)+a\right)^2\sin\left(dx+c\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^2*sin(d*x + c)^m, x)
```

maple [F] time = 0.94, size = 0, normalized size = 0.00

$$\int \left(\sin^m\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x)
[Out] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(dx+c)+a)^2 \sin(dx+c)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^2*sin(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+d\,x)^m \, (a+b\tan(c+d\,x))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^m*(a + b*tan(c + d*x))^2,x)
[Out] int(sin(c + d*x)^m*(a + b*tan(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^2 \sin^m(c+dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**m*(a+b*tan(d*x+c))**2,x)
[Out] Integral((a + b*tan(c + d*x))**2*sin(c + d*x)**m, x)
```

3.81 $\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=109

$$\frac{a\cos(c+dx)\sin^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{b\sin^{m+2}(c+dx)\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(c+dx)\right)}{d(m+2)}$$

```
[Out] b*hypergeom([1, 1+1/2*m],[2+1/2*m],sin(d*x+c)^2)*sin(d*x+c)^(2+m)/d/(2+m)+a
*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sin(d*x+c)
^(1+m)/d/(1+m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {4401, 2643, 2564, 364}

$$\frac{a\cos(c+dx)\sin^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{b\sin^{m+2}(c+dx)\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(c+dx)\right)}{d(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2
]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (b*Hypergeometri
c2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(d*(2 +
 m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx = \int \left(a\sin^m(c+dx)+b\sec(c+dx)\sin^{1+m}(c+dx)\right)dx$$
$$= a\int \sin^m(c+dx)\,dx + b\int \sec(c+dx)\sin^{1+m}(c+dx)\,dx$$
$$= \frac{a\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}} + \frac{b\,\mathrm{Subst}\left(\right.}{}$$
$$= \frac{a\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}} + \frac{b\,{}_2F_1\left(1,\right.}{}$$

Mathematica [A] time = 0.27, size = 109, normalized size = 1.00

$$\frac{a\sqrt{\cos^2(c+dx)}\sec(c+dx)\sin^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)}+\frac{b\sin^{m+2}(c+dx)\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin\right.}{d(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c
+ d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + m))/(d*(1 + m)) + (b*Hypergeometri
c2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(d*(2 +
 m))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((b\tan(dx+c)+a)\sin(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)*sin(d*x + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(dx+c)+a)\sin(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)*sin(d*x + c)^m, x)
```

maple [F] time = 1.40, size = 0, normalized size = 0.00

$$\int (\sin^m(dx+c))(a+b\tan(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^m*(a+b*tan(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)^m*(a+b*tan(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(dx+c)+a)\sin(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)*sin(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(c+d\,x\right)^m\,\left(a+b\,\tan\left(c+d\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^m*(a + b*tan(c + d*x)),x)
```

```
[Out] int(sin(c + d*x)^m*(a + b*tan(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tan{\left (c + d x \right)}\right) \sin^{m}{\left (c + d x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**m*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**m, x)
```

3.82 $\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=765

$$\frac{b2^{m+1}\tan^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^m\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^m F_1\left(\frac{m+2}{2};m+1,1;\frac{m+4}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{a^2\tan}{(b-}\right.}{d(m+2)\sqrt{a^2+b^2}\left(b-\sqrt{a^2+b^2}\right)}$$

```
[Out] 2^(1+m)*hypergeom([1+m, 1/2+1/2*m],[3/2+1/2*m],-tan(1/2*d*x+1/2*c)^2)*tan(1
/2*d*x+1/2*c)*(tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2))^m*(1+tan(1/2*d*
x+1/2*c)^2)^m/a/d/(1+m)+2^(1+m)*b*AppellF1(1+1/2*m,1,1+m,2+1/2*m,a^2*tan(1/
2*d*x+1/2*c)^2/(b-(a^2+b^2)^(1/2))^2,-tan(1/2*d*x+1/2*c)^2)*tan(1/2*d*x+1/2
*c)^2*(tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2))^m*(1+tan(1/2*d*x+1/2*c)
^2)^m/d/(2+m)/(b-(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)-2^(1+m)*b*AppellF1(1+1/2*
m,1,1+m,2+1/2*m,a^2*tan(1/2*d*x+1/2*c)^2/(b+(a^2+b^2)^(1/2))^2,-tan(1/2*d*x
+1/2*c)^2)*tan(1/2*d*x+1/2*c)^2*(tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2
))^m*(1+tan(1/2*d*x+1/2*c)^2)^m/d/(2+m)/(a^2+b^2)^(1/2)/(b+(a^2+b^2)^(1/2))
+2^(1+m)*a*b*AppellF1(3/2+1/2*m,1,1+m,5/2+1/2*m,a^2*tan(1/2*d*x+1/2*c)^2/(b
-(a^2+b^2)^(1/2))^2,-tan(1/2*d*x+1/2*c)^2)*tan(1/2*d*x+1/2*c)^3*(tan(1/2*d*
x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2))^m*(1+tan(1/2*d*x+1/2*c)^2)^m/d/(3+m)/(b-
(a^2+b^2)^(1/2))^2/(a^2+b^2)^(1/2)-2^(1+m)*a*b*AppellF1(3/2+1/2*m,1,1+m,5/2
+1/2*m,a^2*tan(1/2*d*x+1/2*c)^2/(b+(a^2+b^2)^(1/2))^2,-tan(1/2*d*x+1/2*c)^2
)*tan(1/2*d*x+1/2*c)^3*(tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2))^m*(1+t
an(1/2*d*x+1/2*c)^2)^m/d/(3+m)/(a^2+b^2)^(1/2)/(b+(a^2+b^2)^(1/2))^2
```

Rubi [A] time = 4.17, antiderivative size = 765, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {12, 6719, 6728, 364, 959, 510}

$$\frac{ab2^{m+1}\tan^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^m\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^m F_1\left(\frac{m+3}{2};m+1,1;\frac{m+5}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{a^2 t}{(b}\right.}{d(m+3)\sqrt{a^2+b^2}\left(b-\sqrt{a^2+b^2}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^m/(a + b*Tan[c + d*x]),x]
```

```
[Out] (2^(1 + m)*Hypergeometric2F1[(1 + m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/2]
^2]*Tan[(c + d*x)/2]*(Tan[(c + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^m*(1 + Tan
[(c + d*x)/2]^2)^m)/(a*d*(1 + m)) + (2^(1 + m)*b*AppellF1[(2 + m)/2, 1 + m,
 1, (4 + m)/2, -Tan[(c + d*x)/2]^2, (a^2*Tan[(c + d*x)/2]^2)/(b - Sqrt[a^2
+ b^2])^2]*Tan[(c + d*x)/2]^2*(Tan[(c + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^m
*(1 + Tan[(c + d*x)/2]^2)^m)/(Sqrt[a^2 + b^2]*(b - Sqrt[a^2 + b^2])*d*(2 +
m)) - (2^(1 + m)*b*AppellF1[(2 + m)/2, 1 + m, 1, (4 + m)/2, -Tan[(c + d*x)/
2]^2, (a^2*Tan[(c + d*x)/2]^2)/(b + Sqrt[a^2 + b^2])^2]*Tan[(c + d*x)/2]^2*
(Tan[(c + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^m*(1 + Tan[(c + d*x)/2]^2)^m)/(
Sqrt[a^2 + b^2]*(b + Sqrt[a^2 + b^2])*d*(2 + m)) + (2^(1 + m)*a*b*AppellF1[
(3 + m)/2, 1 + m, 1, (5 + m)/2, -Tan[(c + d*x)/2]^2, (a^2*Tan[(c + d*x)/2]^
2)/(b - Sqrt[a^2 + b^2])^2]*Tan[(c + d*x)/2]^3*(Tan[(c + d*x)/2]/(1 + Tan[(
c + d*x)/2]^2))^m*(1 + Tan[(c + d*x)/2]^2)^m)/(Sqrt[a^2 + b^2]*(b - Sqrt[a^
2 + b^2])^2*d*(3 + m)) - (2^(1 + m)*a*b*AppellF1[(3 + m)/2, 1 + m, 1, (5 +
m)/2, -Tan[(c + d*x)/2]^2, (a^2*Tan[(c + d*x)/2]^2)/(b + Sqrt[a^2 + b^2])^2
]*Tan[(c + d*x)/2]^3*(Tan[(c + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^m*(1 + Tan
[(c + d*x)/2]^2)^m)/(Sqrt[a^2 + b^2]*(b + Sqrt[a^2 + b^2])^2*d*(3 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{2^m(1-x^2)\left(\frac{x}{1+x^2}\right)^{1+m}}{x(a+2bx-ax^2)}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$= \frac{2^{1+m}\,\text{Subst}\left(\int \frac{(1-x^2)\left(\frac{x}{1+x^2}\right)^{1+m}}{x(a+2bx-ax^2)}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$= \frac{\left(2^{1+m}\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^m\right)\text{Subst}\left(\int \frac{x^m(1-x^2)(}{a+2b\cdots}\right.}{d}$$

$$= \frac{\left(2^{1+m}\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^m\right)\text{Subst}\left(\int\left(\frac{x^m(1+x^2)}{a}\right.\right.}{d}$$

$$= \frac{\left(2^{1+m}\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^m\right)\text{Subst}\left(\int x^m\left(1+\cdots\right.\right.}{ad}$$

$$= \frac{2^{1+m}\,{}_2F_1\left(\frac{1+m}{2},1+m;\frac{3+m}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\cdots\right.}{ad(1+m)}$$

$$= \frac{2^{1+m}\,{}_2F_1\left(\frac{1+m}{2},1+m;\frac{3+m}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\cdots\right.}{ad(1+m)}$$

$$= \frac{2^{1+m}\,{}_2F_1\left(\frac{1+m}{2},1+m;\frac{3+m}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\cdots\right.}{ad(1+m)}$$

$$= \frac{2^{1+m}\,{}_2F_1\left(\frac{1+m}{2},1+m;\frac{3+m}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\cdots\right.}{ad(1+m)}$$

Mathematica [F] time = 13.22, size = 0, normalized size = 0.00

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^m/(a + b*Tan[c + d*x]),x]
[Out] Integrate[Sin[c + d*x]^m/(a + b*Tan[c + d*x]), x]
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sin(dx+c)^m}{b\tan(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)^m/(b*tan(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^m}{b\tan(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^m/(b*tan(d*x + c) + a), x)
```

maple [F] time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{\sin^m(dx+c)}{a+b\tan(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^m/(a+b*tan(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)^m/(a+b*tan(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^m}{b\tan(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^m/(b*tan(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(c+d\,x)^m}{a+b\tan(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^m/(a + b*tan(c + d*x)),x)
```

```
[Out] int(sin(c + d*x)^m/(a + b*tan(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**m/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**m/(a + b*tan(c + d*x)), x)
```

3.83 $\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=24

$$\mathrm{Int}\left(\sin^m(c+dx)(a+b\tan(c+dx))^n,x\right)$$

```
[Out] CannotIntegrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x)
```

Rubi [A] time = 2.46, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n,x]
[Out] Defer[Int][Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n, x]
```

Rubi steps

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx=\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx$$

Mathematica [A] time = 3.50, size = 0, normalized size = 0.00

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n,x]
[Out] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n, x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((b\tan(dx+c)+a)^n\sin(dx+c)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming c near 0Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Simplification assuming a near 0Simplification assuming a nea
r 0Evaluation time: 0.53Unable to divide, perhaps due to rounding error%%%{
```

-268435456,[0,10,0,10,16,0,4,0]%%%}+%%%{1073741824*i,[0,10,0,10,15,0,5,0]%%
%}+%%%{1610612736,[0,10,0,10,14,0,6,0]%%%}+%%%{-1073741824*i,[0,10,0,10,13,
0,7,0]%%%}+%%%{268435456,[0,10,0,10,12,0,8,0]%%%}+%%%{-2147483648*i,[0,10,0
,10,11,0,9,0]%%%}+%%%{-3221225472,[0,10,0,10,10,0,10,0]%%%}+%%%{2147483648*
i,[0,10,0,10,9,0,11,0]%%%}+%%%{268435456,[0,10,0,10,8,0,12,0]%%%}+%%%{10737
41824*i,[0,10,0,10,7,0,13,0]%%%}+%%%{1610612736,[0,10,0,10,6,0,14,0]%%%}+%%
%{-1073741824*i,[0,10,0,10,5,0,15,0]%%%}+%%%{-268435456,[0,10,0,10,4,0,16,0
]%%%}+%%%{1073741824,[0,9,0,10,16,1,4,0]%%%}+%%%{-4294967296*i,[0,9,0,10,15
,1,5,0]%%%}+%%%{1073741824*i,[0,9,0,10,15,0,5,1]%%%}+%%%{-6710886400,[0,9,0
,10,14,1,6,0]%%%}+%%%{4026531840,[0,9,0,10,14,0,6,1]%%%}+%%%{4831838208*i,[
0,9,0,10,13,1,7,0]%%%}+%%%{-4831838208*i,[0,9,0,10,13,0,7,1]%%%}+%%%{-13421
77280,[0,9,0,10,12,1,8,0]%%%}+%%%{-268435456,[0,9,0,10,12,0,8,1]%%%}+%%%{96
63676416*i,[0,9,0,10,11,1,9,0]%%%}+%%%{-5368709120*i,[0,9,0,10,11,0,9,1]%%%
}+%%%{13421772800,[0,9,0,10,10,1,10,0]%%%}+%%%{-8053063680,[0,9,0,10,10,0,1
0,1]%%%}+%%%{-8589934592*i,[0,9,0,10,9,1,11,0]%%%}+%%%{6442450944*i,[0,9,0,
10,9,0,11,1]%%%}+%%%{-536870912,[0,9,0,10,8,1,12,0]%%%}+%%%{536870912,[0,9,
0,10,8,0,12,1]%%%}+%%%{-5368709120*i,[0,9,0,10,7,1,13,0]%%%}+%%%{4294967296
*i,[0,9,0,10,7,0,13,1]%%%}+%%%{-6710886400,[0,9,0,10,6,1,14,0]%%%}+%%%{4026
531840,[0,9,0,10,6,0,14,1]%%%}+%%%{3758096384*i,[0,9,0,10,5,1,15,0]%%%}+%%%
{-1610612736*i,[0,9,0,10,5,0,15,1]%%%}+%%%{805306368,[0,9,0,10,4,1,16,0]%%%
}+%%%{-268435456,[0,9,0,10,4,0,16,1]%%%}+%%%{-1610612736,[0,8,0,10,16,2,4,0
]%%%}+%%%{536870912,[0,8,0,10,16,0,4,2]%%%}+%%%{6442450944*i,[0,8,0,10,15,2
,5,0]%%%}+%%%{-4294967296*i,[0,8,0,10,15,1,5,1]%%%}+%%%{-2147483648*i,[0,8,
0,10,15,0,5,2]%%%}+%%%{10468982784,[0,8,0,10,14,2,6,0]%%%}+%%%{-16106127360
,[0,8,0,10,14,1,6,1]%%%}+%%%{-1879048192,[0,8,0,10,14,0,6,2]%%%}+%%%{-80530
63680*i,[0,8,0,10,13,2,7,0]%%%}+%%%{19864223744*i,[0,8,0,10,13,1,7,1]%%%}+%
%%{-2147483648*i,[0,8,0,10,13,0,7,2]%%%}+%%%{2348810240,[0,8,0,10,12,2,8,0]
%%%}+%%%{2013265920,[0,8,0,10,12,1,8,1]%%%}+%%%{-3556769792,[0,8,0,10,12,0,
8,2]%%%}+%%%{-16106127360*i,[0,8,0,10,11,2,9,0]%%%}+%%%{22548578304*i,[0,8,
0,10,11,1,9,1]%%%}+%%%{-21206401024,[0,8,0,10,10,2,10,0]%%%}+%%%{3489660928
0,[0,8,0,10,10,1,10,1]%%%}+%%%{-1879048192,[0,8,0,10,10,0,10,2]%%%}+%%%{128
84901888*i,[0,8,0,10,9,2,11,0]%%%}+%%%{-25769803776*i,[0,8,0,10,9,1,11,1]%%
%}+%%%{-402653184,[0,8,0,10,8,2,12,0]%%%}+%%%{268435456,[0,8,0,10,8,1,12,1]
%%%}+%%%{-2550136832,[0,8,0,10,8,0,12,2]%%%}+%%%{9663676416*i,[0,8,0,10,7,2
,13,0]%%%}+%%%{-18253611008*i,[0,8,0,10,7,1,13,1]%%%}+%%%{2147483648*i,[0,8
,0,10,7,0,13,2]%%%}+%%%{10200547328,[0,8,0,10,6,2,14,0]%%%}+%%%{-1556925644
8,[0,8,0,10,6,1,14,1]%%%}+%%%{-1073741824,[0,8,0,10,6,0,14,2]%%%}+%%%{-4831
838208*i,[0,8,0,10,5,2,15,0]%%%}+%%%{5905580032*i,[0,8,0,10,5,1,15,1]%%%}+%
%%{2147483648*i,[0,8,0,10,5,0,15,2]%%%}+%%%{-872415232,[0,8,0,10,4,2,16,0]%
%%}+%%%{939524096,[0,8,0,10,4,1,16,1]%%%}+%%%{738197504,[0,8,0,10,4,0,16,2]
%%%}+%%%{1073741824,[0,7,0,10,16,3,4,0]%%%}+%%%{-2147483648,[0,7,0,10,16,1,
4,2]%%%}+%%%{-4294967296*i,[0,7,0,10,15,3,5,0]%%%}+%%%{6442450944*i,[0,7,0,
10,15,2,5,1]%%%}+%%%{8589934592*i,[0,7,0,10,15,1,5,2]%%%}+%%%{-2147483648*i
,[0,7,0,10,15,0,5,3]%%%}+%%%{-7247757312,[0,7,0,10,14,3,6,0]%%%}+%%%{241591
91040,[0,7,0,10,14,2,6,1]%%%}+%%%{8321499136,[0,7,0,10,14,1,6,2]%%%}+%%%{-8
053063680,[0,7,0,10,14,0,6,3]%%%}+%%%{5905580032*i,[0,7,0,10,13,3,7,0]%%%}+
%%%{-30601641984*i,[0,7,0,10,13,2,7,1]%%%}+%%%{6979321856*i,[0,7,0,10,13,1,
7,2]%%%}+%%%{9126805504*i,[0,7,0,10,13,0,7,3]%%%}+%%%{-1744830464,[0,7,0,10
,12,3,8,0]%%%}+%%%{-4429185024,[0,7,0,10,12,2,8,1]%%%}+%%%{15166603264,[0,7
,0,10,12,1,8,2]%%%}+%%%{-402653184,[0,7,0,10,12,0,8,3]%%%}+%%%{11811160064*
i,[0,7,0,10,11,3,9,0]%%%}+%%%{-35433480192*i,[0,7,0,10,11,2,9,1]%%%}+%%%{-3
221225472*i,[0,7,0,10,11,1,9,2]%%%}+%%%{9663676416*i,[0,7,0,10,11,0,9,3]%%%
}+%%%{15032385536,[0,7,0,10,10,3,10,0]%%%}+%%%{-56371445760,[0,7,0,10,10,2,
10,1]%%%}+%%%{6442450944,[0,7,0,10,10,1,10,2]%%%}+%%%{13421772800,[0,7,0,10
,10,0,10,3]%%%}+%%%{-8589934592*i,[0,7,0,10,9,3,11,0]%%%}+%%%{38654705664*i
,[0,7,0,10,9,2,11,1]%%%}+%%%{-12884901888*i,[0,7,0,10,9,0,11,3]%%%}+%%%{134
2177280,[0,7,0,10,8,3,12,0]%%%}+%%%{-4026531840,[0,7,0,10,8,2,12,1]%%%}+%%%
{9395240960,[0,7,0,10,8,1,12,2]%%%}+%%%{-3489660928,[0,7,0,10,8,0,12,3]%%%}
+%%%{-7516192768*i,[0,7,0,10,7,3,13,0]%%%}+%%%{28991029248*i,[0,7,0,10,7,2,

13,1]%%%}+%%%{-5368709120*i,[0,7,0,10,7,1,13,2]%%%}+%%%{-7516192768*i,[0,7,
0,10,7,0,13,3]%%%}+%%%{-6710886400,[0,7,0,10,6,3,14,0]%%%}+%%%{22548578304,
[0,7,0,10,6,2,14,1]%%%}+%%%{5637144576,[0,7,0,10,6,1,14,2]%%%}+%%%{-8589934
592,[0,7,0,10,6,0,14,3]%%%}+%%%{2684354560*i,[0,7,0,10,5,3,15,0]%%%}+%%%{-8
053063680*i,[0,7,0,10,5,2,15,1]%%%}+%%%{-6979321856*i,[0,7,0,10,5,1,15,2]%%
%}+%%%{3758096384*i,[0,7,0,10,5,0,15,3]%%%}+%%%{402653184,[0,7,0,10,4,3,16,
0]%%%}+%%%{-1207959552,[0,7,0,10,4,2,16,1]%%%}+%%%{-2013265920,[0,7,0,10,4,
1,16,2]%%%}+%%%{671088640,[0,7,0,10,4,0,16,3]%%%}+%%%{-268435456,[0,6,0,10,
16,4,4,0]%%%}+%%%{3221225472,[0,6,0,10,16,2,4,2]%%%}+%%%{-268435456,[0,6,0,
10,16,0,4,4]%%%}+%%%{1073741824*i,[0,6,0,10,15,4,5,0]%%%}+%%%{-4294967296*i
,[0,6,0,10,15,3,5,1]%%%}+%%%{-12884901888*i,[0,6,0,10,15,2,5,2]%%%}+%%%{858
9934592*i,[0,6,0,10,15,1,5,3]%%%}+%%%{1073741824*i,[0,6,0,10,15,0,5,4]%%%}+
%%%{1879048192,[0,6,0,10,14,4,6,0]%%%}+%%%{-16106127360,[0,6,0,10,14,3,6,1]
%%%}+%%%{-13690208256,[0,6,0,10,14,2,6,2]%%%}+%%%{32212254720,[0,6,0,10,14,
1,6,3]%%%}+%%%{-1073741824,[0,6,0,10,14,0,6,4]%%%}+%%%{-1610612736*i,[0,6,0
,10,13,4,7,0]%%%}+%%%{20937965568*i,[0,6,0,10,13,3,7,1]%%%}+%%%{-8053063680
*i,[0,6,0,10,13,2,7,2]%%%}+%%%{-38117834752*i,[0,6,0,10,13,1,7,3]%%%}+%%%{7
516192768*i,[0,6,0,10,13,0,7,4]%%%}+%%%{469762048,[0,6,0,10,12,4,8,0]%%%}+%
%%{3892314112,[0,6,0,10,12,3,8,1]%%%}+%%%{-23890755584,[0,6,0,10,12,2,8,2]%
%%}+%%%{-1207959552,[0,6,0,10,12,1,8,3]%%%}+%%%{6241124352,[0,6,0,10,12,0,8
,4]%%%}+%%%{-3221225472*i,[0,6,0,10,11,4,9,0]%%%}+%%%{24696061952*i,[0,6,0,
10,11,3,9,1]%%%}+%%%{9663676416*i,[0,6,0,10,11,2,9,2]%%%}+%%%{-41875931136*
i,[0,6,0,10,11,1,9,3]%%%}+%%%{6442450944*i,[0,6,0,10,11,0,9,4]%%%}+%%%{-402
6531840,[0,6,0,10,10,4,10,0]%%%}+%%%{40265318400,[0,6,0,10,10,3,10,1]%%%}+%
%%{-6979321856,[0,6,0,10,10,2,10,2]%%%}+%%%{-61740154880,[0,6,0,10,10,1,10,
3]%%%}+%%%{13153337344,[0,6,0,10,10,0,10,4]%%%}+%%%{2147483648*i,[0,6,0,10,
9,4,11,0]%%%}+%%%{-25769803776*i,[0,6,0,10,9,3,11,1]%%%}+%%%{51539607552*i,
[0,6,0,10,9,1,11,3]%%%}+%%%{-6442450944*i,[0,6,0,10,9,0,11,4]%%%}+%%%{-6710
88640,[0,6,0,10,8,4,12,0]%%%}+%%%{5100273664,[0,6,0,10,8,3,12,1]%%%}+%%%{-1
1274289152,[0,6,0,10,8,2,12,2]%%%}+%%%{6710886400,[0,6,0,10,8,1,12,3]%%%}+%
%%{3892314112,[0,6,0,10,8,0,12,4]%%%}+%%%{2147483648*i,[0,6,0,10,7,4,13,0]%
%%}+%%%{-20401094656*i,[0,6,0,10,7,3,13,1]%%%}+%%%{3221225472*i,[0,6,0,10,7
,2,13,2]%%%}+%%%{33285996544*i,[0,6,0,10,7,1,13,3]%%%}+%%%{-7516192768*i,[0
,6,0,10,7,0,13,4]%%%}+%%%{1610612736,[0,6,0,10,6,4,14,0]%%%}+%%%{-144955146
24,[0,6,0,10,6,3,14,1]%%%}+%%%{-9395240960,[0,6,0,10,6,2,14,2]%%%}+%%%{3274
9125632,[0,6,0,10,6,1,14,3]%%%}+%%%{-2952790016,[0,6,0,10,6,0,14,4]%%%}+%%%
{-536870912*i,[0,6,0,10,5,4,15,0]%%%}+%%%{4831838208*i,[0,6,0,10,5,3,15,1]%
%%}+%%%{8053063680*i,[0,6,0,10,5,2,15,2]%%%}+%%%{-13421772800*i,[0,6,0,10,5
,1,15,3]%%%}+%%%{-1073741824*i,[0,6,0,10,5,0,15,4]%%%}+%%%{-67108864,[0,6,0
,10,4,4,16,0]%%%}+%%%{671088640,[0,6,0,10,4,3,16,1]%%%}+%%%{1879048192,[0,6
,0,10,4,2,16,2]%%%}+%%%{-2281701376,[0,6,0,10,4,1,16,3]%%%}+%%%{-738197504,
[0,6,0,10,4,0,16,4]%%%}+%%%{-2147483648,[0,5,0,10,16,3,4,2]%%%}+%%%{1073741
824,[0,5,0,10,16,1,4,4]%%%}+%%%{1073741824*i,[0,5,0,10,15,4,5,1]%%%}+%%%{85
89934592*i,[0,5,0,10,15,3,5,2]%%%}+%%%{-12884901888*i,[0,5,0,10,15,2,5,3]%%
%}+%%%{-4294967296*i,[0,5,0,10,15,1,5,4]%%%}+%%%{1073741824*i,[0,5,0,10,15,
0,5,5]%%%}+%%%{4026531840,[0,5,0,10,14,4,6,1]%%%}+%%%{9932111872,[0,5,0,10,
14,3,6,2]%%%}+%%%{-48318382080,[0,5,0,10,14,2,6,3]%%%}+%%%{3489660928,[0,5,
0,10,14,1,6,4]%%%}+%%%{4026531840,[0,5,0,10,14,0,6,5]%%%}+%%%{-5368709120*i
,[0,5,0,10,13,4,7,1]%%%}+%%%{3758096384*i,[0,5,0,10,13,3,7,2]%%%}+%%%{59592
671232*i,[0,5,0,10,13,2,7,3]%%%}+%%%{-28454158336*i,[0,5,0,10,13,1,7,4]%%%}
+%%%{-3758096384*i,[0,5,0,10,13,0,7,5]%%%}+%%%{-1207959552,[0,5,0,10,12,4,8
,1]%%%}+%%%{16508780544,[0,5,0,10,12,3,8,2]%%%}+%%%{6039797760,[0,5,0,10,12
,2,8,3]%%%}+%%%{-26172456960,[0,5,0,10,12,1,8,4]%%%}+%%%{1610612736,[0,5,0,
10,12,0,8,5]%%%}+%%%{-6442450944*i,[0,5,0,10,11,4,9,1]%%%}+%%%{-9663676416*
i,[0,5,0,10,11,3,9,2]%%%}+%%%{67645734912*i,[0,5,0,10,11,2,9,3]%%%}+%%%{-22
548578304*i,[0,5,0,10,11,1,9,4]%%%}+%%%{-3221225472*i,[0,5,0,10,11,0,9,5]%%
%}+%%%{-10737418240,[0,5,0,10,10,4,10,1]%%%}+%%%{2147483648,[0,5,0,10,10,3,
10,2]%%%}+%%%{104689827840,[0,5,0,10,10,2,10,3]%%%}+%%%{-52613349376,[0,5,0
,10,10,1,10,4]%%%}+%%%{-2684354560,[0,5,0,10,10,0,10,5]%%%}+%%%{6442450944*

i,[0,5,0,10,9,4,11,1]%%%}+%%%{-77309411328*i,[0,5,0,10,9,2,11,3]%%%}+%%%{25
769803776*i,[0,5,0,10,9,1,11,4]%%%}+%%%{6442450944*i,[0,5,0,10,9,0,11,5]%%%
}+%%%{-1879048192,[0,5,0,10,8,4,12,1]%%%}+%%%{4563402752,[0,5,0,10,8,3,12,2
]%%%}+%%%{805306368,[0,5,0,10,8,2,12,3]%%%}+%%%{-16374562816,[0,5,0,10,8,1,
12,4]%%%}+%%%{5368709120,[0,5,0,10,8,0,12,5]%%%}+%%%{5368709120*i,[0,5,0,10
,7,4,13,1]%%%}+%%%{1073741824*i,[0,5,0,10,7,3,13,2]%%%}+%%%{-54760833024*i,
[0,5,0,10,7,2,13,3]%%%}+%%%{26843545600*i,[0,5,0,10,7,1,13,4]%%%}+%%%{21474
83648*i,[0,5,0,10,7,0,13,5]%%%}+%%%{3489660928,[0,5,0,10,6,4,14,1]%%%}+%%%{
6174015488,[0,5,0,10,6,3,14,2]%%%}+%%%{-46707769344,[0,5,0,10,6,2,14,3]%%%}
+%%%{9395240960,[0,5,0,10,6,1,14,4]%%%}+%%%{5100273664,[0,5,0,10,6,0,14,5]%
%%}+%%%{-1073741824*i,[0,5,0,10,5,4,15,1]%%%}+%%%{-3758096384*i,[0,5,0,10,5
,3,15,2]%%%}+%%%{17716740096*i,[0,5,0,10,5,2,15,3]%%%}+%%%{2684354560*i,[0,
5,0,10,5,1,15,4]%%%}+%%%{-2684354560*i,[0,5,0,10,5,0,15,5]%%%}+%%%{-1342177
28,[0,5,0,10,4,4,16,1]%%%}+%%%{-671088640,[0,5,0,10,4,3,16,2]%%%}+%%%{28185
72288,[0,5,0,10,4,2,16,3]%%%}+%%%{1744830464,[0,5,0,10,4,1,16,4]%%%}+%%%{-5
36870912,[0,5,0,10,4,0,16,5]%%%}+%%%{536870912,[0,4,0,10,16,4,4,2]%%%}+%%%{
-1610612736,[0,4,0,10,16,2,4,4]%%%}+%%%{-2147483648*i,[0,4,0,10,15,4,5,2]%%
%}+%%%{8589934592*i,[0,4,0,10,15,3,5,3]%%%}+%%%{6442450944*i,[0,4,0,10,15,2
,5,4]%%%}+%%%{-4294967296*i,[0,4,0,10,15,1,5,5]%%%}+%%%{-2684354560,[0,4,0,
10,14,4,6,2]%%%}+%%%{32212254720,[0,4,0,10,14,3,6,3]%%%}+%%%{-4026531840,[0
,4,0,10,14,2,6,4]%%%}+%%%{-16106127360,[0,4,0,10,14,1,6,5]%%%}+%%%{13421772
80,[0,4,0,10,14,0,6,6]%%%}+%%%{-536870912*i,[0,4,0,10,13,4,7,2]%%%}+%%%{-41
339060224*i,[0,4,0,10,13,3,7,3]%%%}+%%%{40265318400*i,[0,4,0,10,13,2,7,4]%%
%}+%%%{16642998272*i,[0,4,0,10,13,1,7,5]%%%}+%%%{-4294967296*i,[0,4,0,10,13
,0,7,6]%%%}+%%%{-4227858432,[0,4,0,10,12,4,8,2]%%%}+%%%{-6845104128,[0,4,0,
10,12,3,8,3]%%%}+%%%{40667971584,[0,4,0,10,12,2,8,4]%%%}+%%%{-3623878656,[0
,4,0,10,12,1,8,5]%%%}+%%%{-2885681152,[0,4,0,10,12,0,8,6]%%%}+%%%{322122547
2*i,[0,4,0,10,11,4,9,2]%%%}+%%%{-48318382080*i,[0,4,0,10,11,3,9,3]%%%}+%%%{
28991029248*i,[0,4,0,10,11,2,9,4]%%%}+%%%{16106127360*i,[0,4,0,10,11,1,9,5]
%%%}+%%%{-4294967296*i,[0,4,0,10,11,0,9,6]%%%}+%%%{268435456,[0,4,0,10,10,4
,10,2]%%%}+%%%{-77846282240,[0,4,0,10,10,3,10,3]%%%}+%%%{77309411328,[0,4,0
,10,10,2,10,4]%%%}+%%%{18790481920,[0,4,0,10,10,1,10,5]%%%}+%%%{-7784628224
,[0,4,0,10,10,0,10,6]%%%}+%%%{51539607552*i,[0,4,0,10,9,3,11,3]%%%}+%%%{-38
654705664*i,[0,4,0,10,9,2,11,4]%%%}+%%%{-25769803776*i,[0,4,0,10,9,1,11,5]%
%%}+%%%{4294967296*i,[0,4,0,10,9,0,11,6]%%%}+%%%{-134217728,[0,4,0,10,8,4,1
2,2]%%%}+%%%{-7784628224,[0,4,0,10,8,3,12,3]%%%}+%%%{23353884672,[0,4,0,10,
8,2,12,4]%%%}+%%%{-14227079168,[0,4,0,10,8,1,12,5]%%%}+%%%{-1207959552,[0,4
,0,10,8,0,12,6]%%%}+%%%{-1073741824*i,[0,4,0,10,7,4,13,2]%%%}+%%%{397284474
88*i,[0,4,0,10,7,3,13,3]%%%}+%%%{-35433480192*i,[0,4,0,10,7,2,13,4]%%%}+%%%
{-11811160064*i,[0,4,0,10,7,1,13,5]%%%}+%%%{4294967296*i,[0,4,0,10,7,0,13,6
]%%%}+%%%{-1342177280,[0,4,0,10,6,4,14,2]%%%}+%%%{29527900160,[0,4,0,10,6,3
,14,3]%%%}+%%%{-12079595520,[0,4,0,10,6,2,14,4]%%%}+%%%{-18790481920,[0,4,0
,10,6,1,14,5]%%%}+%%%{2684354560,[0,4,0,10,6,0,14,6]%%%}+%%%{536870912*i,[0
,4,0,10,5,4,15,2]%%%}+%%%{-10200547328*i,[0,4,0,10,5,3,15,3]%%%}+%%%{-16106
12736*i,[0,4,0,10,5,2,15,4]%%%}+%%%{9126805504*i,[0,4,0,10,5,1,15,5]%%%}+%%
%{67108864,[0,4,0,10,4,4,16,2]%%%}+%%%{-1476395008,[0,4,0,10,4,3,16,3]%%%}+
%%%{-1207959552,[0,4,0,10,4,2,16,4]%%%}+%%%{1744830464,[0,4,0,10,4,1,16,5]%
%%}+%%%{335544320,[0,4,0,10,4,0,16,6]%%%}+%%%{1073741824,[0,3,0,10,16,3,4,4
]%%%}+%%%{-2147483648*i,[0,3,0,10,15,4,5,3]%%%}+%%%{-4294967296*i,[0,3,0,10
,15,3,5,4]%%%}+%%%{6442450944*i,[0,3,0,10,15,2,5,5]%%%}+%%%{-8053063680,[0,
3,0,10,14,4,6,3]%%%}+%%%{1879048192,[0,3,0,10,14,3,6,4]%%%}+%%%{24159191040
,[0,3,0,10,14,2,6,5]%%%}+%%%{-5100273664,[0,3,0,10,14,1,6,6]%%%}+%%%{107374
18240*i,[0,3,0,10,13,4,7,3]%%%}+%%%{-25232932864*i,[0,3,0,10,13,3,7,4]%%%}+
%%%{-27380416512*i,[0,3,0,10,13,2,7,5]%%%}+%%%{16642998272*i,[0,3,0,10,13,1
,7,6]%%%}+%%%{-536870912*i,[0,3,0,10,13,0,7,7]%%%}+%%%{2415919104,[0,3,0,10
,12,4,8,3]%%%}+%%%{-27783069696,[0,3,0,10,12,3,8,4]%%%}+%%%{1207959552,[0,3
,0,10,12,2,8,5]%%%}+%%%{12213813248,[0,3,0,10,12,1,8,6]%%%}+%%%{-939524096,
[0,3,0,10,12,0,8,7]%%%}+%%%{12884901888*i,[0,3,0,10,11,4,9,3]%%%}+%%%{-1610
6127360*i,[0,3,0,10,11,3,9,4]%%%}+%%%{-28991029248*i,[0,3,0,10,11,2,9,5]%%%

}+%%%{16106127360*i,[0,3,0,10,11,1,9,6]%%%}+%%%{-1073741824*i,[0,3,0,10,11,
0,9,7]%%%}+%%%{21474836480,[0,3,0,10,10,4,10,3]%%%}+%%%{-49392123904,[0,3,0
,10,10,3,10,4]%%%}+%%%{-40265318400,[0,3,0,10,10,2,10,5]%%%}+%%%{3221225472
0,[0,3,0,10,10,1,10,6]%%%}+%%%{-2684354560,[0,3,0,10,10,0,10,7]%%%}+%%%{-12
884901888*i,[0,3,0,10,9,4,11,3]%%%}+%%%{25769803776*i,[0,3,0,10,9,3,11,4]%%
%}+%%%{38654705664*i,[0,3,0,10,9,2,11,5]%%%}+%%%{-17179869184*i,[0,3,0,10,9
,1,11,6]%%%}+%%%{3758096384,[0,3,0,10,8,4,12,3]%%%}+%%%{-13153337344,[0,3,0
,10,8,3,12,4]%%%}+%%%{10468982784,[0,3,0,10,8,2,12,5]%%%}+%%%{6710886400,[0
,3,0,10,8,1,12,6]%%%}+%%%{-2415919104,[0,3,0,10,8,0,12,7]%%%}+%%%{-10737418
240*i,[0,3,0,10,7,4,13,3]%%%}+%%%{20401094656*i,[0,3,0,10,7,3,13,4]%%%}+%%%
{22548578304*i,[0,3,0,10,7,2,13,5]%%%}+%%%{-16106127360*i,[0,3,0,10,7,1,13,
6]%%%}+%%%{1073741824*i,[0,3,0,10,7,0,13,7]%%%}+%%%{-6979321856,[0,3,0,10,6
,4,14,3]%%%}+%%%{7784628224,[0,3,0,10,6,3,14,4]%%%}+%%%{25769803776,[0,3,0,
10,6,2,14,5]%%%}+%%%{-8858370048,[0,3,0,10,6,1,14,6]%%%}+%%%{-536870912,[0,
3,0,10,6,0,14,7]%%%}+%%%{2147483648*i,[0,3,0,10,5,4,15,3]%%%}+%%%{-53687091
2*i,[0,3,0,10,5,3,15,4]%%%}+%%%{-11274289152*i,[0,3,0,10,5,2,15,5]%%%}+%%%{
536870912*i,[0,3,0,10,5,1,15,6]%%%}+%%%{536870912*i,[0,3,0,10,5,0,15,7]%%%}
+%%%{268435456,[0,3,0,10,4,4,16,3]%%%}+%%%{134217728,[0,3,0,10,4,3,16,4]%%%
}+%%%{-2013265920,[0,3,0,10,4,2,16,5]%%%}+%%%{-671088640,[0,3,0,10,4,1,16,6
]%%%}+%%%{134217728,[0,3,0,10,4,0,16,7]%%%}+%%%{-268435456,[0,2,0,10,16,4,4
,4]%%%}+%%%{1073741824*i,[0,2,0,10,15,4,5,4]%%%}+%%%{-4294967296*i,[0,2,0,1
0,15,3,5,5]%%%}+%%%{-268435456,[0,2,0,10,14,4,6,4]%%%}+%%%{-16106127360,[0,
2,0,10,14,3,6,5]%%%}+%%%{7247757312,[0,2,0,10,14,2,6,6]%%%}+%%%{5905580032*
i,[0,2,0,10,13,4,7,4]%%%}+%%%{19864223744*i,[0,2,0,10,13,3,7,5]%%%}+%%%{-24
159191040*i,[0,2,0,10,13,2,7,6]%%%}+%%%{1610612736*i,[0,2,0,10,13,1,7,7]%%%
}+%%%{7046430720,[0,2,0,10,12,4,8,4]%%%}+%%%{2013265920,[0,2,0,10,12,3,8,5]
%%%}+%%%{-19058917376,[0,2,0,10,12,2,8,6]%%%}+%%%{2818572288,[0,2,0,10,12,1
,8,7]%%%}+%%%{-67108864,[0,2,0,10,12,0,8,8]%%%}+%%%{3221225472*i,[0,2,0,10,
11,4,9,4]%%%}+%%%{22548578304*i,[0,2,0,10,11,3,9,5]%%%}+%%%{-22548578304*i,
[0,2,0,10,11,2,9,6]%%%}+%%%{3221225472*i,[0,2,0,10,11,1,9,7]%%%}+%%%{115427
24608,[0,2,0,10,10,4,10,4]%%%}+%%%{34896609280,[0,2,0,10,10,3,10,5]%%%}+%%%
{-48855252992,[0,2,0,10,10,2,10,6]%%%}+%%%{8053063680,[0,2,0,10,10,1,10,7]%
%%}+%%%{-268435456,[0,2,0,10,10,0,10,8]%%%}+%%%{-6442450944*i,[0,2,0,10,9,4
,11,4]%%%}+%%%{-25769803776*i,[0,2,0,10,9,3,11,5]%%%}+%%%{25769803776*i,[0,
2,0,10,9,2,11,6]%%%}+%%%{2281701376,[0,2,0,10,8,4,12,4]%%%}+%%%{268435456,[
0,2,0,10,8,3,12,5]%%%}+%%%{-11274289152,[0,2,0,10,8,2,12,6]%%%}+%%%{7247757
312,[0,2,0,10,8,1,12,7]%%%}+%%%{-402653184,[0,2,0,10,8,0,12,8]%%%}+%%%{-429
4967296*i,[0,2,0,10,7,4,13,4]%%%}+%%%{-18253611008*i,[0,2,0,10,7,3,13,5]%%%
}+%%%{22548578304*i,[0,2,0,10,7,2,13,6]%%%}+%%%{-3221225472*i,[0,2,0,10,7,1
,13,7]%%%}+%%%{-2147483648,[0,2,0,10,6,4,14,4]%%%}+%%%{-15569256448,[0,2,0,
10,6,3,14,5]%%%}+%%%{11542724608,[0,2,0,10,6,2,14,6]%%%}+%%%{1610612736,[0,
2,0,10,6,1,14,7]%%%}+%%%{-268435456,[0,2,0,10,6,0,14,8]%%%}+%%%{536870912*i
,[0,2,0,10,5,4,15,4]%%%}+%%%{5905580032*i,[0,2,0,10,5,3,15,5]%%%}+%%%{-1610
612736*i,[0,2,0,10,5,2,15,6]%%%}+%%%{-1610612736*i,[0,2,0,10,5,1,15,7]%%%}+
%%%{67108864,[0,2,0,10,4,4,16,4]%%%}+%%%{939524096,[0,2,0,10,4,3,16,5]%%%}+
%%%{268435456,[0,2,0,10,4,2,16,6]%%%}+%%%{-402653184,[0,2,0,10,4,1,16,7]%%%
}+%%%{-67108864,[0,2,0,10,4,0,16,8]%%%}+%%%{1073741824*i,[0,1,0,10,15,4,5,5
]%%%}+%%%{4026531840,[0,1,0,10,14,4,6,5]%%%}+%%%{-4563402752,[0,1,0,10,14,3
,6,6]%%%}+%%%{-5368709120*i,[0,1,0,10,13,4,7,5]%%%}+%%%{15569256448*i,[0,1,
0,10,13,3,7,6]%%%}+%%%{-1610612736*i,[0,1,0,10,13,2,7,7]%%%}+%%%{-120795955
2,[0,1,0,10,12,4,8,5]%%%}+%%%{13019119616,[0,1,0,10,12,3,8,6]%%%}+%%%{-2818
572288,[0,1,0,10,12,2,8,7]%%%}+%%%{134217728,[0,1,0,10,12,1,8,8]%%%}+%%%{-6
442450944*i,[0,1,0,10,11,4,9,5]%%%}+%%%{13958643712*i,[0,1,0,10,11,3,9,6]%%
%}+%%%{-3221225472*i,[0,1,0,10,11,2,9,7]%%%}+%%%{-10737418240,[0,1,0,10,10,
4,10,5]%%%}+%%%{32212254720,[0,1,0,10,10,3,10,6]%%%}+%%%{-8053063680,[0,1,0
,10,10,2,10,7]%%%}+%%%{536870912,[0,1,0,10,10,1,10,8]%%%}+%%%{6442450944*i,
[0,1,0,10,9,4,11,5]%%%}+%%%{-17179869184*i,[0,1,0,10,9,3,11,6]%%%}+%%%{-187
9048192,[0,1,0,10,8,4,12,5]%%%}+%%%{7247757312,[0,1,0,10,8,3,12,6]%%%}+%%%{
-7247757312,[0,1,0,10,8,2,12,7]%%%}+%%%{805306368,[0,1,0,10,8,1,12,8]%%%}+%

```
%%{5368709120*i,[0,1,0,10,7,4,13,5]%%%}+%%%{-13958643712*i,[0,1,0,10,7,3,13
,6]%%%}+%%%{3221225472*i,[0,1,0,10,7,2,13,7]%%%}+%%%{3489660928,[0,1,0,10,6
,4,14,5]%%%}+%%%{-7247757312,[0,1,0,10,6,3,14,6]%%%}+%%%{-1610612736,[0,1,0
,10,6,2,14,7]%%%}+%%%{536870912,[0,1,0,10,6,1,14,8]%%%}+%%%{-1073741824*i,[
0,1,0,10,5,4,15,5]%%%}+%%%{1610612736*i,[0,1,0,10,5,3,15,6]%%%}+%%%{1610612
736*i,[0,1,0,10,5,2,15,7]%%%}+%%%{-134217728,[0,1,0,10,4,4,16,5]%%%}+%%%{13
4217728,[0,1,0,10,4,3,16,6]%%%}+%%%{402653184,[0,1,0,10,4,2,16,7]%%%}+%%%{1
34217728,[0,1,0,10,4,1,16,8]%%%}+%%%{1073741824,[0,0,0,10,14,4,6,6]%%%}+%%%
{-3758096384*i,[0,0,0,10,13,4,7,6]%%%}+%%%{536870912*i,[0,0,0,10,13,3,7,7]%
%%}+%%%{-3288334336,[0,0,0,10,12,4,8,6]%%%}+%%%{939524096,[0,0,0,10,12,3,8,
7]%%%}+%%%{-67108864,[0,0,0,10,12,2,8,8]%%%}+%%%{-3221225472*i,[0,0,0,10,11
,4,9,6]%%%}+%%%{1073741824*i,[0,0,0,10,11,3,9,7]%%%}+%%%{-7784628224,[0,0,0
,10,10,4,10,6]%%%}+%%%{2684354560,[0,0,0,10,10,3,10,7]%%%}+%%%{-268435456,[
0,0,0,10,10,2,10,8]%%%}+%%%{4294967296*i,[0,0,0,10,9,4,11,6]%%%}+%%%{-14763
95008,[0,0,0,10,8,4,12,6]%%%}+%%%{2415919104,[0,0,0,10,8,3,12,7]%%%}+%%%{-4
02653184,[0,0,0,10,8,2,12,8]%%%}+%%%{3221225472*i,[0,0,0,10,7,4,13,6]%%%}+%
%%{-1073741824*i,[0,0,0,10,7,3,13,7]%%%}+%%%{1879048192,[0,0,0,10,6,4,14,6]
%%%}+%%%{536870912,[0,0,0,10,6,3,14,7]%%%}+%%%{-268435456,[0,0,0,10,6,2,14,
8]%%%}+%%%{-536870912*i,[0,0,0,10,5,4,15,6]%%%}+%%%{-536870912*i,[0,0,0,10,
5,3,15,7]%%%}+%%%{-67108864,[0,0,0,10,4,4,16,6]%%%}+%%%{-134217728,[0,0,0,1
0,4,3,16,7]%%%}+%%%{-67108864,[0,0,0,10,4,2,16,8]%%%} / %%%{1024,[0,4,0,4,8
,0,0,0]%%%}+%%%{-4096*i,[0,4,0,4,7,0,1,0]%%%}+%%%{-8192,[0,4,0,4,6,0,2,0]%%
%}+%%%{12288*i,[0,4,0,4,5,0,3,0]%%%}+%%%{14336,[0,4,0,4,4,0,4,0]%%%}+%%%{-1
2288*i,[0,4,0,4,3,0,5,0]%%%}+%%%{-8192,[0,4,0,4,2,0,6,0]%%%}+%%%{4096*i,[0,
4,0,4,1,0,7,0]%%%}+%%%{1024,[0,4,0,4,0,0,8,0]%%%}+%%%{-2048,[0,3,0,4,8,1,0,
0]%%%}+%%%{8192*i,[0,3,0,4,7,1,1,0]%%%}+%%%{-4096*i,[0,3,0,4,7,0,1,1]%%%}+%
%%{17408,[0,3,0,4,6,1,2,0]%%%}+%%%{-15360,[0,3,0,4,6,0,2,1]%%%}+%%%{-26624*
i,[0,3,0,4,5,1,3,0]%%%}+%%%{26624*i,[0,3,0,4,5,0,3,1]%%%}+%%%{-29696,[0,3,0
,4,4,1,4,0]%%%}+%%%{31744,[0,3,0,4,4,0,4,1]%%%}+%%%{24576*i,[0,3,0,4,3,1,5,
0]%%%}+%%%{-28672*i,[0,3,0,4,3,0,5,1]%%%}+%%%{15360,[0,3,0,4,2,1,6,0]%%%}+%
%%{-17408,[0,3,0,4,2,0,6,1]%%%}+%%%{-6144*i,[0,3,0,4,1,1,7,0]%%%}+%%%{6144*
i,[0,3,0,4,1,0,7,1]%%%}+%%%{-1024,[0,3,0,4,0,1,8,0]%%%}+%%%{1024,[0,3,0,4,0
,0,8,1]%%%}+%%%{1024,[0,2,0,4,8,2,0,0]%%%}+%%%{-4096*i,[0,2,0,4,7,2,1,0]%%%
}+%%%{8192*i,[0,2,0,4,7,1,1,1]%%%}+%%%{-9216,[0,2,0,4,6,2,2,0]%%%}+%%%{3072
0,[0,2,0,4,6,1,2,1]%%%}+%%%{-5120,[0,2,0,4,6,0,2,2]%%%}+%%%{14336*i,[0,2,0,
4,5,2,3,0]%%%}+%%%{-55296*i,[0,2,0,4,5,1,3,1]%%%}+%%%{16384*i,[0,2,0,4,5,0,
3,2]%%%}+%%%{15616,[0,2,0,4,4,2,4,0]%%%}+%%%{-67072,[0,2,0,4,4,1,4,1]%%%}+%
%%{21760,[0,2,0,4,4,0,4,2]%%%}+%%%{-12288*i,[0,2,0,4,3,2,5,0]%%%}+%%%{57344
*i,[0,2,0,4,3,1,5,1]%%%}+%%%{-16384*i,[0,2,0,4,3,0,5,2]%%%}+%%%{-6656,[0,2,
0,4,2,2,6,0]%%%}+%%%{31744,[0,2,0,4,2,1,6,1]%%%}+%%%{-6656,[0,2,0,4,2,0,6,2
]%%%}+%%%{2048*i,[0,2,0,4,1,2,7,0]%%%}+%%%{-10240*i,[0,2,0,4,1,1,7,1]%%%}+%
%%{256,[0,2,0,4,0,2,8,0]%%%}+%%%{-1536,[0,2,0,4,0,1,8,1]%%%}+%%%{-768,[0,2,
0,4,0,0,8,2]%%%}+%%%{-4096*i,[0,1,0,4,7,2,1,1]%%%}+%%%{-15360,[0,1,0,4,6,2,
2,1]%%%}+%%%{9216,[0,1,0,4,6,1,2,2]%%%}+%%%{28672*i,[0,1,0,4,5,2,3,1]%%%}+%
%%{-30720*i,[0,1,0,4,5,1,3,2]%%%}+%%%{2048*i,[0,1,0,4,5,0,3,3]%%%}+%%%{3532
8,[0,1,0,4,4,2,4,1]%%%}+%%%{-43008,[0,1,0,4,4,1,4,2]%%%}+%%%{3584,[0,1,0,4,
4,0,4,3]%%%}+%%%{-28672*i,[0,1,0,4,3,2,5,1]%%%}+%%%{32768*i,[0,1,0,4,3,1,5,
2]%%%}+%%%{-14336,[0,1,0,4,2,2,6,1]%%%}+%%%{13312,[0,1,0,4,2,1,6,2]%%%}+%%%
{3072,[0,1,0,4,2,0,6,3]%%%}+%%%{4096*i,[0,1,0,4,1,2,7,1]%%%}+%%%{-2048*i,[0
,1,0,4,1,1,7,2]%%%}+%%%{-2048*i,[0,1,0,4,1,0,7,3]%%%}+%%%{512,[0,1,0,4,0,2,
8,1]%%%}+%%%{-512,[0,1,0,4,0,0,8,3]%%%}+%%%{-4096,[0,0,0,4,6,2,2,2]%%%}+%%%
{14336*i,[0,0,0,4,5,2,3,2]%%%}+%%%{-2048*i,[0,0,0,4,5,1,3,3]%%%}+%%%{20736,
[0,0,0,4,4,2,4,2]%%%}+%%%{-3584,[0,0,0,4,4,1,4,3]%%%}+%%%{256,[0,0,0,4,4,0,
4,4]%%%}+%%%{-16384*i,[0,0,0,4,3,2,5,2]%%%}+%%%{-7680,[0,0,0,4,2,2,6,2]%%%}
+%%%{-3072,[0,0,0,4,2,1,6,3]%%%}+%%%{512,[0,0,0,4,2,0,6,4]%%%}+%%%{2048*i,[
0,0,0,4,1,2,7,2]%%%}+%%%{2048*i,[0,0,0,4,1,1,7,3]%%%}+%%%{256,[0,0,0,4,0,2,
8,2]%%%}+%%%{512,[0,0,0,4,0,1,8,3]%%%}+%%%{256,[0,0,0,4,0,0,8,4]%%%} Error:
 Bad Argument Value
```

maple [A] time = 1.55, size = 0, normalized size = 0.00

$$\int (\sin^m (dx + c)) (a + b \tan (dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x)
[Out] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \tan (dx + c) + a)^n \sin (dx + c)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \sin (c + d\,x)^m \, (a + b \tan (c + d\,x))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^m*(a + b*tan(c + d*x))^n,x)
[Out] int(sin(c + d*x)^m*(a + b*tan(c + d*x))^n, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan (c + dx))^n \sin^m (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**m*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*sin(c + d*x)**m, x)
```

3.84 $\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=435

$$\frac{\cos^4(c+dx)(a\tan(c+dx)+b)(a+b\tan(c+dx))^{n+1}}{4d\left(a^2+b^2\right)}-\frac{\cos^2(c+dx)\left(a\left(5a^2+b^2(2n+3)\right)\tan(c+dx)+b\left(a^2(\right.\right.}{8d\left(a^2+b^2\right)^2}$$

```
[Out] -1/16*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+(-b^2)^(1/2)))*(a*b^2*n*
(5*a^2+b^2*(3+2*n))-(3*a^4+a^2*b^2*(-n^2+6*n+6)+b^4*(n^2+4*n+3))*(-b^2)^(1/
2))*(a+b*tan(d*x+c))^(1+n)/b/(a^2+b^2)^2/d/(1+n)/(a+(-b^2)^(1/2))-1/16*hype
rgeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)))*(a*b^2*n*(5*a^2+b^2
*(3+2*n))+(3*a^4+a^2*b^2*(-n^2+6*n+6)+b^4*(n^2+4*n+3))*(-b^2)^(1/2))*(a+b*t
an(d*x+c))^(1+n)/b/(a^2+b^2)^2/d/(1+n)/(a-(-b^2)^(1/2))+1/4*cos(d*x+c)^4*(b
+a*tan(d*x+c))*(a+b*tan(d*x+c))^(1+n)/(a^2+b^2)/d-1/8*cos(d*x+c)^2*(a+b*tan
(d*x+c))^(1+n)*(b*(a^2*(7-n)+b^2*(5+n))+a*(5*a^2+b^2*(3+2*n))*tan(d*x+c))/(
a^2+b^2)^2/d
```

Rubi [A] time = 0.80, antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3516, 1649, 831, 68}

$$-\frac{\left(\sqrt{-b^2}\left(a^2b^2\left(-n^2+6n+6\right)+3a^4+b^4\left(n^2+4n+3\right)\right)+ab^2n\left(5a^2+b^2(2n+3)\right)\right)(a+b\tan(c+dx))^{n+1}\,{}_2F}{16bd(n+1)\left(a^2+b^2\right)^2\left(a-\sqrt{-b^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] -((a*b^2*n*(5*a^2 + b^2*(3 + 2*n)) + Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 + 6*n -
 n^2) + b^4*(3 + 4*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan
[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(16*b*(a^2 + b^2
)^2*(a - Sqrt[-b^2])*d*(1 + n)) - ((a*b^2*n*(5*a^2 + b^2*(3 + 2*n)) - Sqrt[
-b^2]*(3*a^4 + a^2*b^2*(6 + 6*n - n^2) + b^4*(3 + 4*n + n^2)))*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c
+ d*x])^(1 + n))/(16*b*(a^2 + b^2)^2*(a + Sqrt[-b^2])*d*(1 + n)) + (Cos[c +
 d*x]^4*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/(4*(a^2 + b^2)*d
) - (Cos[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n)*(b*(a^2*(7 - n) + b^2*(5 +
 n)) + a*(5*a^2 + b^2*(3 + 2*n))*Tan[c + d*x]))/(8*(a^2 + b^2)^2*d)
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
```

```
x], x, 1]}, -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^(p + 1)*(a*(e*f - d*g) +
(c*d*f + a*e*g)*x))/(2*a*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*a*(p + 1)
*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*(p +
1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p + 3) - a*e*(d*g*m - e*f*(m + 2*p + 3))
+ e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, c, d, e, m},
x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[m, 0]
&& RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{x^4(a+x)^n}{\left(b^2+x^2\right)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d} - \frac{\mathrm{Subst}\left(\int \frac{(a+x)^n(}{}\right.}{}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d} - \frac{\cos^2(c+dx)(a}{}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d} - \frac{\cos^2(c+dx)(a}{}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d} - \frac{\cos^2(c+dx)(a}{}$$

$$= -\frac{\left(ab^2n\left(5a^2+b^2(3+2n)\right)+\sqrt{-b^2}\left(3a^4+a^2b^2\left(6+6n-n^2\right)+b^4\left(3+\right.\right.\right.}{16b\left(a^2+b^2\right)^2\left(a-\right.}$$

Mathematica [B] time = 6.60, size = 910, normalized size = 2.09

$$\frac{b\left(\frac{{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{n+1}}{2\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)(n+1)} - \frac{{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{n+1}}{2\sqrt{-b^2}\left(a+\sqrt{-b^2}\right)(n+1)} + \frac{\cos^4(c+dx)\left(b^2+a\tan(c+dx)b\right)(a+b\tan(}{4b^2\left(a^2+b^2\right)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*((Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2
])]*(a + b*Tan[c + d*x])^(1 + n))/(2*Sqrt[-b^2]*(a - Sqrt[-b^2])*(1 + n)) -
 (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]
*(a + b*Tan[c + d*x])^(1 + n))/(2*Sqrt[-b^2]*(a + Sqrt[-b^2])*(1 + n)) - (C
os[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n)*(b^2 + a*b*Tan[c + d*x]))/(b^2*(
a^2 + b^2)) + (Cos[c + d*x]^4*(a + b*Tan[c + d*x])^(1 + n)*(b^2 + a*b*Tan[c
 + d*x]))/(4*b^2*(a^2 + b^2)) + (((Sqrt[-b^2]*(a^2 + b^2*(1 - n)) - a*b^2*n
)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]
*(a + b*Tan[c + d*x])^(1 + n))/(b^2*(a - Sqrt[-b^2])*(1 + n)) - ((a^2*Sqrt[
-b^2] - (-b^2)^(3/2)*(1 - n) + a*b^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n,
(a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(b^2*(
a + Sqrt[-b^2])*(1 + n)))/(2*(a^2 + b^2)) - (b^2*((Cos[c + d*x]^2*(a + b*Ta
n[c + d*x])^(1 + n)*(b^2*(-3*a^2 - b^2*(3 - n)) + a^2*b^2*(2 - n) + b*(a*(-
3*a^2 - b^2*(3 - n)) - a*b^2*(2 - n))*Tan[c + d*x]))/(2*b^4*(a^2 + b^2)) -
(((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n - Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 - 2*n -
 n^2) + b^4*(3 - 4*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan
[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(2*b^2*(a - Sqrt
[-b^2])*(1 + n)) + ((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n + Sqrt[-b^2]*(3*a^4 +
a^2*b^2*(6 - 2*n - n^2) + b^4*(3 - 4*n + n^2)))*Hypergeometric2F1[1, 1 + n,
 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n)
)/(2*b^2*(a + Sqrt[-b^2])*(1 + n)))/(2*b^2*(a^2 + b^2))))/(4*(a^2 + b^2))))
/d
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right)\left(b\tan\left(dx+c\right)+a\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(b*tan(d*x + c) + a)^n, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.46Unable to divide, perha
ps due to rounding error%%%{1048576,[2,0,10,17,20,20]%%%}+%%%{-10485760,[2,
0,10,17,20,18]%%%}+%%%{47185920,[2,0,10,17,20,16]%%%}+%%%{-125829120,[2,0,1
0,17,20,14]%%%}+%%%{220200960,[2,0,10,17,20,12]%%%}+%%%{-264241152,[2,0,10,
17,20,10]%%%}+%%%{220200960,[2,0,10,17,20,8]%%%}+%%%{-125829120,[2,0,10,17,
20,6]%%%}+%%%{47185920,[2,0,10,17,20,4]%%%}+%%%{-10485760,[2,0,10,17,20,2]%
%%}+%%%{1048576,[2,0,10,17,20,0]%%%}+%%%{3145728*i,[2,0,10,16,21,20]%%%}+%%
%{-31457280*i,[2,0,10,16,21,18]%%%}+%%%{141557760*i,[2,0,10,16,21,16]%%%}+%
%%{-377487360*i,[2,0,10,16,21,14]%%%}+%%%{660602880*i,[2,0,10,16,21,12]%%%}
+%%%{-792723456*i,[2,0,10,16,21,10]%%%}+%%%{660602880*i,[2,0,10,16,21,8]%%%
}+%%%{-377487360*i,[2,0,10,16,21,6]%%%}+%%%{141557760*i,[2,0,10,16,21,4]%%%
}+%%%{-31457280*i,[2,0,10,16,21,2]%%%}+%%%{3145728*i,[2,0,10,16,21,0]%%%}+%
%%{4194304,[2,0,10,15,22,20]%%%}+%%%{-41943040,[2,0,10,15,22,18]%%%}+%%%{18
8743680,[2,0,10,15,22,16]%%%}+%%%{-503316480,[2,0,10,15,22,14]%%%}+%%%{8808
03840,[2,0,10,15,22,12]%%%}+%%%{-1056964608,[2,0,10,15,22,10]%%%}+%%%{88080
3840,[2,0,10,15,22,8]%%%}+%%%{-503316480,[2,0,10,15,22,6]%%%}+%%%{188743680
,[2,0,10,15,22,4]%%%}+%%%{-41943040,[2,0,10,15,22,2]%%%}+%%%{4194304,[2,0,1
0,15,22,0]%%%}+%%%{20971520*i,[2,0,10,14,23,20]%%%}+%%%{-209715200*i,[2,0,1
0,14,23,18]%%%}+%%%{943718400*i,[2,0,10,14,23,16]%%%}+%%%{-2516582400*i,[2,
```

```
0,10,14,23,14]%%%}+%%%{4404019200*i,[2,0,10,14,23,12]%%%}+%%%{-5284823040*i
,[2,0,10,14,23,10]%%%}+%%%{4404019200*i,[2,0,10,14,23,8]%%%}+%%%{-251658240
0*i,[2,0,10,14,23,6]%%%}+%%%{943718400*i,[2,0,10,14,23,4]%%%}+%%%{-20971520
0*i,[2,0,10,14,23,2]%%%}+%%%{20971520*i,[2,0,10,14,23,0]%%%}+%%%{58720256*i
,[2,0,10,12,25,20]%%%}+%%%{-587202560*i,[2,0,10,12,25,18]%%%}+%%%{264241152
0*i,[2,0,10,12,25,16]%%%}+%%%{-7046430720*i,[2,0,10,12,25,14]%%%}+%%%{12331
253760*i,[2,0,10,12,25,12]%%%}+%%%{-14797504512*i,[2,0,10,12,25,10]%%%}+%%%
{12331253760*i,[2,0,10,12,25,8]%%%}+%%%{-7046430720*i,[2,0,10,12,25,6]%%%}+
%%%{2642411520*i,[2,0,10,12,25,4]%%%}+%%%{-587202560*i,[2,0,10,12,25,2]%%%}
+%%%{58720256*i,[2,0,10,12,25,0]%%%}+%%%{-29360128,[2,0,10,11,26,20]%%%}+%%
%{293601280,[2,0,10,11,26,18]%%%}+%%%{-1321205760,[2,0,10,11,26,16]%%%}+%%%
{3523215360,[2,0,10,11,26,14]%%%}+%%%{-6165626880,[2,0,10,11,26,12]%%%}+%%%
{7398752256,[2,0,10,11,26,10]%%%}+%%%{-6165626880,[2,0,10,11,26,8]%%%}+%%%{
3523215360,[2,0,10,11,26,6]%%%}+%%%{-1321205760,[2,0,10,11,26,4]%%%}+%%%{29
3601280,[2,0,10,11,26,2]%%%}+%%%{-29360128,[2,0,10,11,26,0]%%%}+%%%{8808038
4*i,[2,0,10,10,27,20]%%%}+%%%{-880803840*i,[2,0,10,10,27,18]%%%}+%%%{396361
7280*i,[2,0,10,10,27,16]%%%}+%%%{-10569646080*i,[2,0,10,10,27,14]%%%}+%%%{1
8496880640*i,[2,0,10,10,27,12]%%%}+%%%{-22196256768*i,[2,0,10,10,27,10]%%%}
+%%%{18496880640*i,[2,0,10,10,27,8]%%%}+%%%{-10569646080*i,[2,0,10,10,27,6]
%%%}+%%%{3963617280*i,[2,0,10,10,27,4]%%%}+%%%{-880803840*i,[2,0,10,10,27,2
]%%%}+%%%{88080384*i,[2,0,10,10,27,0]%%%}+%%%{-73400320,[2,0,10,9,28,20]%%%
}+%%%{734003200,[2,0,10,9,28,18]%%%}+%%%{-3303014400,[2,0,10,9,28,16]%%%}+%
%%{8808038400,[2,0,10,9,28,14]%%%}+%%%{-15414067200,[2,0,10,9,28,12]%%%}+%%
%{18496880640,[2,0,10,9,28,10]%%%}+%%%{-15414067200,[2,0,10,9,28,8]%%%}+%%%
{8808038400,[2,0,10,9,28,6]%%%}+%%%{-3303014400,[2,0,10,9,28,4]%%%}+%%%{734
003200,[2,0,10,9,28,2]%%%}+%%%{-73400320,[2,0,10,9,28,0]%%%}+%%%{73400320*i
,[2,0,10,8,29,20]%%%}+%%%{-734003200*i,[2,0,10,8,29,18]%%%}+%%%{3303014400*
i,[2,0,10,8,29,16]%%%}+%%%{-8808038400*i,[2,0,10,8,29,14]%%%}+%%%{154140672
00*i,[2,0,10,8,29,12]%%%}+%%%{-18496880640*i,[2,0,10,8,29,10]%%%}+%%%{15414
067200*i,[2,0,10,8,29,8]%%%}+%%%{-8808038400*i,[2,0,10,8,29,6]%%%}+%%%{3303
014400*i,[2,0,10,8,29,4]%%%}+%%%{-734003200*i,[2,0,10,8,29,2]%%%}+%%%{73400
320*i,[2,0,10,8,29,0]%%%}+%%%{-88080384,[2,0,10,7,30,20]%%%}+%%%{880803840,
[2,0,10,7,30,18]%%%}+%%%{-3963617280,[2,0,10,7,30,16]%%%}+%%%{10569646080,[
2,0,10,7,30,14]%%%}+%%%{-18496880640,[2,0,10,7,30,12]%%%}+%%%{22196256768,[
2,0,10,7,30,10]%%%}+%%%{-18496880640,[2,0,10,7,30,8]%%%}+%%%{10569646080,[2
,0,10,7,30,6]%%%}+%%%{-3963617280,[2,0,10,7,30,4]%%%}+%%%{880803840,[2,0,10
,7,30,2]%%%}+%%%{-88080384,[2,0,10,7,30,0]%%%}+%%%{29360128*i,[2,0,10,6,31,
20]%%%}+%%%{-293601280*i,[2,0,10,6,31,18]%%%}+%%%{1321205760*i,[2,0,10,6,31
,16]%%%}+%%%{-3523215360*i,[2,0,10,6,31,14]%%%}+%%%{6165626880*i,[2,0,10,6,
31,12]%%%}+%%%{-7398752256*i,[2,0,10,6,31,10]%%%}+%%%{6165626880*i,[2,0,10,
6,31,8]%%%}+%%%{-3523215360*i,[2,0,10,6,31,6]%%%}+%%%{1321205760*i,[2,0,10,
6,31,4]%%%}+%%%{-293601280*i,[2,0,10,6,31,2]%%%}+%%%{29360128*i,[2,0,10,6,3
1,0]%%%}+%%%{-58720256,[2,0,10,5,32,20]%%%}+%%%{587202560,[2,0,10,5,32,18]%
%%}+%%%{-2642411520,[2,0,10,5,32,16]%%%}+%%%{7046430720,[2,0,10,5,32,14]%%%
}+%%%{-12331253760,[2,0,10,5,32,12]%%%}+%%%{14797504512,[2,0,10,5,32,10]%%%
}+%%%{-12331253760,[2,0,10,5,32,8]%%%}+%%%{7046430720,[2,0,10,5,32,6]%%%}+%
%%{-2642411520,[2,0,10,5,32,4]%%%}+%%%{587202560,[2,0,10,5,32,2]%%%}+%%%{-5
8720256,[2,0,10,5,32,0]%%%}+%%%{-20971520,[2,0,10,3,34,20]%%%}+%%%{20971520
0,[2,0,10,3,34,18]%%%}+%%%{-943718400,[2,0,10,3,34,16]%%%}+%%%{2516582400,[
2,0,10,3,34,14]%%%}+%%%{-4404019200,[2,0,10,3,34,12]%%%}+%%%{5284823040,[2,
0,10,3,34,10]%%%}+%%%{-4404019200,[2,0,10,3,34,8]%%%}+%%%{2516582400,[2,0,1
0,3,34,6]%%%}+%%%{-943718400,[2,0,10,3,34,4]%%%}+%%%{209715200,[2,0,10,3,34
,2]%%%}+%%%{-20971520,[2,0,10,3,34,0]%%%}+%%%{-4194304*i,[2,0,10,2,35,20]%%
%}+%%%{41943040*i,[2,0,10,2,35,18]%%%}+%%%{-188743680*i,[2,0,10,2,35,16]%%%
}+%%%{503316480*i,[2,0,10,2,35,14]%%%}+%%%{-880803840*i,[2,0,10,2,35,12]%%%
}+%%%{1056964608*i,[2,0,10,2,35,10]%%%}+%%%{-880803840*i,[2,0,10,2,35,8]%%%
}+%%%{503316480*i,[2,0,10,2,35,6]%%%}+%%%{-188743680*i,[2,0,10,2,35,4]%%%}+
%%%{41943040*i,[2,0,10,2,35,2]%%%}+%%%{-4194304*i,[2,0,10,2,35,0]%%%}+%%%{-
3145728,[2,0,10,1,36,20]%%%}+%%%{31457280,[2,0,10,1,36,18]%%%}+%%%{-1415577
```

60,[2,0,10,1,36,16]%%%}+%%%{377487360,[2,0,10,1,36,14]%%%}+%%%{-660602880,[
2,0,10,1,36,12]%%%}+%%%{792723456,[2,0,10,1,36,10]%%%}+%%%{-660602880,[2,0,
10,1,36,8]%%%}+%%%{377487360,[2,0,10,1,36,6]%%%}+%%%{-141557760,[2,0,10,1,3
6,4]%%%}+%%%{31457280,[2,0,10,1,36,2]%%%}+%%%{-3145728,[2,0,10,1,36,0]%%%}+
%%%{-1048576*i,[2,0,10,0,37,20]%%%}+%%%{10485760*i,[2,0,10,0,37,18]%%%}+%%%
{-47185920*i,[2,0,10,0,37,16]%%%}+%%%{125829120*i,[2,0,10,0,37,14]%%%}+%%%{
-220200960*i,[2,0,10,0,37,12]%%%}+%%%{264241152*i,[2,0,10,0,37,10]%%%}+%%%{
-220200960*i,[2,0,10,0,37,8]%%%}+%%%{125829120*i,[2,0,10,0,37,6]%%%}+%%%{-4
7185920*i,[2,0,10,0,37,4]%%%}+%%%{10485760*i,[2,0,10,0,37,2]%%%}+%%%{-10485
76*i,[2,0,10,0,37,0]%%%}+%%%{-6291456,[1,0,10,17,20,20]%%%}+%%%{62914560,[1
,0,10,17,20,18]%%%}+%%%{-283115520,[1,0,10,17,20,16]%%%}+%%%{754974720,[1,0
,10,17,20,14]%%%}+%%%{-1321205760,[1,0,10,17,20,12]%%%}+%%%{1585446912,[1,0
,10,17,20,10]%%%}+%%%{-1321205760,[1,0,10,17,20,8]%%%}+%%%{754974720,[1,0,1
0,17,20,6]%%%}+%%%{-283115520,[1,0,10,17,20,4]%%%}+%%%{62914560,[1,0,10,17,
20,2]%%%}+%%%{-6291456,[1,0,10,17,20,0]%%%}+%%%{-20971520*i,[1,0,10,16,21,2
0]%%%}+%%%{209715200*i,[1,0,10,16,21,18]%%%}+%%%{-943718400*i,[1,0,10,16,21
,16]%%%}+%%%{2516582400*i,[1,0,10,16,21,14]%%%}+%%%{-4404019200*i,[1,0,10,1
6,21,12]%%%}+%%%{5284823040*i,[1,0,10,16,21,10]%%%}+%%%{-4404019200*i,[1,0,
10,16,21,8]%%%}+%%%{2516582400*i,[1,0,10,16,21,6]%%%}+%%%{-943718400*i,[1,0
,10,16,21,4]%%%}+%%%{209715200*i,[1,0,10,16,21,2]%%%}+%%%{-20971520*i,[1,0,
10,16,21,0]%%%}+%%%{-16777216,[1,0,10,15,22,20]%%%}+%%%{167772160,[1,0,10,1
5,22,18]%%%}+%%%{-754974720,[1,0,10,15,22,16]%%%}+%%%{2013265920,[1,0,10,15
,22,14]%%%}+%%%{-3523215360,[1,0,10,15,22,12]%%%}+%%%{4227858432,[1,0,10,15
,22,10]%%%}+%%%{-3523215360,[1,0,10,15,22,8]%%%}+%%%{2013265920,[1,0,10,15,
22,6]%%%}+%%%{-754974720,[1,0,10,15,22,4]%%%}+%%%{167772160,[1,0,10,15,22,2
]%%%}+%%%{-16777216,[1,0,10,15,22,0]%%%}+%%%{-125829120*i,[1,0,10,14,23,20]
%%%}+%%%{1258291200*i,[1,0,10,14,23,18]%%%}+%%%{-5662310400*i,[1,0,10,14,23
,16]%%%}+%%%{15099494400*i,[1,0,10,14,23,14]%%%}+%%%{-26424115200*i,[1,0,10
,14,23,12]%%%}+%%%{31708938240*i,[1,0,10,14,23,10]%%%}+%%%{-26424115200*i,[
1,0,10,14,23,8]%%%}+%%%{15099494400*i,[1,0,10,14,23,6]%%%}+%%%{-5662310400*
i,[1,0,10,14,23,4]%%%}+%%%{1258291200*i,[1,0,10,14,23,2]%%%}+%%%{-125829120
*i,[1,0,10,14,23,0]%%%}+%%%{41943040,[1,0,10,13,24,20]%%%}+%%%{-419430400,[
1,0,10,13,24,18]%%%}+%%%{1887436800,[1,0,10,13,24,16]%%%}+%%%{-5033164800,[
1,0,10,13,24,14]%%%}+%%%{8808038400,[1,0,10,13,24,12]%%%}+%%%{-10569646080,
[1,0,10,13,24,10]%%%}+%%%{8808038400,[1,0,10,13,24,8]%%%}+%%%{-5033164800,[
1,0,10,13,24,6]%%%}+%%%{1887436800,[1,0,10,13,24,4]%%%}+%%%{-419430400,[1,0
,10,13,24,2]%%%}+%%%{41943040,[1,0,10,13,24,0]%%%}+%%%{-310378496*i,[1,0,10
,12,25,20]%%%}+%%%{3103784960*i,[1,0,10,12,25,18]%%%}+%%%{-13967032320*i,[1
,0,10,12,25,16]%%%}+%%%{37245419520*i,[1,0,10,12,25,14]%%%}+%%%{-6517948416
0*i,[1,0,10,12,25,12]%%%}+%%%{78215380992*i,[1,0,10,12,25,10]%%%}+%%%{-6517
9484160*i,[1,0,10,12,25,8]%%%}+%%%{37245419520*i,[1,0,10,12,25,6]%%%}+%%%{-
13967032320*i,[1,0,10,12,25,4]%%%}+%%%{3103784960*i,[1,0,10,12,25,2]%%%}+%%
%{-310378496*i,[1,0,10,12,25,0]%%%}+%%%{251658240,[1,0,10,11,26,20]%%%}+%%%
{-2516582400,[1,0,10,11,26,18]%%%}+%%%{11324620800,[1,0,10,11,26,16]%%%}+%%
%{-30198988800,[1,0,10,11,26,14]%%%}+%%%{52848230400,[1,0,10,11,26,12]%%%}+
%%%{-63417876480,[1,0,10,11,26,10]%%%}+%%%{52848230400,[1,0,10,11,26,8]%%%}
+%%%{-30198988800,[1,0,10,11,26,6]%%%}+%%%{11324620800,[1,0,10,11,26,4]%%%}
+%%%{-2516582400,[1,0,10,11,26,2]%%%}+%%%{251658240,[1,0,10,11,26,0]%%%}+%%
%{-394264576*i,[1,0,10,10,27,20]%%%}+%%%{3942645760*i,[1,0,10,10,27,18]%%%}
+%%%{-17741905920*i,[1,0,10,10,27,16]%%%}+%%%{47311749120*i,[1,0,10,10,27,1
4]%%%}+%%%{-82795560960*i,[1,0,10,10,27,12]%%%}+%%%{99354673152*i,[1,0,10,1
0,27,10]%%%}+%%%{-82795560960*i,[1,0,10,10,27,8]%%%}+%%%{47311749120*i,[1,0
,10,10,27,6]%%%}+%%%{-17741905920*i,[1,0,10,10,27,4]%%%}+%%%{3942645760*i,[
1,0,10,10,27,2]%%%}+%%%{-394264576*i,[1,0,10,10,27,0]%%%}+%%%{482344960,[1,
0,10,9,28,20]%%%}+%%%{-4823449600,[1,0,10,9,28,18]%%%}+%%%{21705523200,[1,0
,10,9,28,16]%%%}+%%%{-57881395200,[1,0,10,9,28,14]%%%}+%%%{101292441600,[1,
0,10,9,28,12]%%%}+%%%{-121550929920,[1,0,10,9,28,10]%%%}+%%%{101292441600,[
1,0,10,9,28,8]%%%}+%%%{-57881395200,[1,0,10,9,28,6]%%%}+%%%{21705523200,[1,
0,10,9,28,4]%%%}+%%%{-4823449600,[1,0,10,9,28,2]%%%}+%%%{482344960,[1,0,10,

9,28,0]%%%}+%%%{-251658240*i,[1,0,10,8,29,20]%%%}+%%%{2516582400*i,[1,0,10,
8,29,18]%%%}+%%%{-11324620800*i,[1,0,10,8,29,16]%%%}+%%%{30198988800*i,[1,0
,10,8,29,14]%%%}+%%%{-52848230400*i,[1,0,10,8,29,12]%%%}+%%%{63417876480*i,
[1,0,10,8,29,10]%%%}+%%%{-52848230400*i,[1,0,10,8,29,8]%%%}+%%%{30198988800
*i,[1,0,10,8,29,6]%%%}+%%%{-11324620800*i,[1,0,10,8,29,4]%%%}+%%%{251658240
0*i,[1,0,10,8,29,2]%%%}+%%%{-251658240*i,[1,0,10,8,29,0]%%%}+%%%{486539264,
[1,0,10,7,30,20]%%%}+%%%{-4865392640,[1,0,10,7,30,18]%%%}+%%%{21894266880,[
1,0,10,7,30,16]%%%}+%%%{-58384711680,[1,0,10,7,30,14]%%%}+%%%{102173245440,
[1,0,10,7,30,12]%%%}+%%%{-122607894528,[1,0,10,7,30,10]%%%}+%%%{10217324544
0,[1,0,10,7,30,8]%%%}+%%%{-58384711680,[1,0,10,7,30,6]%%%}+%%%{21894266880,
[1,0,10,7,30,4]%%%}+%%%{-4865392640,[1,0,10,7,30,2]%%%}+%%%{486539264,[1,0,
10,7,30,0]%%%}+%%%{-41943040*i,[1,0,10,6,31,20]%%%}+%%%{419430400*i,[1,0,10
,6,31,18]%%%}+%%%{-1887436800*i,[1,0,10,6,31,16]%%%}+%%%{5033164800*i,[1,0,
10,6,31,14]%%%}+%%%{-8808038400*i,[1,0,10,6,31,12]%%%}+%%%{10569646080*i,[1
,0,10,6,31,10]%%%}+%%%{-8808038400*i,[1,0,10,6,31,8]%%%}+%%%{5033164800*i,[
1,0,10,6,31,6]%%%}+%%%{-1887436800*i,[1,0,10,6,31,4]%%%}+%%%{419430400*i,[1
,0,10,6,31,2]%%%}+%%%{-41943040*i,[1,0,10,6,31,0]%%%}+%%%{276824064,[1,0,10
,5,32,20]%%%}+%%%{-2768240640,[1,0,10,5,32,18]%%%}+%%%{12457082880,[1,0,10,
5,32,16]%%%}+%%%{-33218887680,[1,0,10,5,32,14]%%%}+%%%{58133053440,[1,0,10,
5,32,12]%%%}+%%%{-69759664128,[1,0,10,5,32,10]%%%}+%%%{58133053440,[1,0,10,
5,32,8]%%%}+%%%{-33218887680,[1,0,10,5,32,6]%%%}+%%%{12457082880,[1,0,10,5,
32,4]%%%}+%%%{-2768240640,[1,0,10,5,32,2]%%%}+%%%{276824064,[1,0,10,5,32,0]
%%%}+%%%{41943040*i,[1,0,10,4,33,20]%%%}+%%%{-419430400*i,[1,0,10,4,33,18]%
%%}+%%%{1887436800*i,[1,0,10,4,33,16]%%%}+%%%{-5033164800*i,[1,0,10,4,33,14
]%%%}+%%%{8808038400*i,[1,0,10,4,33,12]%%%}+%%%{-10569646080*i,[1,0,10,4,33
,10]%%%}+%%%{8808038400*i,[1,0,10,4,33,8]%%%}+%%%{-5033164800*i,[1,0,10,4,3
3,6]%%%}+%%%{1887436800*i,[1,0,10,4,33,4]%%%}+%%%{-419430400*i,[1,0,10,4,33
,2]%%%}+%%%{41943040*i,[1,0,10,4,33,0]%%%}+%%%{83886080,[1,0,10,3,34,20]%%%
}+%%%{-838860800,[1,0,10,3,34,18]%%%}+%%%{3774873600,[1,0,10,3,34,16]%%%}+%
%%{-10066329600,[1,0,10,3,34,14]%%%}+%%%{17616076800,[1,0,10,3,34,12]%%%}+%
%%{-21139292160,[1,0,10,3,34,10]%%%}+%%%{17616076800,[1,0,10,3,34,8]%%%}+%%
%{-10066329600,[1,0,10,3,34,6]%%%}+%%%{3774873600,[1,0,10,3,34,4]%%%}+%%%{-
838860800,[1,0,10,3,34,2]%%%}+%%%{83886080,[1,0,10,3,34,0]%%%}+%%%{25165824
*i,[1,0,10,2,35,20]%%%}+%%%{-251658240*i,[1,0,10,2,35,18]%%%}+%%%{113246208
0*i,[1,0,10,2,35,16]%%%}+%%%{-3019898880*i,[1,0,10,2,35,14]%%%}+%%%{5284823
040*i,[1,0,10,2,35,12]%%%}+%%%{-6341787648*i,[1,0,10,2,35,10]%%%}+%%%{52848
23040*i,[1,0,10,2,35,8]%%%}+%%%{-3019898880*i,[1,0,10,2,35,6]%%%}+%%%{11324
62080*i,[1,0,10,2,35,4]%%%}+%%%{-251658240*i,[1,0,10,2,35,2]%%%}+%%%{251658
24*i,[1,0,10,2,35,0]%%%}+%%%{10485760,[1,0,10,1,36,20]%%%}+%%%{-104857600,[
1,0,10,1,36,18]%%%}+%%%{471859200,[1,0,10,1,36,16]%%%}+%%%{-1258291200,[1,0
,10,1,36,14]%%%}+%%%{2202009600,[1,0,10,1,36,12]%%%}+%%%{-2642411520,[1,0,1
0,1,36,10]%%%}+%%%{2202009600,[1,0,10,1,36,8]%%%}+%%%{-1258291200,[1,0,10,1
,36,6]%%%}+%%%{471859200,[1,0,10,1,36,4]%%%}+%%%{-104857600,[1,0,10,1,36,2]
%%%}+%%%{10485760,[1,0,10,1,36,0]%%%}+%%%{4194304*i,[1,0,10,0,37,20]%%%}+%%
%{-41943040*i,[1,0,10,0,37,18]%%%}+%%%{188743680*i,[1,0,10,0,37,16]%%%}+%%%
{-503316480*i,[1,0,10,0,37,14]%%%}+%%%{880803840*i,[1,0,10,0,37,12]%%%}+%%%
{-1056964608*i,[1,0,10,0,37,10]%%%}+%%%{880803840*i,[1,0,10,0,37,8]%%%}+%%%
{-503316480*i,[1,0,10,0,37,6]%%%}+%%%{188743680*i,[1,0,10,0,37,4]%%%}+%%%{-
41943040*i,[1,0,10,0,37,2]%%%}+%%%{4194304*i,[1,0,10,0,37,0]%%%}+%%%{-16777
216,[0,0,10,19,18,18]%%%}+%%%{150994944,[0,0,10,19,18,16]%%%}+%%%{-60397977
6,[0,0,10,19,18,14]%%%}+%%%{1409286144,[0,0,10,19,18,12]%%%}+%%%{-211392921
6,[0,0,10,19,18,10]%%%}+%%%{2113929216,[0,0,10,19,18,8]%%%}+%%%{-1409286144
,[0,0,10,19,18,6]%%%}+%%%{603979776,[0,0,10,19,18,4]%%%}+%%%{-150994944,[0,
0,10,19,18,2]%%%}+%%%{16777216,[0,0,10,19,18,0]%%%}+%%%{-50331648*i,[0,0,10
,18,19,18]%%%}+%%%{452984832*i,[0,0,10,18,19,16]%%%}+%%%{-1811939328*i,[0,0
,10,18,19,14]%%%}+%%%{4227858432*i,[0,0,10,18,19,12]%%%}+%%%{-6341787648*i,
[0,0,10,18,19,10]%%%}+%%%{6341787648*i,[0,0,10,18,19,8]%%%}+%%%{-4227858432
*i,[0,0,10,18,19,6]%%%}+%%%{1811939328*i,[0,0,10,18,19,4]%%%}+%%%{-45298483
2*i,[0,0,10,18,19,2]%%%}+%%%{50331648*i,[0,0,10,18,19,0]%%%}+%%%{1048576,[0

,0,10,17,20,20]%%%}+%%%{-16777216,[0,0,10,17,20,19]%%%}+%%%{-77594624,[0,0,
10,17,20,18]%%%}+%%%{150994944,[0,0,10,17,20,17]%%%}+%%%{651165696,[0,0,10,
17,20,16]%%%}+%%%{-603979776,[0,0,10,17,20,15]%%%}+%%%{-2541748224,[0,0,10,
17,20,14]%%%}+%%%{1409286144,[0,0,10,17,20,13]%%%}+%%%{5857345536,[0,0,10,1
7,20,12]%%%}+%%%{-2113929216,[0,0,10,17,20,11]%%%}+%%%{-8719958016,[0,0,10,
17,20,10]%%%}+%%%{2113929216,[0,0,10,17,20,9]%%%}+%%%{8675917824,[0,0,10,17
,20,8]%%%}+%%%{-1409286144,[0,0,10,17,20,7]%%%}+%%%{-5762973696,[0,0,10,17,
20,6]%%%}+%%%{603979776,[0,0,10,17,20,5]%%%}+%%%{2463105024,[0,0,10,17,20,4
]%%%}+%%%{-150994944,[0,0,10,17,20,3]%%%}+%%%{-614465536,[0,0,10,17,20,2]%%
%}+%%%{16777216,[0,0,10,17,20,1]%%%}+%%%{68157440,[0,0,10,17,20,0]%%%}+%%%{
1048576*i,[0,0,10,16,21,20]%%%}+%%%{-50331648*i,[0,0,10,16,21,19]%%%}+%%%{-
346030080*i,[0,0,10,16,21,18]%%%}+%%%{452984832*i,[0,0,10,16,21,17]%%%}+%%%
{3067084800*i,[0,0,10,16,21,16]%%%}+%%%{-1811939328*i,[0,0,10,16,21,15]%%%}
+%%%{-12205424640*i,[0,0,10,16,21,14]%%%}+%%%{4227858432*i,[0,0,10,16,21,13
]%%%}+%%%{28405923840*i,[0,0,10,16,21,12]%%%}+%%%{-6341787648*i,[0,0,10,16,
21,11]%%%}+%%%{-42542825472*i,[0,0,10,16,21,10]%%%}+%%%{6341787648*i,[0,0,1
0,16,21,9]%%%}+%%%{42498785280*i,[0,0,10,16,21,8]%%%}+%%%{-4227858432*i,[0,
0,10,16,21,7]%%%}+%%%{-28311552000*i,[0,0,10,16,21,6]%%%}+%%%{1811939328*i,
[0,0,10,16,21,5]%%%}+%%%{12126781440*i,[0,0,10,16,21,4]%%%}+%%%{-452984832*
i,[0,0,10,16,21,3]%%%}+%%%{-3030384640*i,[0,0,10,16,21,2]%%%}+%%%{50331648*
i,[0,0,10,16,21,1]%%%}+%%%{336592896*i,[0,0,10,16,21,0]%%%}+%%%{8388608,[0,
0,10,15,22,20]%%%}+%%%{-58720256,[0,0,10,15,22,19]%%%}+%%%{-92274688,[0,0,1
0,15,22,18]%%%}+%%%{528482304,[0,0,10,15,22,17]%%%}+%%%{452984832,[0,0,10,1
5,22,16]%%%}+%%%{-2113929216,[0,0,10,15,22,15]%%%}+%%%{-1308622848,[0,0,10,
15,22,14]%%%}+%%%{4932501504,[0,0,10,15,22,13]%%%}+%%%{2466250752,[0,0,10,1
5,22,12]%%%}+%%%{-7398752256,[0,0,10,15,22,11]%%%}+%%%{-3170893824,[0,0,10,
15,22,10]%%%}+%%%{7398752256,[0,0,10,15,22,9]%%%}+%%%{2818572288,[0,0,10,15
,22,8]%%%}+%%%{-4932501504,[0,0,10,15,22,7]%%%}+%%%{-1711276032,[0,0,10,15,
22,6]%%%}+%%%{2113929216,[0,0,10,15,22,5]%%%}+%%%{679477248,[0,0,10,15,22,4
]%%%}+%%%{-528482304,[0,0,10,15,22,3]%%%}+%%%{-159383552,[0,0,10,15,22,2]%%
%}+%%%{58720256,[0,0,10,15,22,1]%%%}+%%%{16777216,[0,0,10,15,22,0]%%%}+%%%{
8388608*i,[0,0,10,14,23,20]%%%}+%%%{-310378496*i,[0,0,10,14,23,19]%%%}+%%%{
-1048576000*i,[0,0,10,14,23,18]%%%}+%%%{2793406464*i,[0,0,10,14,23,17]%%%}+
%%%{9059696640*i,[0,0,10,14,23,16]%%%}+%%%{-11173625856*i,[0,0,10,14,23,15]
%%%}+%%%{-35735470080*i,[0,0,10,14,23,14]%%%}+%%%{26071793664*i,[0,0,10,14,
23,13]%%%}+%%%{82795560960*i,[0,0,10,14,23,12]%%%}+%%%{-39107690496*i,[0,0,
10,14,23,11]%%%}+%%%{-123664859136*i,[0,0,10,14,23,10]%%%}+%%%{39107690496*
i,[0,0,10,14,23,9]%%%}+%%%{123312537600*i,[0,0,10,14,23,8]%%%}+%%%{-2607179
3664*i,[0,0,10,14,23,7]%%%}+%%%{-82040586240*i,[0,0,10,14,23,6]%%%}+%%%{111
73625856*i,[0,0,10,14,23,5]%%%}+%%%{35106324480*i,[0,0,10,14,23,4]%%%}+%%%{
-2793406464*i,[0,0,10,14,23,3]%%%}+%%%{-8766095360*i,[0,0,10,14,23,2]%%%}+%
%%{310378496*i,[0,0,10,14,23,1]%%%}+%%%{973078528*i,[0,0,10,14,23,0]%%%}+%%
%{29360128,[0,0,10,13,24,20]%%%}+%%%{25165824,[0,0,10,13,24,19]%%%}+%%%{150
994944,[0,0,10,13,24,18]%%%}+%%%{-226492416,[0,0,10,13,24,17]%%%}+%%%{-2680
160256,[0,0,10,13,24,16]%%%}+%%%{905969664,[0,0,10,13,24,15]%%%}+%%%{124822
48704,[0,0,10,13,24,14]%%%}+%%%{-2113929216,[0,0,10,13,24,13]%%%}+%%%{-3118
0455936,[0,0,10,13,24,12]%%%}+%%%{3170893824,[0,0,10,13,24,11]%%%}+%%%{4862
0371968,[0,0,10,13,24,10]%%%}+%%%{-3170893824,[0,0,10,13,24,9]%%%}+%%%{-498
53497344,[0,0,10,13,24,8]%%%}+%%%{2113929216,[0,0,10,13,24,7]%%%}+%%%{33822
867456,[0,0,10,13,24,6]%%%}+%%%{-905969664,[0,0,10,13,24,5]%%%}+%%%{-146842
58304,[0,0,10,13,24,4]%%%}+%%%{226492416,[0,0,10,13,24,3]%%%}+%%%{370776473
6,[0,0,10,13,24,2]%%%}+%%%{-25165824,[0,0,10,13,24,1]%%%}+%%%{-415236096,[0
,0,10,13,24,0]%%%}+%%%{29360128*i,[0,0,10,12,25,20]%%%}+%%%{-796917760*i,[0
,0,10,12,25,19]%%%}+%%%{-1845493760*i,[0,0,10,12,25,18]%%%}+%%%{7172259840*
i,[0,0,10,12,25,17]%%%}+%%%{15288238080*i,[0,0,10,12,25,16]%%%}+%%%{-286890
39360*i,[0,0,10,12,25,15]%%%}+%%%{-59391344640*i,[0,0,10,12,25,14]%%%}+%%%{
66941091840*i,[0,0,10,12,25,13]%%%}+%%%{136524595200*i,[0,0,10,12,25,12]%%%
}+%%%{-100411637760*i,[0,0,10,12,25,11]%%%}+%%%{-202937204736*i,[0,0,10,12,
25,10]%%%}+%%%{100411637760*i,[0,0,10,12,25,9]%%%}+%%%{201704079360*i,[0,0,

10,12,25,8]%%%}+%%%{-66941091840*i,[0,0,10,12,25,7]%%%}+%%%{-133882183680*i
,[0,0,10,12,25,6]%%%}+%%%{28689039360*i,[0,0,10,12,25,5]%%%}+%%%{5718933504
0*i,[0,0,10,12,25,4]%%%}+%%%{-7172259840*i,[0,0,10,12,25,3]%%%}+%%%{-142606
33600*i,[0,0,10,12,25,2]%%%}+%%%{796917760*i,[0,0,10,12,25,1]%%%}+%%%{15812
52608*i,[0,0,10,12,25,0]%%%}+%%%{58720256,[0,0,10,11,26,20]%%%}+%%%{4445962
24,[0,0,10,11,26,19]%%%}+%%%{612368384,[0,0,10,11,26,18]%%%}+%%%{-400136601
6,[0,0,10,11,26,17]%%%}+%%%{-8153726976,[0,0,10,11,26,16]%%%}+%%%{160054640
64,[0,0,10,11,26,15]%%%}+%%%{36138123264,[0,0,10,11,26,14]%%%}+%%%{-3734608
2816,[0,0,10,11,26,13]%%%}+%%%{-88432705536,[0,0,10,11,26,12]%%%}+%%%{56019
124224,[0,0,10,11,26,11]%%%}+%%%{136348434432,[0,0,10,11,26,10]%%%}+%%%{-56
019124224,[0,0,10,11,26,9]%%%}+%%%{-138814685184,[0,0,10,11,26,8]%%%}+%%%{3
7346082816,[0,0,10,11,26,7]%%%}+%%%{93717528576,[0,0,10,11,26,6]%%%}+%%%{-1
6005464064,[0,0,10,11,26,5]%%%}+%%%{-40542142464,[0,0,10,11,26,4]%%%}+%%%{4
001366016,[0,0,10,11,26,3]%%%}+%%%{10208935936,[0,0,10,11,26,2]%%%}+%%%{-44
4596224,[0,0,10,11,26,1]%%%}+%%%{-1140850688,[0,0,10,11,26,0]%%%}+%%%{58720
256*i,[0,0,10,10,27,20]%%%}+%%%{-1082130432*i,[0,0,10,10,27,19]%%%}+%%%{-20
88763392*i,[0,0,10,10,27,18]%%%}+%%%{9739173888*i,[0,0,10,10,27,17]%%%}+%%%
{16156459008*i,[0,0,10,10,27,16]%%%}+%%%{-38956695552*i,[0,0,10,10,27,15]%%
%}+%%%{-61102620672*i,[0,0,10,10,27,14]%%%}+%%%{90898956288*i,[0,0,10,10,27
,13]%%%}+%%%{138462363648*i,[0,0,10,10,27,12]%%%}+%%%{-136348434432*i,[0,0,
10,10,27,11]%%%}+%%%{-203994169344*i,[0,0,10,10,27,10]%%%}+%%%{136348434432
*i,[0,0,10,10,27,9]%%%}+%%%{201527918592*i,[0,0,10,10,27,8]%%%}+%%%{-908989
56288*i,[0,0,10,10,27,7]%%%}+%%%{-133177540608*i,[0,0,10,10,27,6]%%%}+%%%{3
8956695552*i,[0,0,10,10,27,5]%%%}+%%%{56698601472*i,[0,0,10,10,27,4]%%%}+%%
%{-9739173888*i,[0,0,10,10,27,3]%%%}+%%%{-14101250048*i,[0,0,10,10,27,2]%%%
}+%%%{1082130432*i,[0,0,10,10,27,1]%%%}+%%%{1560281088*i,[0,0,10,10,27,0]%%
%}+%%%{73400320,[0,0,10,9,28,20]%%%}+%%%{964689920,[0,0,10,9,28,19]%%%}+%%%
{884998144,[0,0,10,9,28,18]%%%}+%%%{-8682209280,[0,0,10,9,28,17]%%%}+%%%{-1
1267997696,[0,0,10,9,28,16]%%%}+%%%{34728837120,[0,0,10,9,28,15]%%%}+%%%{49
476009984,[0,0,10,9,28,14]%%%}+%%%{-81033953280,[0,0,10,9,28,13]%%%}+%%%{-1
20582045696,[0,0,10,9,28,12]%%%}+%%%{121550929920,[0,0,10,9,28,11]%%%}+%%%{
185497288704,[0,0,10,9,28,10]%%%}+%%%{-121550929920,[0,0,10,9,28,9]%%%}+%%%
{-188580102144,[0,0,10,9,28,8]%%%}+%%%{81033953280,[0,0,10,9,28,7]%%%}+%%%{
127188074496,[0,0,10,9,28,6]%%%}+%%%{-34728837120,[0,0,10,9,28,5]%%%}+%%%{-
54981033984,[0,0,10,9,28,4]%%%}+%%%{8682209280,[0,0,10,9,28,3]%%%}+%%%{1383
7008896,[0,0,10,9,28,2]%%%}+%%%{-964689920,[0,0,10,9,28,1]%%%}+%%%{-1545601
024,[0,0,10,9,28,0]%%%}+%%%{73400320*i,[0,0,10,8,29,20]%%%}+%%%{-796917760*
i,[0,0,10,8,29,19]%%%}+%%%{-1581252608*i,[0,0,10,8,29,18]%%%}+%%%{717225984
0*i,[0,0,10,8,29,17]%%%}+%%%{10928259072*i,[0,0,10,8,29,16]%%%}+%%%{-286890
39360*i,[0,0,10,8,29,15]%%%}+%%%{-39309017088*i,[0,0,10,8,29,14]%%%}+%%%{66
941091840*i,[0,0,10,8,29,13]%%%}+%%%{86583017472*i,[0,0,10,8,29,12]%%%}+%%%
{-100411637760*i,[0,0,10,8,29,11]%%%}+%%%{-125250306048*i,[0,0,10,8,29,10]%
%%}+%%%{100411637760*i,[0,0,10,8,29,9]%%%}+%%%{122167492608*i,[0,0,10,8,29,
8]%%%}+%%%{-66941091840*i,[0,0,10,8,29,7]%%%}+%%%{-79976988672*i,[0,0,10,8,
29,6]%%%}+%%%{28689039360*i,[0,0,10,8,29,5]%%%}+%%%{33803993088*i,[0,0,10,8
,29,4]%%%}+%%%{-7172259840*i,[0,0,10,8,29,3]%%%}+%%%{-8359247872*i,[0,0,10,
8,29,2]%%%}+%%%{796917760*i,[0,0,10,8,29,1]%%%}+%%%{920649728*i,[0,0,10,8,2
9,0]%%%}+%%%{58720256,[0,0,10,7,30,20]%%%}+%%%{1031798784,[0,0,10,7,30,19]%
%%}+%%%{729808896,[0,0,10,7,30,18]%%%}+%%%{-9286189056,[0,0,10,7,30,17]%%%}
+%%%{-9210691584,[0,0,10,7,30,16]%%%}+%%%{37144756224,[0,0,10,7,30,15]%%%}+
%%%{40365981696,[0,0,10,7,30,14]%%%}+%%%{-86671097856,[0,0,10,7,30,13]%%%}+
%%%{-98297708544,[0,0,10,7,30,12]%%%}+%%%{130006646784,[0,0,10,7,30,11]%%%}
+%%%{151145938944,[0,0,10,7,30,10]%%%}+%%%{-130006646784,[0,0,10,7,30,9]%%%
}+%%%{-153612189696,[0,0,10,7,30,8]%%%}+%%%{86671097856,[0,0,10,7,30,7]%%%}
+%%%{103582531584,[0,0,10,7,30,6]%%%}+%%%{-37144756224,[0,0,10,7,30,5]%%%}+
%%%{-44770000896,[0,0,10,7,30,4]%%%}+%%%{9286189056,[0,0,10,7,30,3]%%%}+%%%
{11265900544,[0,0,10,7,30,2]%%%}+%%%{-1031798784,[0,0,10,7,30,1]%%%}+%%%{-1
258291200,[0,0,10,7,30,0]%%%}+%%%{58720256*i,[0,0,10,6,31,20]%%%}+%%%{-2600
46848*i,[0,0,10,6,31,19]%%%}+%%%{-796917760*i,[0,0,10,6,31,18]%%%}+%%%{2340

421632*i,[0,0,10,6,31,17]%%%}+%%%{4529848320*i,[0,0,10,6,31,16]%%%}+%%%{-93
61686528*i,[0,0,10,6,31,15]%%%}+%%%{-14596177920*i,[0,0,10,6,31,14]%%%}+%%%
{21843935232*i,[0,0,10,6,31,13]%%%}+%%%{29947330560*i,[0,0,10,6,31,12]%%%}+
%%%{-32765902848*i,[0,0,10,6,31,11]%%%}+%%%{-41221619712*i,[0,0,10,6,31,10]
%%%}+%%%{32765902848*i,[0,0,10,6,31,9]%%%}+%%%{38755368960*i,[0,0,10,6,31,8
]%%%}+%%%{-21843935232*i,[0,0,10,6,31,7]%%%}+%%%{-24662507520*i,[0,0,10,6,3
1,6]%%%}+%%%{9361686528*i,[0,0,10,6,31,5]%%%}+%%%{10192158720*i,[0,0,10,6,3
1,4]%%%}+%%%{-2340421632*i,[0,0,10,6,31,3]%%%}+%%%{-2474639360*i,[0,0,10,6,
31,2]%%%}+%%%{260046848*i,[0,0,10,6,31,1]%%%}+%%%{268435456*i,[0,0,10,6,31,
0]%%%}+%%%{29360128,[0,0,10,5,32,20]%%%}+%%%{612368384,[0,0,10,5,32,19]%%%}
+%%%{369098752,[0,0,10,5,32,18]%%%}+%%%{-5511315456,[0,0,10,5,32,17]%%%}+%%
%{-4643094528,[0,0,10,5,32,16]%%%}+%%%{22045261824,[0,0,10,5,32,15]%%%}+%%%
{20333985792,[0,0,10,5,32,14]%%%}+%%%{-51438944256,[0,0,10,5,32,13]%%%}+%%%
{-49501175808,[0,0,10,5,32,12]%%%}+%%%{77158416384,[0,0,10,5,32,11]%%%}+%%%
{76101451776,[0,0,10,5,32,10]%%%}+%%%{-77158416384,[0,0,10,5,32,9]%%%}+%%%{
-77334577152,[0,0,10,5,32,8]%%%}+%%%{51438944256,[0,0,10,5,32,7]%%%}+%%%{52
143587328,[0,0,10,5,32,6]%%%}+%%%{-22045261824,[0,0,10,5,32,5]%%%}+%%%{-225
35995392,[0,0,10,5,32,4]%%%}+%%%{5511315456,[0,0,10,5,32,3]%%%}+%%%{5670699
008,[0,0,10,5,32,2]%%%}+%%%{-612368384,[0,0,10,5,32,1]%%%}+%%%{-633339904,[
0,0,10,5,32,0]%%%}+%%%{29360128*i,[0,0,10,4,33,20]%%%}+%%%{25165824*i,[0,0,
10,4,33,19]%%%}+%%%{-251658240*i,[0,0,10,4,33,18]%%%}+%%%{-226492416*i,[0,0
,10,4,33,17]%%%}+%%%{943718400*i,[0,0,10,4,33,16]%%%}+%%%{905969664*i,[0,0,
10,4,33,15]%%%}+%%%{-2013265920*i,[0,0,10,4,33,14]%%%}+%%%{-2113929216*i,[0
,0,10,4,33,13]%%%}+%%%{2642411520*i,[0,0,10,4,33,12]%%%}+%%%{3170893824*i,[
0,0,10,4,33,11]%%%}+%%%{-2113929216*i,[0,0,10,4,33,10]%%%}+%%%{-3170893824*
i,[0,0,10,4,33,9]%%%}+%%%{880803840*i,[0,0,10,4,33,8]%%%}+%%%{2113929216*i,
[0,0,10,4,33,7]%%%}+%%%{-905969664*i,[0,0,10,4,33,5]%%%}+%%%{-188743680*i,[
0,0,10,4,33,4]%%%}+%%%{226492416*i,[0,0,10,4,33,3]%%%}+%%%{83886080*i,[0,0,
10,4,33,2]%%%}+%%%{-25165824*i,[0,0,10,4,33,1]%%%}+%%%{-12582912*i,[0,0,10,
4,33,0]%%%}+%%%{8388608,[0,0,10,3,34,20]%%%}+%%%{192937984,[0,0,10,3,34,19]
%%%}+%%%{109051904,[0,0,10,3,34,18]%%%}+%%%{-1736441856,[0,0,10,3,34,17]%%%
}+%%%{-1358954496,[0,0,10,3,34,16]%%%}+%%%{6945767424,[0,0,10,3,34,15]%%%}+
%%%{5939134464,[0,0,10,3,34,14]%%%}+%%%{-16206790656,[0,0,10,3,34,13]%%%}+%
%%{-14445182976,[0,0,10,3,34,12]%%%}+%%%{24310185984,[0,0,10,3,34,11]%%%}+%
%%{22196256768,[0,0,10,3,34,10]%%%}+%%%{-24310185984,[0,0,10,3,34,9]%%%}+%%
%{-22548578304,[0,0,10,3,34,8]%%%}+%%%{16206790656,[0,0,10,3,34,7]%%%}+%%%{
15200157696,[0,0,10,3,34,6]%%%}+%%%{-6945767424,[0,0,10,3,34,5]%%%}+%%%{-65
68280064,[0,0,10,3,34,4]%%%}+%%%{1736441856,[0,0,10,3,34,3]%%%}+%%%{1652555
776,[0,0,10,3,34,2]%%%}+%%%{-192937984,[0,0,10,3,34,1]%%%}+%%%{-184549376,[
0,0,10,3,34,0]%%%}+%%%{8388608*i,[0,0,10,2,35,20]%%%}+%%%{41943040*i,[0,0,1
0,2,35,19]%%%}+%%%{-41943040*i,[0,0,10,2,35,18]%%%}+%%%{-377487360*i,[0,0,1
0,2,35,17]%%%}+%%%{1509949440*i,[0,0,10,2,35,15]%%%}+%%%{503316480*i,[0,0,1
0,2,35,14]%%%}+%%%{-3523215360*i,[0,0,10,2,35,13]%%%}+%%%{-1761607680*i,[0,
0,10,2,35,12]%%%}+%%%{5284823040*i,[0,0,10,2,35,11]%%%}+%%%{3170893824*i,[0
,0,10,2,35,10]%%%}+%%%{-5284823040*i,[0,0,10,2,35,9]%%%}+%%%{-3523215360*i,
[0,0,10,2,35,8]%%%}+%%%{3523215360*i,[0,0,10,2,35,7]%%%}+%%%{2516582400*i,[
0,0,10,2,35,6]%%%}+%%%{-1509949440*i,[0,0,10,2,35,5]%%%}+%%%{-1132462080*i,
[0,0,10,2,35,4]%%%}+%%%{377487360*i,[0,0,10,2,35,3]%%%}+%%%{293601280*i,[0,
0,10,2,35,2]%%%}+%%%{-41943040*i,[0,0,10,2,35,1]%%%}+%%%{-33554432*i,[0,0,1
0,2,35,0]%%%}+%%%{1048576,[0,0,10,1,36,20]%%%}+%%%{25165824,[0,0,10,1,36,19
]%%%}+%%%{14680064,[0,0,10,1,36,18]%%%}+%%%{-226492416,[0,0,10,1,36,17]%%%}
+%%%{-179306496,[0,0,10,1,36,16]%%%}+%%%{905969664,[0,0,10,1,36,15]%%%}+%%%
{780140544,[0,0,10,1,36,14]%%%}+%%%{-2113929216,[0,0,10,1,36,13]%%%}+%%%{-1
893728256,[0,0,10,1,36,12]%%%}+%%%{3170893824,[0,0,10,1,36,11]%%%}+%%%{2906
652672,[0,0,10,1,36,10]%%%}+%%%{-3170893824,[0,0,10,1,36,9]%%%}+%%%{-295069
2864,[0,0,10,1,36,8]%%%}+%%%{2113929216,[0,0,10,1,36,7]%%%}+%%%{1988100096,
[0,0,10,1,36,6]%%%}+%%%{-905969664,[0,0,10,1,36,5]%%%}+%%%{-858783744,[0,0,
10,1,36,4]%%%}+%%%{226492416,[0,0,10,1,36,3]%%%}+%%%{216006656,[0,0,10,1,36
,2]%%%}+%%%{-25165824,[0,0,10,1,36,1]%%%}+%%%{-24117248,[0,0,10,1,36,0]%%%}

```
+%%%{1048576*i,[0,0,10,0,37,20]%%%}+%%%{8388608*i,[0,0,10,0,37,19]%%%}+%%%{
-2097152*i,[0,0,10,0,37,18]%%%}+%%%{-75497472*i,[0,0,10,0,37,17]%%%}+%%%{-2
8311552*i,[0,0,10,0,37,16]%%%}+%%%{301989888*i,[0,0,10,0,37,15]%%%}+%%%{176
160768*i,[0,0,10,0,37,14]%%%}+%%%{-704643072*i,[0,0,10,0,37,13]%%%}+%%%{-48
4442112*i,[0,0,10,0,37,12]%%%}+%%%{1056964608*i,[0,0,10,0,37,11]%%%}+%%%{79
2723456*i,[0,0,10,0,37,10]%%%}+%%%{-1056964608*i,[0,0,10,0,37,9]%%%}+%%%{-8
36763648*i,[0,0,10,0,37,8]%%%}+%%%{704643072*i,[0,0,10,0,37,7]%%%}+%%%{5788
13952*i,[0,0,10,0,37,6]%%%}+%%%{-301989888*i,[0,0,10,0,37,5]%%%}+%%%{-25480
3968*i,[0,0,10,0,37,4]%%%}+%%%{75497472*i,[0,0,10,0,37,3]%%%}+%%%{65011712*
i,[0,0,10,0,37,2]%%%}+%%%{-8388608*i,[0,0,10,0,37,1]%%%}+%%%{-7340032*i,[0,
0,10,0,37,0]%%%} / %%%{1024,[0,0,5,9,10,10]%%%}+%%%{-5120,[0,0,5,9,10,8]%%%
}+%%%{10240,[0,0,5,9,10,6]%%%}+%%%{-10240,[0,0,5,9,10,4]%%%}+%%%{5120,[0,0,
5,9,10,2]%%%}+%%%{-1024,[0,0,5,9,10,0]%%%}+%%%{1024*i,[0,0,5,8,11,10]%%%}+%
%%{-5120*i,[0,0,5,8,11,8]%%%}+%%%{10240*i,[0,0,5,8,11,6]%%%}+%%%{-10240*i,[
0,0,5,8,11,4]%%%}+%%%{5120*i,[0,0,5,8,11,2]%%%}+%%%{-1024*i,[0,0,5,8,11,0]%
%%}+%%%{4096,[0,0,5,7,12,10]%%%}+%%%{-20480,[0,0,5,7,12,8]%%%}+%%%{40960,[0
,0,5,7,12,6]%%%}+%%%{-40960,[0,0,5,7,12,4]%%%}+%%%{20480,[0,0,5,7,12,2]%%%}
+%%%{-4096,[0,0,5,7,12,0]%%%}+%%%{4096*i,[0,0,5,6,13,10]%%%}+%%%{-20480*i,[
0,0,5,6,13,8]%%%}+%%%{40960*i,[0,0,5,6,13,6]%%%}+%%%{-40960*i,[0,0,5,6,13,4
]%%%}+%%%{20480*i,[0,0,5,6,13,2]%%%}+%%%{-4096*i,[0,0,5,6,13,0]%%%}+%%%{614
4,[0,0,5,5,14,10]%%%}+%%%{-30720,[0,0,5,5,14,8]%%%}+%%%{61440,[0,0,5,5,14,6
]%%%}+%%%{-61440,[0,0,5,5,14,4]%%%}+%%%{30720,[0,0,5,5,14,2]%%%}+%%%{-6144,
[0,0,5,5,14,0]%%%}+%%%{6144*i,[0,0,5,4,15,10]%%%}+%%%{-30720*i,[0,0,5,4,15,
8]%%%}+%%%{61440*i,[0,0,5,4,15,6]%%%}+%%%{-61440*i,[0,0,5,4,15,4]%%%}+%%%{3
0720*i,[0,0,5,4,15,2]%%%}+%%%{-6144*i,[0,0,5,4,15,0]%%%}+%%%{4096,[0,0,5,3,
16,10]%%%}+%%%{-20480,[0,0,5,3,16,8]%%%}+%%%{40960,[0,0,5,3,16,6]%%%}+%%%{-
40960,[0,0,5,3,16,4]%%%}+%%%{20480,[0,0,5,3,16,2]%%%}+%%%{-4096,[0,0,5,3,16
,0]%%%}+%%%{4096*i,[0,0,5,2,17,10]%%%}+%%%{-20480*i,[0,0,5,2,17,8]%%%}+%%%{
40960*i,[0,0,5,2,17,6]%%%}+%%%{-40960*i,[0,0,5,2,17,4]%%%}+%%%{20480*i,[0,0
,5,2,17,2]%%%}+%%%{-4096*i,[0,0,5,2,17,0]%%%}+%%%{1024,[0,0,5,1,18,10]%%%}+
%%%{-5120,[0,0,5,1,18,8]%%%}+%%%{10240,[0,0,5,1,18,6]%%%}+%%%{-10240,[0,0,5
,1,18,4]%%%}+%%%{5120,[0,0,5,1,18,2]%%%}+%%%{-1024,[0,0,5,1,18,0]%%%}+%%%{1
024*i,[0,0,5,0,19,10]%%%}+%%%{-5120*i,[0,0,5,0,19,8]%%%}+%%%{10240*i,[0,0,5
,0,19,6]%%%}+%%%{-10240*i,[0,0,5,0,19,4]%%%}+%%%{5120*i,[0,0,5,0,19,2]%%%}+
%%%{-1024*i,[0,0,5,0,19,0]%%%} Error: Bad Argument Value
```

maple [F] time = 3.52, size = 0, normalized size = 0.00

$$\int \left(\sin^4(dx+c)\right)(a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
[Out] int(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(dx+c)+a)^n \sin(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin(c+d\,x)^4\,(a+b\tan(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + b*tan(c + d*x))^n,x)
[Out] int(sin(c + d*x)^4*(a + b*tan(c + d*x))^n, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4*(a+b*tan(d*x+c))**n,x)
[Out] Timed out
```

3.85 $\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=276

$$-\frac{\left(\sqrt{-b^2}\left(a^2+b^2(n+1)\right)+ab^2n\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{4bd(n+1)\left(a^2+b^2\right)\left(a-\sqrt{-b^2}\right)}-\frac{\left(ab^2n-\sqrt{-b^2}\left(a^2+b\right.\right.}{}$$

```
[Out] -1/4*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+(-b^2)^(1/2)))*(a*b^2*n-(
a^2+b^2*(1+n))*(-b^2)^(1/2))*(a+b*tan(d*x+c))^(1+n)/b/(a^2+b^2)/d/(1+n)/(a+
(-b^2)^(1/2))-1/4*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)
))*(a*b^2*n+(a^2+b^2*(1+n))*(-b^2)^(1/2))*(a+b*tan(d*x+c))^(1+n)/b/(a^2+b^2
)/d/(1+n)/(a-(-b^2)^(1/2))-1/2*cos(d*x+c)^2*(b+a*tan(d*x+c))*(a+b*tan(d*x+c
))^(1+n)/(a^2+b^2)/d
```

Rubi [A] time = 0.37, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3516, 1649, 831, 68}

$$-\frac{\left(\sqrt{-b^2}\left(a^2+b^2(n+1)\right)+ab^2n\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{4bd(n+1)\left(a^2+b^2\right)\left(a-\sqrt{-b^2}\right)}-\frac{\left(ab^2n-\sqrt{-b^2}\left(a^2+b\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] -((a*b^2*n + Sqrt[-b^2]*(a^2 + b^2*(1 + n)))*Hypergeometric2F1[1, 1 + n, 2
+ n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(
4*b*(a^2 + b^2)*(a - Sqrt[-b^2])*d*(1 + n)) - ((a*b^2*n - Sqrt[-b^2]*(a^2 +
 b^2*(1 + n)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a +
 Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(4*b*(a^2 + b^2)*(a + Sqrt[-b^2
])*d*(1 + n)) - (Cos[c + d*x]^2*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])^(
1 + n))/(2*(a^2 + b^2)*d)
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^(p + 1)*(a*(e*f - d*g) +
(c*d*f + a*e*g)*x))/(2*a*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*a*(p + 1)
*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*(p +
 1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p + 3) - a*e*(d*g*m - e*f*(m + 2*p + 3))
 + e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, c, d, e, m},
 x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[m, 0]
```

```
 && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^2(a+x)^n}{\left(b^2+x^2\right)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2\left(a^2+b^2\right)d} - \frac{\text{Subst}\left(\int \frac{(a}{}\right.}{}$$

$$= -\frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2\left(a^2+b^2\right)d} - \frac{\text{Subst}\left(\int\left(\right.\right.}{}$$

$$= -\frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2\left(a^2+b^2\right)d} - \frac{\left(ab^2n-\sqrt{-}\right.}{}$$

$$= -\frac{\left(ab^2n+\sqrt{-b^2}\left(a^2+b^2(1+n)\right)\right)\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a}{4b\left(a^2+b^2\right)\left(a-\sqrt{-b^2}\right)d(1+n)}$$

Mathematica [A] time = 1.15, size = 270, normalized size = 0.98

$$\frac{(a+b\tan(c+dx))^{n+1}\left(2b(n+1)\left(a^2+b^2\right)\cos(c+dx)(a\sin(c+dx)+b\cos(c+dx))+\left(a^3\sqrt{-b^2}+a^2b^2(n-1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (((a^3*Sqrt[-b^2] + a^2*b^2*(-1 + n) - b^4*(1 + n) - a*(-b^2)^(3/2)*(1 + 2*
n))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]
)] - (a^3*Sqrt[-b^2] - a^2*b^2*(-1 + n) + b^4*(1 + n) - a*(-b^2)^(3/2)*(1 +
 2*n))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b
^2])] + 2*b*(a^2 + b^2)*(1 + n)*Cos[c + d*x]*(b*Cos[c + d*x] + a*Sin[c + d*
x]))*(a + b*Tan[c + d*x])^(1 + n))/(4*b*(a^2 + b^2)*(-a + Sqrt[-b^2])*(a +
Sqrt[-b^2])*d*(1 + n))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(dx+c\right)^2-1\right)\left(b\tan\left(dx+c\right)+a\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*tan(d*x + c) + a)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(dx+c)+a)^n \sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

maple [F] time = 2.00, size = 0, normalized size = 0.00

$$\int \left(\sin^2(dx+c)\right)(a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
[Out] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(dx+c)+a)^n \sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin(c+d\,x)^2\,(a+b\tan(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b*tan(c + d*x))^n,x)
[Out] int(sin(c + d*x)^2*(a + b*tan(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^n \sin^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*sin(c + d*x)**2, x)
```

3.86 $\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=48

$$\frac{b(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{b\tan(c+dx)}{a}+1\right)}{a^2d(n+1)}$$

```
[Out] b*hypergeom([2, 1+n],[2+n],1+b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^(1+n)/a^2/d/(
1+n)
```

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3516, 65}

$$\frac{b(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{b\tan(c+dx)}{a}+1\right)}{a^2d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Tan[c
+ d*x])^(1 + n))/(a^2*d*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx &= \frac{b\,\text{Subst}\left(\int \frac{(a+x)^n}{x^2}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{b\,{}_2F_1\left(2,1+n;2+n;1+\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^{1+n}}{a^2d(1+n)}\end{aligned}$$

Mathematica [A] time = 0.94, size = 48, normalized size = 1.00

$$\frac{b(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{b\tan(c+dx)}{a}+1\right)}{a^2d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Tan[c
+ d*x])^(1 + n))/(a^2*d*(1 + n))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left((b\tan(dx+c)+a)^n\csc(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(dx+c)+a)^n\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

maple [F] time = 0.87, size = 0, normalized size = 0.00

$$\int \left(\csc^2(dx+c)\right)(a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
[Out] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(dx+c)+a)^n\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\tan(c+d\,x))^n}{\sin(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^n/sin(c + d*x)^2,x)
[Out] int((a + b*tan(c + d*x))^n/sin(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^n\csc^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*csc(c + d*x)**2, x)
```

3.87 $\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=140

$$\frac{b(2-n)\cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{6a^2d}+\frac{b\left(6a^2+b^2\left(n^2-3n+2\right)\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n\right.}{6a^4d(n+1)}$$

```
[Out] 1/6*b*(2-n)*cot(d*x+c)^2*(a+b*tan(d*x+c))^(1+n)/a^2/d-1/3*cot(d*x+c)^3*(a+b
*tan(d*x+c))^(1+n)/a/d+1/6*b*(6*a^2+b^2*(n^2-3*n+2))*hypergeom([2, 1+n],[2+
n],1+b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^(1+n)/a^4/d/(1+n)
```

Rubi [A] time = 0.13, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3516, 950, 78, 65}

$$\frac{b\left(6a^2+b^2\left(n^2-3n+2\right)\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{b\tan(c+dx)}{a}+1\right)}{6a^4d(n+1)}+\frac{b(2-n)\cot^2(c+dx)(a+}{6a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*(2 - n)*Cot[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n))/(6*a^2*d) - (Cot[c
+ d*x]^3*(a + b*Tan[c + d*x])^(1 + n))/(3*a*d) + (b*(6*a^2 + b^2*(2 - 3*n +
 n^2))*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Ta
n[c + d*x])^(1 + n))/(6*a^4*d*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 950

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + c*x^2)^p, d + e*x
, x], R = PolynomialRemainder[(a + c*x^2)^p, d + e*x, x]}, Simp[(R*(d + e*x
)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)), x] + Dist[1/((m + 1)*(e
*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*ExpandToSum[(m + 1)*(e*f - d*
g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)^n(b^2+x^2)}{x^4}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cot^3(c+dx)(a+b\tan(c+dx))^{1+n}}{3ad} - \frac{b\,\text{Subst}\left(\int \frac{(a+x)^n(b^2(2-n)-3ax)}{x^3}\,dx, x\right.}{3ad}$$

$$= \frac{b(2-n)\cot^2(c+dx)(a+b\tan(c+dx))^{1+n}}{6a^2d} - \frac{\cot^3(c+dx)(a+b\tan(c+}{3ad}$$

$$= \frac{b(2-n)\cot^2(c+dx)(a+b\tan(c+dx))^{1+n}}{6a^2d} - \frac{\cot^3(c+dx)(a+b\tan(c+}{3ad}$$

Mathematica [A] time = 1.31, size = 78, normalized size = 0.56

$$\frac{b(a+b\tan(c+dx))^{n+1}\left(a^2\,{}_2F_1\left(2,n+1;n+2;\frac{b\tan(c+dx)}{a}+1\right)+b^2\,{}_2F_1\left(4,n+1;n+2;\frac{b\tan(c+dx)}{a}+1\right)\right)}{a^4d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*(a^2*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a] + b^2*Hy
pergeometric2F1[4, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a])*(a + b*Tan[c + d*
x])^(1 + n))/(a^4*d*(1 + n))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left((b\tan(dx+c)+a)^n\csc(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(dx+c)+a)^n\csc(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \left(\csc^4(dx+c)\right)(a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \tan (dx + c) + a)^n \csc (dx + c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \tan (c + d\,x))^n}{\sin (c + d\,x)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^n/sin(c + d*x)^4,x)
```

```
[Out] int((a + b*tan(c + d*x))^n/sin(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

3.88 $\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\sin^3(c+dx)(a+b\tan(c+dx))^n,x\right)$$

```
[Out] CannotIntegrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

Rubi [A] time = 1.98, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
[Out] Defer[Int][Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]
```

Rubi steps

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

Mathematica [A] time = 3.21, size = 0, normalized size = 0.00

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
[Out] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos(dx+c)^2-1\right)(b\tan(dx+c)+a)^n\sin(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*tan(d*x + c) + a)^n*sin(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(dx+c)+a)^n\sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

maple [A] time = 1.87, size = 0, normalized size = 0.00

$$\int \left(\sin^3(dx+c)\right)(a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(dx+c)+a)^n \sin(dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \sin(c+d\,x)^3 \,(a+b\tan(c+d\,x))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b*tan(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)^3*(a + b*tan(c + d*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

3.89 $\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=22

$$\text{Int}\left(\sin(c+dx)(a+b\tan(c+dx))^n,x\right)$$

```
[Out] CannotIntegrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Rubi [A] time = 0.86, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.000, Rules used = {}

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] Defer[Int][Sin[c + d*x]*(a + b*Tan[c + d*x])^n, x]
```

Rubi steps

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx = \int \sin(c+dx)(a+b\tan(c+dx))^n\,dx$$

Mathematica [A] time = 2.25, size = 0, normalized size = 0.00

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^n, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left((b\tan(dx+c)+a)^n\sin(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*sin(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(dx+c)+a)^n\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c), x)
```

maple [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \sin(dx+c)(a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(dx+c)+a)^n \sin(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \sin(c+d\,x)\ (a+b\tan(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*tan(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)*(a + b*tan(c + d*x))^n, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^n \sin(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*sin(c + d*x), x)
```

3.90 $\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=22

$$\text{Int}\left(\csc(c+dx)(a+b\tan(c+dx))^n,x\right)$$

```
[Out] CannotIntegrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Rubi [A] time = 0.49, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^n,x]
[Out] Defer[Int][Csc[c + d*x]*(a + b*Tan[c + d*x])^n, x]
```

Rubi steps

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx=\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx$$

Mathematica [A] time = 1.59, size = 0, normalized size = 0.00

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^n,x]
[Out] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^n, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left((b\tan\left(dx+c\right)+a)^n\csc\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n*csc(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan\left(dx+c\right)+a)^n\csc\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c), x)
```

maple [A] time = 0.87, size = 0, normalized size = 0.00

$$\int \csc\left(dx+c\right)\left(a+b\tan\left(dx+c\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(dx+c)+a)^n \csc(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(a+b\tan(c+d\,x))^n}{\sin(c+d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^n/sin(c + d*x),x)
```

```
[Out] int((a + b*tan(c + d*x))^n/sin(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^n \csc(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*csc(c + d*x), x)
```

3.91 $\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\csc^3(c+dx)(a+b\tan(c+dx))^n,x\right)$$

[Out] CannotIntegrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x)

Rubi [A] time = 1.70, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is Not applicable to the result.

[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]

[Out] Defer[Int][Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]

Rubi steps

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx=\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

Mathematica [A] time = 15.43, size = 0, normalized size = 0.00

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]

[Out] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left((b\tan(dx+c)+a)^n\csc(dx+c)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")

[Out] integral((b*tan(d*x + c) + a)^n*csc(d*x + c)^3, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(dx+c)+a)^n\csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")

[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^3, x)

maple [A] time = 0.75, size = 0, normalized size = 0.00

$$\int \left(\csc^3(dx+c)\right)(a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
[Out] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(dx+c)+a)^n \csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\tan(c+d\,x))^n}{\sin(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^n/sin(c + d*x)^3,x)
[Out] int((a + b*tan(c + d*x))^n/sin(c + d*x)^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^n \csc^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*csc(c + d*x)**3, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]



(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```